13 Scattering

We formulated the scattering problem. In this case incoming light induces currents in the object, which in turn create a radiation field. We will work with small objects and weak scattering where the effect of the induced radiation fields can be neglected in determining the currents. The external incoming field will induce acceleration in the case of light-electron scattering, or induce time-dependent dipole moments (i.e. currents) in the case of light scattering off a sphere.

(a) The Electric field can be written

$$\boldsymbol{E} = \boldsymbol{E}_{\rm inc} + \boldsymbol{E}_{\rm scat} \tag{13.1}$$

where

$$\boldsymbol{E}_{\rm inc}(t, \boldsymbol{r}) = E_o\, \boldsymbol{\epsilon}_o e^{ikz - i\omega t} \tag{13.2}$$

while the scattered field falls of as $1/r$

$$\boldsymbol{E}_{\rm scat}(t, \boldsymbol{r}) \to \boldsymbol{C}(\boldsymbol{k}) \frac{e^{ikr - i\omega t}}{r} \tag{13.3}$$

$\boldsymbol{E}_{\rm scat}$ (in the far field) might as well be called $\boldsymbol{E}_{\rm rad}$. The constant is proportional for E_o for linear response and so the far field of the scattered field is written in terms of the scattering amplitude, $\boldsymbol{f}(\boldsymbol{k})$.

$$\boldsymbol{E}_{\rm scat}(t, \boldsymbol{r}) \to E_o \boldsymbol{f}(\boldsymbol{k}) \frac{e^{ikr - i\omega t}}{r} \tag{13.4}$$

(b) The radiation field $\boldsymbol{E}_{\rm scat}$ can be decomposed into polarizations

$$\boldsymbol{E}_{\rm scat} = E_1 \boldsymbol{\epsilon}_1 + E_2 \boldsymbol{\epsilon}_2 \tag{13.5}$$

Using the orthogonality of the polarization vectors

$$\boldsymbol{\epsilon}_a^* \cdot \boldsymbol{\epsilon}_b = \delta_{ab} \, , \tag{13.6}$$

we have, *e.g.*

$$E_1 = \boldsymbol{\epsilon}_1^* \cdot \boldsymbol{E}_{\rm scat} \qquad E_2 = \boldsymbol{\epsilon}_2^* \cdot \boldsymbol{E}_{\rm scat} \, . \tag{13.7}$$

The time averaged power radiated per solid angle with polarization $\boldsymbol{\epsilon}_1$ is

$$\overline{\frac{dP}{d\Omega}(\boldsymbol{\epsilon}_1; \boldsymbol{\epsilon}_o)} = \frac{c}{2} |r\, \boldsymbol{\epsilon}_1^* \cdot \boldsymbol{E}_{\rm scat}|^2 \tag{13.8}$$

and similarly for $\boldsymbol{\epsilon}_2$. This will in general depend on the incoming polarization, $\boldsymbol{\epsilon}_o$, of the light.

(c) The cross section is the time averaged radiated power divided by the (time-averaged) input flux

$$\frac{d\sigma(\boldsymbol{\epsilon}; \boldsymbol{\epsilon}_o)}{d\Omega} = \frac{\overline{\frac{dP}{d\Omega}(\boldsymbol{\epsilon}_1; \boldsymbol{\epsilon}_o)}}{\frac{c}{2}|E_o|^2} = |\boldsymbol{\epsilon}_1^* \cdot \boldsymbol{f}(\boldsymbol{k})|^2 \tag{13.9}$$

(d) We studied Thomson scattering (light-electron scatering) and found that the cross section was proportional to the classical electron radius squared

$$\sigma_T = \frac{8\pi}{3} r_e^2 \qquad r_e^2 = \left(\frac{q^2}{4\pi mc^2} \right)^2 \tag{13.10}$$

You should feel comfortable deriving this result and estimating the answer without looking up numbers.

(e) We also studied dipole scattering were we found that the cross section increases as ω^4. You should feel comfortable deriving this result.