Model Dielectric Overview

We will present the Lorentz / Harmonic oscillator model for the dielectric response. Why?

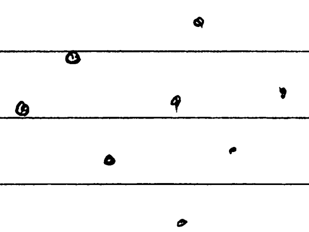
① The model captures certain qualitative features of dielectrics and metals.

Later we will see this characteristic response is a conequence of causality

② The model gives simple and correct estimates for the size of medium parameters such as the susceptibility χ_e in insulators and σ_0 in metals.

Simple Model For the Dielectric Response - Lorentz

Consider a model consisting of n electrons per volume harmonically bound to atomic centers in an external field $E(t) = E_\omega e^{-i\omega t}$



Atomic electrons bound to protons

$$\vec{E}(t) = E_\omega e^{-i\omega t}$$

Then the external field makes a current

$$(1) \qquad m\frac{d^2x}{dt^2} + m\eta\frac{dx}{dt} + m\omega_0^2 x = E_\omega e^{-i\omega t}$$

Here we have added a damping term which could represent collisions other atoms. The units are such that η represents the frequency of collisions. Below we define the collision time

$$\eta \equiv \frac{1}{\tau_c}$$

Then solving Eq (1) we have $x(t) = x_\omega e^{-i\omega t}$

$$x_\omega = \frac{eE_\omega/m}{[-\omega^2 + \omega_0^2 - i\omega\eta]}$$

The the current $\vec{j}(t) = e\,n\,\overset{\overset{-i\omega x_\omega e^{-i\omega t}}{\downarrow}}{v(t)} = j_\omega e^{-i\omega t}$ is

$$j_\omega = \frac{(ne^2/m)}{[-\omega^2 + \omega_0^2 - i\omega\eta]}(-i\omega E_\omega)$$

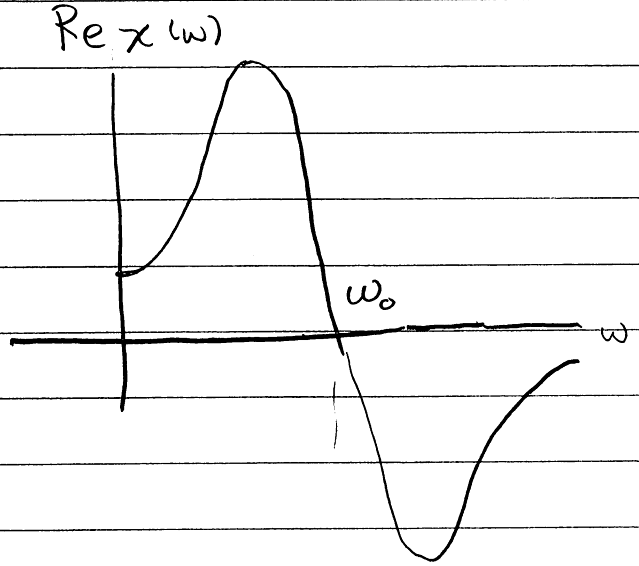
So the susceptibility $j(\omega) = \chi_e(\omega)\,(-i\omega E(\omega))$ is

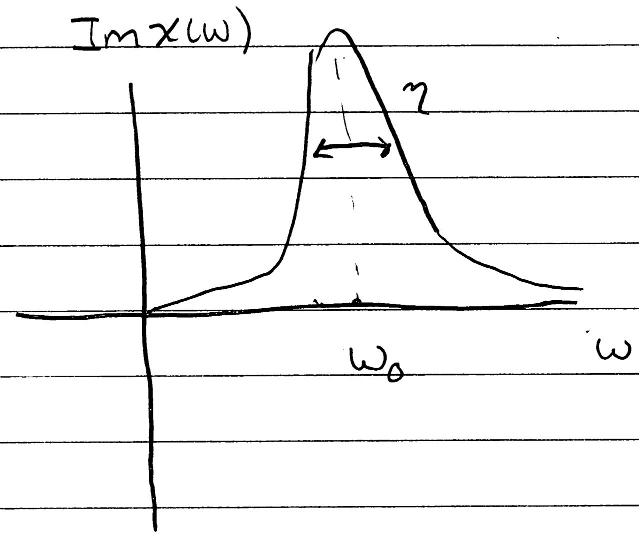
$$\chi_e(\omega) = \frac{ne^2/m}{[-\omega^2 + \omega_0^2 - i\omega\eta]}$$

Then writing in Real + i Imaginary parts

$$\chi_e(\omega) = \underbrace{\frac{(ne^2/m)\,(\omega_0^2 - \omega^2)}{[(\omega^2 - \omega_0^2)^2 + (\omega\eta)^2]}}_{\mathrm{Re}\,\chi} + \underbrace{i\frac{(ne^2/m)\,(\omega\eta)}{[(\omega^2 - \omega_0^2)^2 + (\omega\eta)^2]}}_{\mathrm{Im}\,\chi}$$

