Last Time

We started with the wave eqn

$$-\Box \vec{A} = \vec{J}/c$$

Then the solution in the far field limit

$$\vec{A} = \frac{1}{4\pi r}\int_{r_0} \vec{J}(T, r_0)/c$$

Where $T = \underbrace{t - \frac{r}{c}}_{\equiv t_e} + \underbrace{\frac{\vec{n}\cdot\vec{r}_0}{c}}_{\text{small}}$

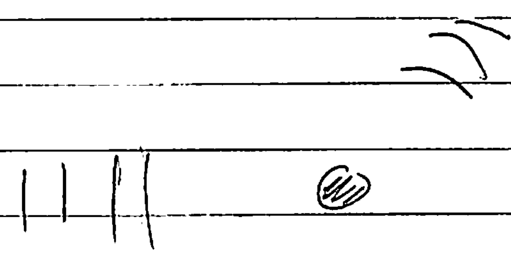
$$\vec{E}_{rad} = n \times n \times \frac{1}{c}\frac{\partial A_{rad}}{\partial t}$$

Then

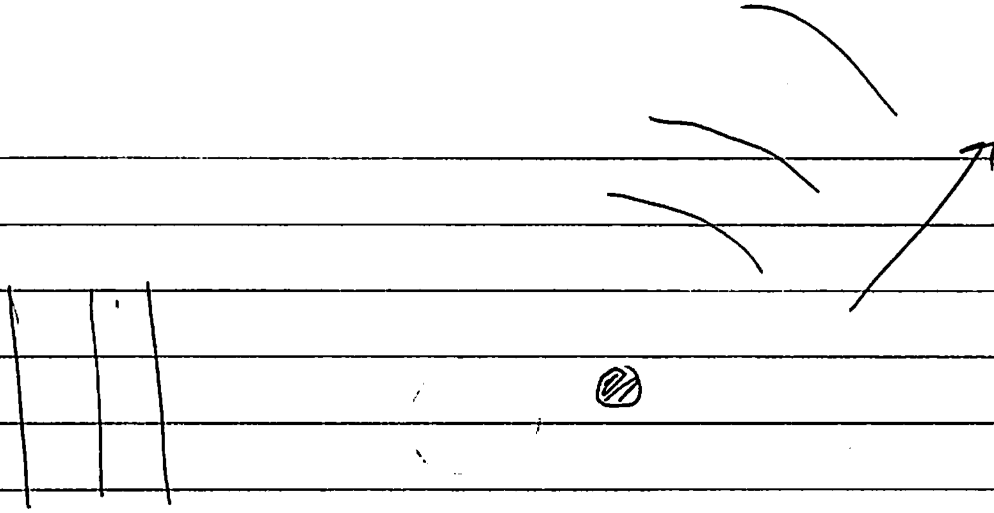
for a non-rel particle in the far field

$$\vec{E}_{rad} = \frac{q}{4\pi r c^2}\underbrace{\vec{n}\times\vec{n}\times\vec{a}(t_e)}_{[-\vec{a} + \vec{n}(\vec{n}\cdot\vec{a})]}$$

Then we formulated the scattering problem starting with light electron scattering:



The incident wave induces currents which radiate away light



$$\vec{E}_{scatt} = E_0 \vec{f} \frac{e^{i\vec{k}\cdot\vec{r} - i\omega t}}{r}$$

$$\vec{E} = \vec{E}_{inc} + \vec{E}_{scatt}$$

$$= E_0 \vec{\varepsilon}_0 e^{ikz - i\omega t} + \vec{E}_{scatt}$$

We want the piece of $\vec{E}_{scatt}$ that decreases like $\frac{1}{r}$, call it $\vec{f}$, the scatt amplitude:

$$\vec{E}_{scatt} \xrightarrow[r\to\infty]{} E_0 \vec{f}(\vec{k}) \frac{e^{i\vec{k}\vec{r} - i\omega t}}{r}$$

Then the cross section is:

$$\frac{d\sigma}{d\Omega}(\varepsilon;\varepsilon_0) = \frac{\frac{c}{2}\left| r\, \vec{\varepsilon}^{\,*}\cdot \vec{E}_{scatt}\right|^2}{\frac{1}{2}c|E_0|^2} = \frac{\text{average power radiated (into angle } \theta \text{ with pol } \varepsilon) \;\; " \frac{d\bar{P}(\varepsilon)}{d\Omega}}{\text{incoming power/area}}$$

$$= \left|\vec{\varepsilon}^{\,*}\cdot \vec{f}(\vec{k})\right|^2$$

Discussed the Thomson cross section $\gamma + e \rightarrow \gamma + e$

- The incoming wave causes accel of electron, which radiates

$$\sigma_T = \frac{8\pi}{3} r_e^2 \qquad r_e = \frac{e^2}{4\pi mc^2}$$

$$= 2.8\,\text{fm}$$

$$= 0.66 \text{ Barns}$$

$$= 0.66 \times 10^{-24}\,\text{cm}^2$$

<u>In class problem</u> on $\gamma + e^- \rightarrow \gamma + e$

① Start from $A_{rad} = \frac{1}{4\pi r} \int_{r_0} \frac{J(T, r_0)}{c}$ and $E_{rad} = \frac{n \times n \times \partial A}{c\, \partial t}$

Show that $E_{rad} = \frac{q}{4\pi r c^2}\, n \times n \times \vec{a}(t_e)$

② Show that the cross section for incoming light of polarization $\vec{\varepsilon}_0$, to produce radiation ω polarization $\vec{\varepsilon}$ is

$$\frac{d\sigma}{d\Omega} = r_e^2\, |\vec{\varepsilon}^* \cdot \vec{\varepsilon}_0|^2$$

Start by showing that the time averaged power power radiated is

$$\frac{dP(\varepsilon)}{d\Omega} = \frac{q^2}{16\pi^2 c^3} \frac{|\varepsilon^* \cdot \vec{a}|^2}{2}$$

Whats the acceleration?

Whats the time-averaged power radiated

Solution

The acceleration caused by the incoming light is

$$\vec{a} = \frac{\vec{F}}{m} = \frac{q E_0 \vec{\varepsilon}_0}{m} e^{-i\omega t}$$

So

$$\varepsilon^* \cdot \vec{E}_{rad} = \vec{\varepsilon}^* \cdot \frac{q}{4\pi r c^2} \overbrace{n \times n \times \vec{a}(t_e)}^{[-\vec{a} + \vec{n}(\vec{n}\cdot\vec{a})]}$$

Since $\vec{\varepsilon}^* \cdot \vec{n} = 0$ we have the instantaneous power

$$\frac{dP(\varepsilon)}{d\Omega} = c\left(r\vec{\varepsilon}\cdot\vec{E}_{rad}^{(t_e)}\right)^2 = \frac{q^2}{16\pi^2 c^3}\left(\vec{\varepsilon}\cdot\vec{a}(t_e)\right)^2$$

(assumes $\vec{\varepsilon}$ is real!)

So the time aver power

comes from time average

$$\frac{d\overline{P(\varepsilon)}}{d\Omega} = \left[\frac{q^2}{16\pi^2 c^3}\left|\vec{\varepsilon}^* \cdot \frac{q E_0 \vec{\varepsilon}_0}{m}\right|^2 \frac{1}{2}\right]$$

and

$$\frac{d\sigma}{d\Omega} = \frac{d\overline{P}/d\Omega}{\frac{1}{2} c |E_0|^2} = \left(\frac{q^2}{4\pi m c^2}\right)^2 |\varepsilon^* \cdot \varepsilon_0|^2$$