6.2 Magnetic Matter

Basic equations

(a) We are considering materials in the presence of a magnetic field. We write $\boldsymbol{j}_{\rm mat}$ (the medium (material) currents) as an expansion in terms of the derivatives in the magnetic field. For weak fields, and an isotropic medium , the lowest term in the derivative expansion, for a parity and time-reversal invariant material is

$$\frac{\boldsymbol{j}_{\rm mat}}{c} = \chi_m^B \nabla \times \boldsymbol{B} \tag{6.33}$$

where we have inserted a factor of c for later convenience.

(b) The current takes the form

$$\frac{\boldsymbol{j}_{\rm mat}}{c} = \nabla \times \boldsymbol{M} \tag{6.34}$$

i) $\boldsymbol{M}$ is known as the magnetization, and can be interpreted as the magnetic dipole moment per volume.

ii) We have worked with linear response for an isotropic medium where

$$\boldsymbol{M} = \chi_m^B \boldsymbol{B} \tag{6.35}$$

This is most often what we will assume.

iii) Usually people work with $\boldsymbol{H}$ (see the next items (c), (d) for the definition of $\boldsymbol{H}$) not $\boldsymbol{B}$ [1]

$$\boldsymbol{M} = \chi_m \boldsymbol{H} \tag{6.36}$$

iv) For not-that soft ferromagnets $\boldsymbol{M}(\boldsymbol{B})$ can be a very non-linear function of $\boldsymbol{B}$. This will need to be specified (usually by experiment) before any problems can be solved. Usually this is expressed as the magnetic field as a function of $\boldsymbol{H}$

$$\boldsymbol{B}(\boldsymbol{H}) \tag{6.37}$$

where $\boldsymbol{H}$ is small (of order gauss) and $\boldsymbol{B}$ is large (of order Tesla)

(c) After specifying the currents in matter, Maxwell equations take the form

$$\nabla \times \boldsymbol{B} = \nabla \times \boldsymbol{M} + \frac{\boldsymbol{j}_{ext}}{c} \tag{6.38}$$

$$\nabla \cdot \boldsymbol{B} = 0 \tag{6.39}$$

or

$$\nabla \times \boldsymbol{H} = \frac{\boldsymbol{j}_{ext}}{c} \tag{6.40}$$

$$\nabla \cdot \boldsymbol{B} = 0 \tag{6.41}$$

where [2]

$$\boldsymbol{H} = \boldsymbol{B} - \boldsymbol{M} \tag{6.43}$$

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[1] There are a couple of reasons for this. One reason is because the parallel components of H are continuous across the sample. But, ultimately it is $\boldsymbol{B}$ which is the curl $\boldsymbol{A}$, and it is ultimately the average current which responds to the gauge potential, through a retarded medium current-current correlation function that we wish to categorize.

[2] In the MKS system one has $\boldsymbol{H}_{MKS} = \frac{1}{\mu_o}\boldsymbol{B}_{MKS} - M_{MKS}$ so that $\boldsymbol{B}$ and $\boldsymbol{H}$ have different units. In a system of units where $\varepsilon_o = 1$ (so $1/\mu_o = c^2$) we have $H_{HL} = H_{MKS}/c$, $\boldsymbol{M}_{HL} = \boldsymbol{M}_{MKS}/c$ or since $1/c = \sqrt{\mu_o}$:

$$\boldsymbol{H}_{HL} = \sqrt{\mu_o} H_{MKS} \qquad \boldsymbol{M}_{HL} = \sqrt{\mu_o}\boldsymbol{M}_{MKS} \tag{6.42}$$

(d) For linear materials :

$$\boldsymbol{B} = \mu \boldsymbol{H} = \frac{1}{1 - \chi_m^B} \boldsymbol{H} = (1 + \chi_m) \boldsymbol{H} \tag{6.44}$$

Implying the definitions

$$\mu \equiv \frac{1}{1 - \chi_m^B} \equiv (1 + \chi_m) \tag{6.45}$$

Solving magnetostatic problems with linear magnetic media:

All of the methods described in Sect. (6.1) will work with minor modifications due to the boundary conditions described below

(a) For linear materials in the coulomb gauge we get

$$\nabla \times \boldsymbol{H} = \mu \frac{\boldsymbol{j}_{\text{ext}}}{c} \tag{6.46}$$

$$\nabla \cdot \boldsymbol{B} = 0 \tag{6.47}$$

and with $\boldsymbol{B} = \nabla \times \boldsymbol{A}$ and constant μ we find

$$-\nabla^2 \boldsymbol{A} = \mu \frac{\boldsymbol{j}_{\text{ext}}}{c} \tag{6.48}$$

which can be solved using the methods of magnetostatics.

(b) To solve magneto static equations we have boundary conditions:

$$\boldsymbol{n} \times (\boldsymbol{H}_2 - \boldsymbol{H}_1) = \frac{\boldsymbol{K}_{\text{ext}}}{c} \tag{6.49}$$

$$\boldsymbol{n} \cdot (\boldsymbol{B}_2 - \boldsymbol{B}_1) = 0 \tag{6.50}$$

i.e. if there are no external currents then the parallel components of $\boldsymbol{H}$ are continuous and the perpendicular components of $\boldsymbol{B}$ are continuous.

(c) At an interface the there are bound currents which are generated

$$\boldsymbol{n} \times (\boldsymbol{M}_2 - \boldsymbol{M}_1) = \frac{\boldsymbol{K}_{\text{mat}}}{c} \tag{6.51}$$