# 5 Ohms Law and Conduction

## 5.1 Steady current and Ohms Law: Lecture 17

(a) For steady currents

$$\nabla \cdot \boldsymbol{j} = 0 \tag{5.1}$$

(b) For steady currents in ohmic matter

$$\boldsymbol{j} = \sigma \boldsymbol{E} \tag{5.2}$$

(c) $\sigma$ has units of $1/s$. Note that in MKS units $\sigma_{MKS}$ has the uninformative unit 1/ohm m:

$$\sigma_{HL} = \frac{\sigma_{MKS}}{\varepsilon_o} \tag{5.3}$$

For $\sigma_{MKS} = 10^7\,(\text{ohm m})^{-1}$ we find $\sigma \sim 10^{18}\,\text{s}^{-1}$.

(d) To find the flow of current we need to solve the electrostatics problem

$$-\nabla \cdot (\sigma \boldsymbol{E}) = 0 \tag{5.4}$$

$$\nabla \times \boldsymbol{E} = 0 \tag{5.5}$$

or for homogeneous material ($\sigma = \text{const}$)

$$-\sigma \nabla^2 \Phi = 0 \tag{5.6}$$

We see that we are supposed to solve the Laplace equation. However the boundary conditions are rather different.

(e) A point source of current is represented by a delta function $I\delta^3(\boldsymbol{r} - \boldsymbol{r}_o)$. While a sink of current is represented by a delta function of opposite sign $-I\delta^3(\boldsymbol{r} - \boldsymbol{r}_o)$.

(f) Eq. (5.4) and Eq. (5.6) need boundary conditions. At an interface current should be conserved so

$$\boldsymbol{n} \cdot (\boldsymbol{j}_2 - \boldsymbol{j}_1) = 0 \tag{5.7}$$

or

$$\sigma_2 \frac{\partial \Phi_2}{\partial n} = \sigma_1 \frac{\partial \Phi_1}{\partial n} \tag{5.8}$$

Most often this is used to say that the normal component of the Electric field at a metal-insulator interface should be zero:

$$\boldsymbol{n} \cdot \boldsymbol{E} = 0 \qquad \text{at metal-insulator interface} \tag{5.9}$$

(g) In general the input current (or normal derivatives of the potential) must be specified at all the boundaries in order to have a well posed boundary value problem that can be solved (at least numerically.)

(h) In general the input currents $I_a = I_1, I_2, \ldots$ on a set conductors will be will be specified, specifying the normal derivatives on all of the surfaces. Then you solve for the potential. The voltages of a given electrode relative to ground is $V_a$ , and you will find that $V_a = \sum_b R_{ab} I_b$. $R_{ab}$ is the resistance matrix.

## 5.2 Basic physics of metals, Drude model of conductivity: Lecture 22

This section really lies outside of electrodynamics. But it helps to understand what is going on.

(a) The electrons in the metal under go scatterings with impurities and other defects on a time scale $\tau_c$. For copper:

$$\tau_c \sim 10^{-14} s \tag{5.10}$$

(b) A typical coulomb oscillation / orbital frequency is set by the plasma frequency

$$\omega_p = \sqrt{\frac{ne^2}{m}} \tag{5.11}$$

For copper $\boldsymbol{\omega_p}$ is of order a typical quantum frequency and scales like:

$$\omega_p \sim \Big( \frac{1}{m} \underbrace{\frac{e^2}{a_o^3 m}}_{\text{spring const}} \Big)^{1/2} \tag{5.12}$$

$$\sim \left( \frac{27.2\,\mathrm{e}V}{\hbar} \right) \tag{5.13}$$

$$\sim 10^{-16}\ 1/\mathrm{s} \tag{5.14}$$

In the second to last line we ignored all $4\pi$ factors and used Bohr model identities

$$\frac{1}{2}\left( \frac{e^2}{4\pi a_o} \right) = \frac{\hbar^2}{2ma_o^2} = 13.6\,\mathrm{e}V \tag{5.15}$$

which you can remember by noting that (minus) coulomb potential energy is twice the kinetic energy$=p^2/2m$ and knowing $p_{bohr} = \hbar/a_o$ as expected by the uncertainty principle.

(c) Since the distances between collisions are long compared to the Debroglie wavelength, and the time between collisions is long compared to a typical inverse quantum frequency, we are justified in using classical transport

$$\omega_p \tau_c \sim 100 \gg 1 \tag{5.16}$$

(d) In the Drude model the magnitude of the driving force $F_E = eE_{\rm ext}$ equals the magnitude drag force $F_{\rm drag} = m\boldsymbol{v}/\tau_c$, leading to an estimate of the conductivity

$$\sigma = \frac{ne^2\tau_c}{m} = \omega_p^2\,\tau_c \tag{5.17}$$

The estimates given show

$$\sigma \sim 10^{18}\ \mathrm{s}^{-1} \tag{5.18}$$

for a metal like copper.