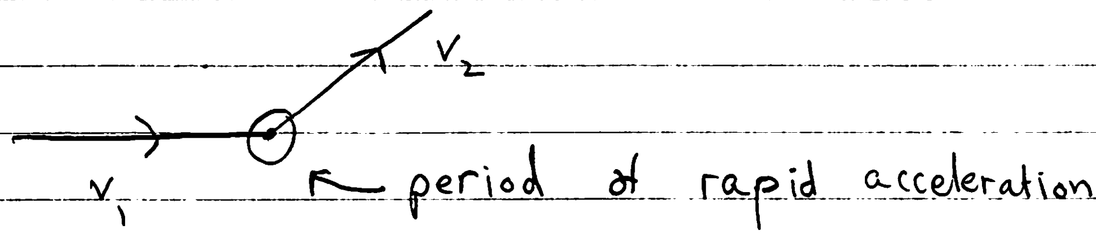
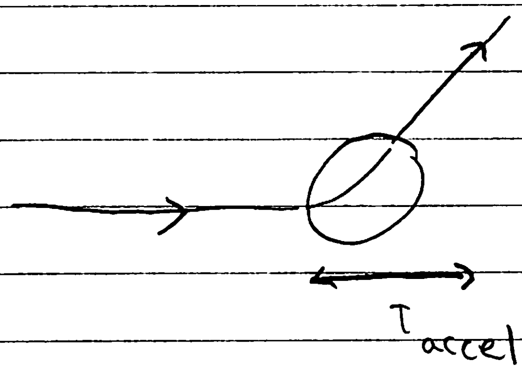
Radiation During Collisions

Consider a charged particle that gets a kick



In general we could imagine the particle gets rapidly accelerated over time τ_{accel}:



We want to compute the fourier transform of the radiation field

$$E_{rad}(\omega) = q/4\pi r c^2 \; e^{i\omega r/c} \, (-i\omega) \int_{-\infty}^{\infty} dT \, e^{i\omega T - i\vec{k}\cdot\vec{r}_*} \, \vec{n}\times\vec{n}\times\vec{v}(T)$$

It is easier to use a form which makes the acceleration explicit:

$$E_{rad}(\omega) = \frac{q}{4\pi r c^2} \, e^{i\omega r/c} \int_{-\infty}^{\infty} e^{i\omega(T - n\cdot r_*/c)} \, \frac{d}{dT} \frac{n\times n\times \vec{V}}{(1-\vec{n}\cdot\vec{\beta})} \, dT$$

The integrand vanishes except over a short period of $\Delta T \sim \tau_{accel}$. Over this period of time the phase is essentially constant, provided the frequency is not too large.

$$\Delta\phi = \overbrace{\omega\,\Delta T\left(1 - n\cdot\frac{\Delta r_*}{\Delta T}\right)}^{\text{change in phase}} \ll 1$$

Then we integrate

$$E_{rad}(\omega) \simeq \frac{q}{4\pi r c^2}\, e^{i\omega r/c} \int_{-\infty}^{\infty} \underbrace{e^{i\phi}}_{\text{constant, change } \Delta\phi \ll 1} \frac{d}{dT}\frac{n\times n\times v}{(1-n\beta)}\, dT$$

Or

$$\boxed{E_{rad}(\omega) = \frac{q}{4\pi r c^2}\, e^{i\omega r/c}\, e^{i\phi}\left[\frac{n\times n\times v_2}{(1-n\cdot\beta_2)} - \frac{n\times n\times v_1}{(1-n\,\beta_1)}\right]}$$

Thus during a collision expect a distribution of energy:

$$2\pi\frac{dW}{d\omega\,d\Omega} = \frac{q^2}{16\pi^2 c^3}\left|\frac{n\times n\times v_2}{(1-n\cdot\beta_2)} - \frac{n\times n\times v_1}{(1-n\cdot\beta_1)}\right|^2$$

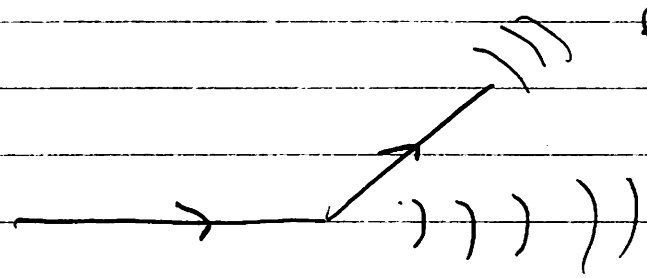
Qualitative pg. 1

Lets Look at the qualitative features

① There are two collinear factors

$$\frac{1}{(1-n\cdot v_2/c)} \quad \text{and} \quad \frac{1}{(1-n\cdot v_1/c)}$$

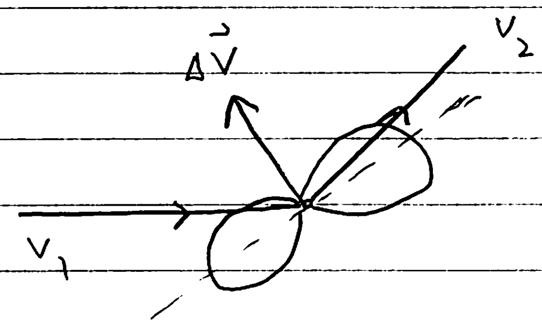
As long as v_1 and v_2 are separated by a wide angle, then the radiation will be peaked in the v_1 and v_2 directions



