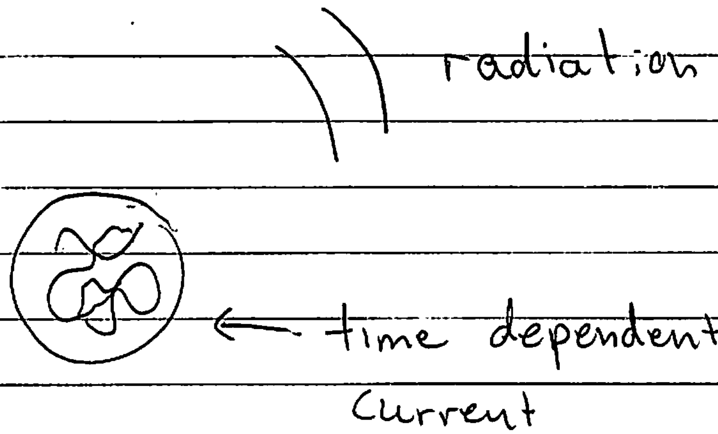
The Polarization of the radiation.



Setup

$$-\Box\varphi = \rho$$

$$-\Box\vec{A} = \vec{J}/c$$

Solve

$$\vec{A}_{rad} = \frac{1}{4\pi r}\int_{r_0} \vec{J}(T, r_0)/c$$

where

$$T(r_0) = \overbrace{t - \frac{r}{c}}^{\equiv t_e} + \overbrace{\frac{n\cdot r_0}{c}}^{\text{small for non-rel systems}}$$

For a point-like charge

$$\vec{J} = q\, v(T)\, \delta^3(\vec{r} - \vec{r}_*(T(r_0)))$$

we integrate over $\vec{r}_0$ and find

$$A_{rad} = \frac{1}{4\pi r} \frac{q \vec{V}(T(r_*))}{(1 - n \cdot \beta(T(r_*)))}$$ ← denominator comes from Jacobian

$$T(\vec{r}_*) \equiv \underbrace{t - \frac{r}{c}}_{\equiv t_e} + \frac{n}{c} \cdot r_*(T)$$

$$\delta^3(r_0 - r_*(T(r_0)))$$

$$= \frac{\delta^3(r_0 - r_*(T(r_*))}{(1 - n \cdot \frac{v}{c}(T))}$$

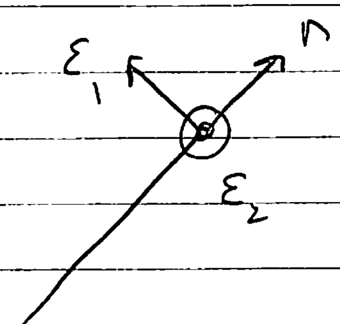
So for non-rel systemes:

$$A_{rad} = \frac{1}{4\pi r} q \vec{V}(t_e)/c$$

And

$$\vec{E}_{rad} = \frac{1}{4\pi r c^2} q \, \vec{n} \times \vec{n} \times \frac{\partial \vec{A}_{rad}}{\partial t} = \frac{q}{4\pi r c^2} \vec{n} \times \vec{n} \times \vec{a}(t_e)$$

In general, decompose the outgoing radiation into its polarized components



$$\vec{E} = E_1 \vec{\varepsilon}_1 + E_2 \vec{\varepsilon}_2$$ ← ε_1 and ε_2 canbe complex

e.g for circular radiation, $\vec{\varepsilon} = (1, i, 0)$

Then

$$E_1 = \vec{\varepsilon}_1^{\,*} \cdot \vec{E}$$

$$E_2 = \vec{\varepsilon}_2^{\,*} \cdot \vec{E}$$

Then the power radiated into light of a definite polarization

$$\frac{dP}{d\Omega}(\varepsilon_1) = c\,|r\, \vec{\varepsilon}_1^{\,*} \cdot \vec{E}\,|$$ ← energy per solid angle with polarization ε_1

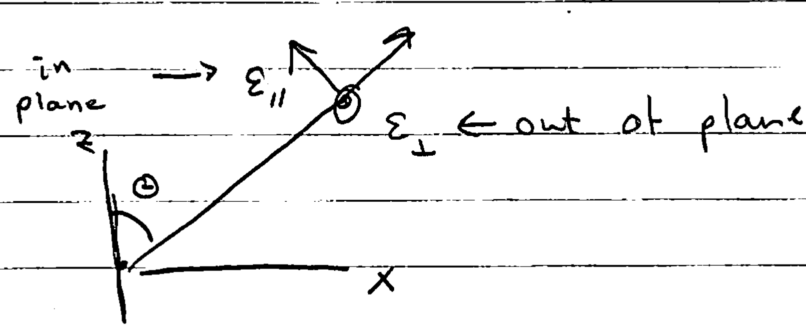
$$\frac{dP}{d\Omega}(\varepsilon_2) = c\,|r\, \vec{\varepsilon}_2^{\,*} \cdot \vec{E}\,|^2$$

and

$$dP/d\Omega = c\,|r\vec{E}\,|^2 = c\,|E_1|^2 + c\,|E_2|^2 = \frac{dP(\varepsilon_1)}{d\Omega} + \frac{dP(\varepsilon_2)}{d\Omega}$$