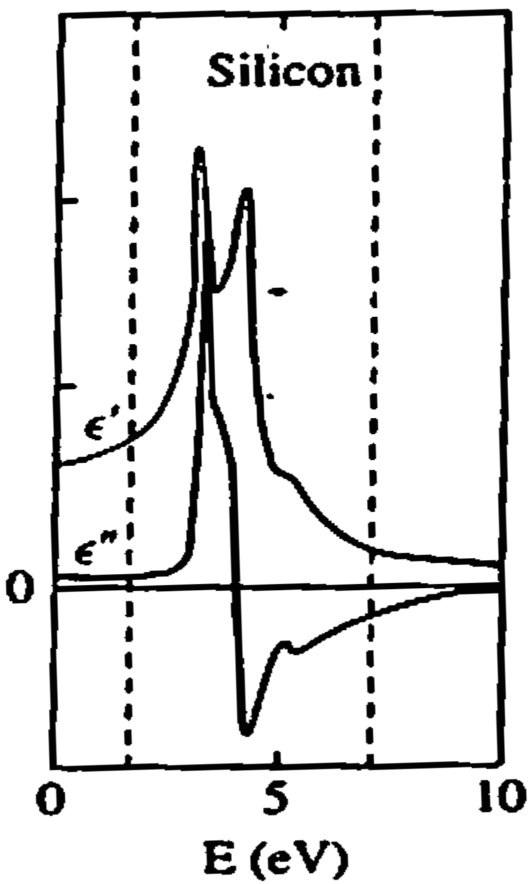
Thus we have $\varepsilon(\omega) = 1 + \chi_e(\omega)$





This is often a reasonable model (see handout). Part of its succes is because the model respects causality

Comparison of the Lorentz Model to the dielectric response of silicon



Zangwill adapted from Wooten (1972)

Comparison with the low frequency limit

① Insulator:

In the limit $\omega \to 0$ find

$$\lim_{\omega \to 0} \chi_e(\omega) = \frac{ne^2}{m\omega_0^2} \equiv \frac{\omega_p^2}{\omega_0^2} \qquad \omega_p^2 = \frac{ne^2}{m}$$

The quantity ne^2/m is known as the (squared) plasma frequency. Lets estimate its value for typical solids and liquids:

$$n \sim \frac{1}{a_0^3} \qquad \hbar\omega_0 \sim \frac{\hbar^2}{ma_0^2} \sim \frac{e^2}{a_0} \qquad \text{(Think Bohr model)}$$

a typical atomic frequency

Then

$$\omega_p^2 \equiv \frac{ne^2}{m} \sim \frac{1}{ma_0^2}\frac{e^2}{a_0} \sim \frac{1}{\hbar^2}\left(\frac{\hbar^2}{ma_0^2}\frac{e^2}{a_0}\right) \sim \omega_0^2$$

Thus, expect that the static succeptibility is of order unity.

$$\chi_e \equiv \frac{\omega_p^2}{\omega_0^2} \sim 1$$

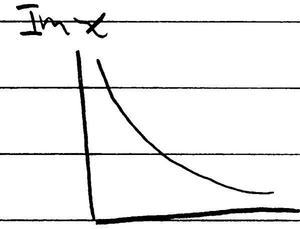
This is what is found:

e.g. quartz $\chi_e = 0.46$

water $\chi_e = 0.33$

(2) Conductors:

Conductors have no binding frequency $\omega_0 = 0$. Define the conductivity by the low frequency limit



$$\lim_{\omega \to 0} \chi_e(\omega) \equiv \frac{i\sigma_0}{\omega}$$

Comparison with the model

$$\lim_{\omega \to 0} \chi_e(\omega) = \frac{i}{\omega} ne^2/m\eta$$

So

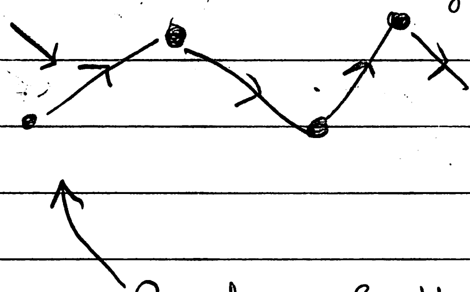
$$\sigma_0 = \omega_p^2 \tau_c \quad \text{with} \quad \omega_p^2 \equiv \frac{ne^2}{m} \qquad \tau_c \equiv \frac{1}{\eta}$$

time between collisions with dattice defects and impurites

Thus the density of electrons in Cu determines the typical debroglie wavelength and oscillation freqenency $\omega_p \sim 10^{15}$ Hz $\sim v_F k_F$, of the conduction electrons. These conduction electrons form wave packets which behave quasi-classically ocassionally scattering off lattice defects and impurities. For Copper $\sigma_{Cu} \sim 10^{17}$ Hz we find

$\omega_p \tau_c \sim 100$. Thus you should have the following quasi-classical picture of scattering of defects for the conduction of electrons in metals.

electron wave packet flying



Random Scatterings with defects

This classical quasi-particle kinetic picture is well justified because the typical debroglie wavelength is short compared to the spacings between collisions, l_{mfp}

$$k_{Debgroglie}\, l_{mfp} \gg 1$$

or since $\tau_c \sim l_{mfp} / V_F$ and $\omega_p = k_{Debroglie} V_F$,

$$\omega_p \tau_c \gg 1.$$