Example

- A charged particle oscillating up and down on the z-axis



Now

$$\vec{E} = \frac{q}{4\pi r c^2}\, n \times n \times \vec{a}$$

So

$$\vec{E} = \frac{q}{4\pi r c^2} \underbrace{\left[-\vec{a} + \vec{n}(\vec{n}\cdot\vec{a}) \right]}_{\vec{E} \text{ is in the } x-z \text{ plane}}$$

In general $\vec{E}$ is polarized in the plane of the acceleration and the observation vector $\vec{n}$

$$\vec{E} = E_{\parallel} \vec{\varepsilon}_{\parallel} + \vec{E}_{\perp} \vec{\varepsilon}_{\perp}$$

So

$\vec{\varepsilon}_{\parallel} \cdot \vec{E}$ = something

$\vec{\varepsilon}_{\perp} \cdot \vec{E} = 0$ ← $\varepsilon_{\perp}$ is orthogonal to $\vec{a}$ and $\vec{n}$

Then

$$\frac{dP_{\perp}}{d\Omega} = 0$$

while
$$\vec{\varepsilon}_{\perp} \cdot \vec{E} = \frac{q}{4\pi r c^2} \left[-\vec{a} \cdot \vec{\varepsilon}_{\parallel} + \cancel{\vec{n} \cdot \vec{\varepsilon}_{\parallel}}^{\,0} (n \cdot a) \right]$$

$$= \frac{q}{4\pi r c^2} (-a \sin\theta)$$

and

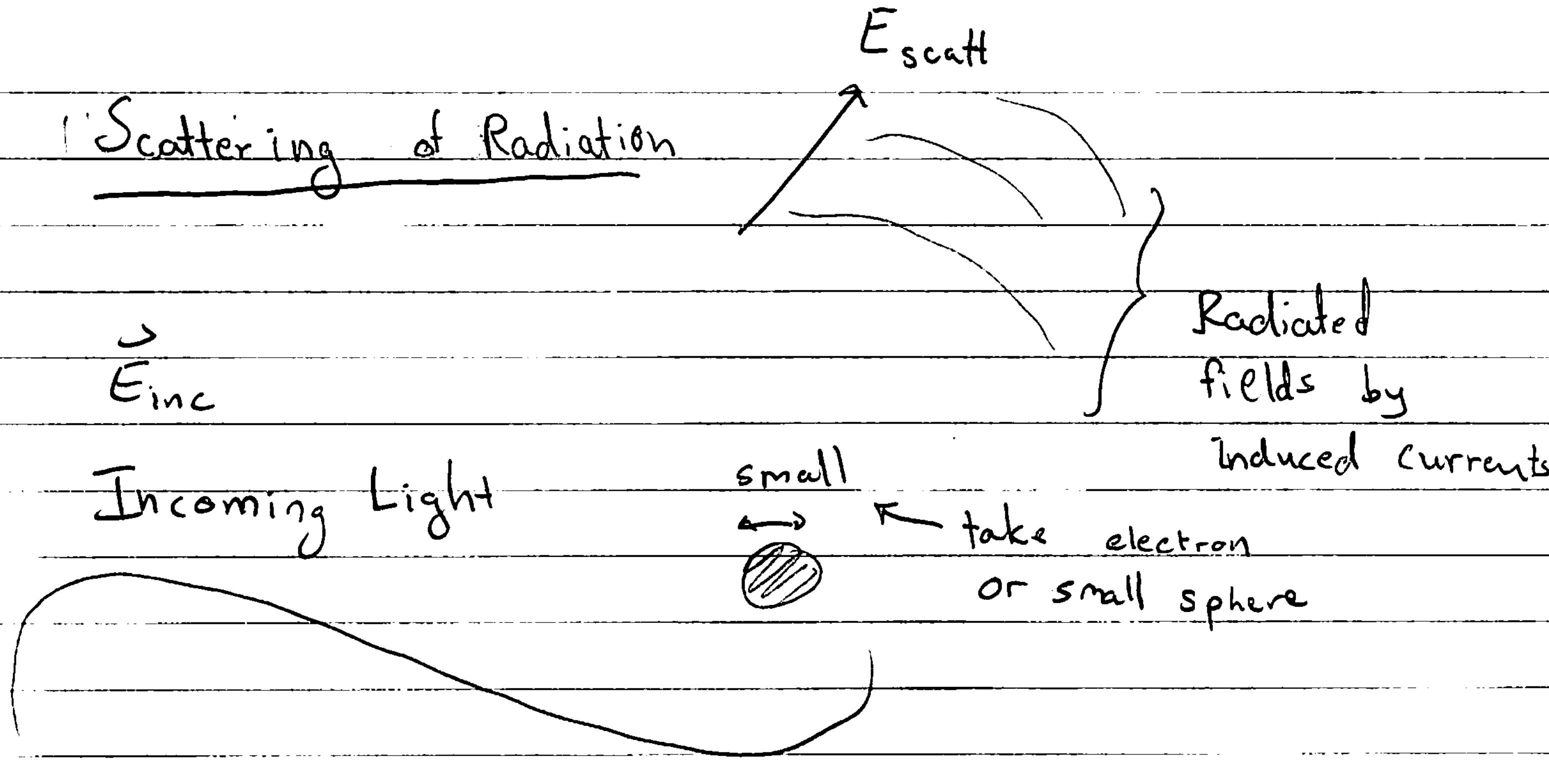
$$\frac{dP_{\parallel}}{d\Omega} = \frac{q}{16\pi^2 c^3} a^2 \sin^2\theta$$

Now so

$$\frac{dP}{d\Omega} = \frac{dP_{\parallel}}{d\Omega} + \frac{dP_{\perp}}{d\Omega}$$

$$\frac{dP}{d\Omega} = \left(\frac{q^2 a^2 \sin^2\theta}{16\pi^2 c^3}\right) + 0$$

Scattering of Radiation



- Initially study the scattering of radiation by small objects $\lambda \gg a$

- Why does light scatter? the light induces currents in the object. The induced currents radiate.

So if you wanted to compute how much light was scattered. You should first compute how the incoming field accelerates the charges, and then compute how the accelerated charges radiate

- For an extended object the acceleration in one point can create fields at another which influences the othe points, leading to a complicated standing wave pattern. Treat small (point-like) objects first.

Suggests two approximations / simplifications

① Small objects $^{\lambda \gg a}$, the external fields can be considered constant, the scattering radiation field from one point of object can be neglected at another. (

dipole

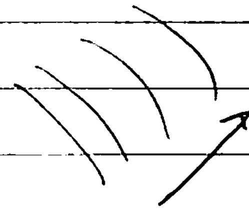
scattering

② Weak scattering. The induced fields, $\vec{E}_{scatt}$, are small compared to $\vec{E}_{inc}$.

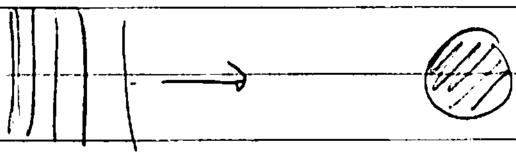
Born Approxiximation

What we want to compute - Cross Sections

$$\vec{E}_{inc} = \varepsilon_0 E_0 e^{ikz - i\omega t}$$



$$\vec{E}_{scatt}(r) \xrightarrow[r \to \infty]{} \text{Const } \frac{e^{i\vec{k}\cdot r - i\omega t}}{r}$$



$$\text{Const} \equiv E_0 \vec{f}(\vec{k})$$

$$\vec{E} = \vec{E}_{inc}(\vec{r}) + \vec{E}_{scatt}(\vec{r})$$

"Scattering amplitude"

The incoming energly is:

$$\overline{S \cdot \hat{z}} = \frac{1}{2} c |E_{inc}|^2 \leftarrow \text{time averaged energy per area per time}$$

While the outgoing energy is:

$$\overline{\frac{dP(\varepsilon)}{d\Omega}} = \overline{\vec{S}_{\varepsilon} \cdot \vec{n}} = \frac{1}{2} c \left| r \vec{\varepsilon}^* \cdot \vec{E}_{Scatt} \right|^2 \leftarrow \text{time averaged energy per time per solid angle.}$$

So then define the cross section:

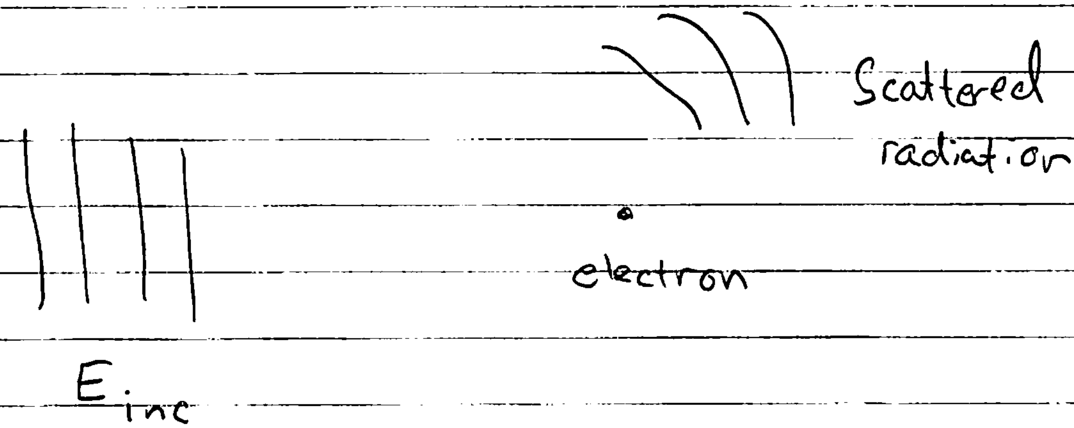
$$\frac{d\sigma(\vec{\varepsilon}; \vec{\varepsilon}_0)}{d\Omega} = \frac{\frac{1}{2} c \left| r \vec{\varepsilon}^* E_{scatt} \right|^2}{\frac{1}{2} c |E_{inc}|^2} = \left| \vec{\varepsilon} \cdot \vec{f}(k) \right|^2$$

energy scattered per solid angle (with polarization $\vec{\varepsilon}$) per incoming flux, with polarization $\vec{\varepsilon}_0$

Thomson - Scattering - Total Cross - Section

Lets compute the total cross - section

for Light electron scattering.



$$\sigma = \frac{\text{Time Power Radiated}}{\frac{1}{2}c\,|E_{inc}|^2} = \frac{\overline{P}}{\frac{1}{2}c\,|E_{inc}|^2}$$

time average

$$\overline{P} = \int \sum_{\varepsilon_a = \varepsilon_\parallel + \varepsilon_2} \overline{\frac{dP(\varepsilon_a)}{d\Omega}} \, d\Omega$$

$$= \int \frac{d\overline{P}}{d\Omega} \, d\Omega = \frac{q^2}{4\pi} \frac{2}{3c^3} \overline{a^2(t_e)}$$

So, could have defined all of this with cross sections (since $|E_{inc}|^2$ const)

$$\sigma = \int \sum_a \frac{d\sigma(\varepsilon_a; \varepsilon_o)}{d\Omega} \, d\Omega$$

The force on the electron is determined by $\vec{E}_{inc}$

$$\vec{a} = \frac{q\vec{E}_{inc}}{m} e^{-i\omega t}$$

$$\overline{a^2} = \frac{q^2}{m^2}\frac{1}{2}|E_{inc}|^2$$

Then

$$\sigma = \frac{\frac{q^2}{4\pi}\frac{2}{3c^3}\frac{q^2}{m^2}\frac{1}{2}|E_{inc}|^2}{\frac{1}{2}c|E_{inc}|^2}$$

$$\sigma = \frac{8\pi}{3}\Big(\underbrace{\frac{q^2}{4\pi mc^2}}\Big)^2 \equiv \frac{8\pi}{3} r_e^2$$

This is known as the classical electron radius

$$r_e = \frac{q^2}{4\pi mc^2}$$

$$r_e = \underbrace{\left(\frac{q^2}{4\pi\hbar c}\right)}_{1/137}\underbrace{\left(\frac{\hbar}{mc}\right)}_{\text{Compton wavelength}/(2\pi)} = \alpha\,\lambdabar_c$$

Compton wavelength

$$\frac{\hbar c}{m_e c^2} = \frac{197 \text{ eV} \cdot \text{nm}}{0.5 \text{ meV}} = 0.3 \times 10^{-12} \text{ m}$$

$$r_c = 2.8 \times 10^{-15} \text{ m}$$

$$r_e \simeq 2.8 \text{ fm}$$

And the Thomson - Cross Section is

$$\frac{8\pi}{3} r_e^2 = 66 \text{ fm}^2$$

$$= 660 \text{ milli-barn}$$

$$\simeq 0.660 \times 10^{-24} \text{ cm}^2$$

$$\simeq 0.660 \text{ barns}$$