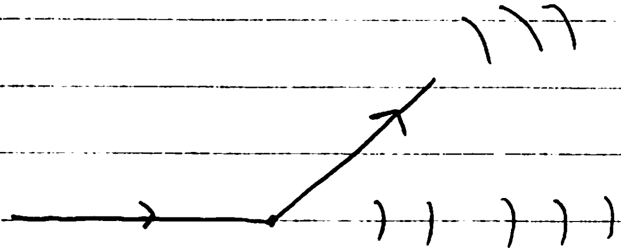
② In the non-relativistic limit find, neglecting the denominators, that

$$\boxed{2\pi \frac{dW}{d\omega\, d\Omega} = \frac{q^2}{16\pi^2 c^3} \left| n\times n\times (v_2-v_1)\right|^2}$$

Kind of Larmour like



(3) Independent of Frequency



Now

$$2\pi \frac{dW}{d\omega\, d\Omega} = \pi \frac{dI}{d\omega\, d\Omega} = \pi (\hbar\omega) \frac{dN_\gamma}{d\omega\, d\Omega} = \text{independent of frequency}$$

So

$$\pi \frac{dN_\gamma}{d\Omega} = \left(\frac{d\omega}{\omega}\right)\left(\frac{q^2}{16\pi^2 \hbar c}\right) \left| \frac{n \times n \times \beta_2}{(1 - n\cdot\beta_2)} - \frac{n \times n \times \beta_1}{(1 - n \cdot \beta_1)} \right|^2$$

So you see a distribution of radiated photons which is extremely characteristic:

$$dN \propto \frac{d\omega}{\omega}$$

The yield soft photons, $\int_0^{\Lambda} \frac{d\omega}{\omega}$, is infinite in the infrared, but the energy they carry is finite

$$\Delta E \sim \int_0^{\Lambda} \frac{d\omega}{\omega} \hbar\omega \sim \text{finite}$$ ← more next time

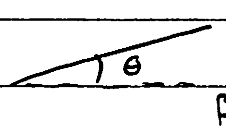
Analysis of Cone

Near one of the directions (say β_1)

$$\frac{dN}{d\omega\, d\Omega} \approx \frac{\alpha}{4\pi^2}\frac{1}{\omega}\left|\frac{n\times n\times\beta_1}{(1-n\cdot\beta_1)}\right|^2$$

(not valid for $\theta \sim 1$)

Then, $n\times n\times\beta \simeq \beta\sin\theta \simeq \sin\theta \simeq \theta$,



and $1/(1-n\cdot\beta) \simeq \dfrac{2\gamma^2}{(1+(\gamma\theta)^2)}$

So

$$\frac{dN}{d\omega\, d\Omega} = \frac{\alpha}{\pi^2}\frac{\gamma^2}{\omega}\frac{(\gamma\theta)^2}{(1+(\gamma\theta)^2)^2}$$

(Eq ★★)

Then we see a characteristic $1/\omega$ distribution. We integrate over the cone, to find, using:

$$d\Omega = \sin\theta\, d\theta \simeq 2\pi\,\theta\, d\theta$$

that,

$$dN_\gamma = \frac{2\alpha}{\pi}\frac{d\omega}{\omega}\frac{(\gamma\theta)^2}{(1+(\gamma\theta)^2)^2}(\gamma\theta)\,d(\gamma\theta)$$

For $\gamma\to\infty$, but θ fixed, i.e. $\gamma\theta \gg 1$ we find

$$\boxed{dN_\gamma = \frac{2\alpha}{\pi}\frac{d\omega}{\omega}\frac{d\theta}{\theta}}$$

Analysis of cone pg. 2

To evaluate the number of photons in the cone we use a logarithmic approximation

$$dN_\gamma = \frac{2\alpha}{\pi}\frac{d\omega}{\omega}\int_{\theta_{min}}^{\theta_{max}}\frac{d\theta}{\theta}$$

$$= \frac{2\alpha}{\pi}\frac{d\omega}{\omega}\log\frac{\theta_{max}}{\theta_{min}}$$

We should set the limits of integration where the approximation breaks down. The upper limit is $\theta_{max} \sim 1$. At this point we can no longer make small angle approximations.

$$\left|\frac{n\times n\times V_2}{(1-n\cdot V_2)} - \frac{n\times n\times V_1}{(1-n\cdot V_1)}\right|$$

Similarly for the lower limit we set $\theta_{min} \sim V\gamma$. At this point we should return to Eq ★★ on the previous page. In a logarithmic approximation we find,

$\leftarrow \theta_{max}/\theta_{min}$

$$dN_\gamma = \frac{2\alpha}{\pi}\frac{d\omega}{\omega}\log\gamma$$

$$dN = \frac{2\alpha}{\pi}\frac{d\omega}{\omega}\log\left(\frac{E}{mc^2}\right)$$

Then to find the total # of photons we integrate

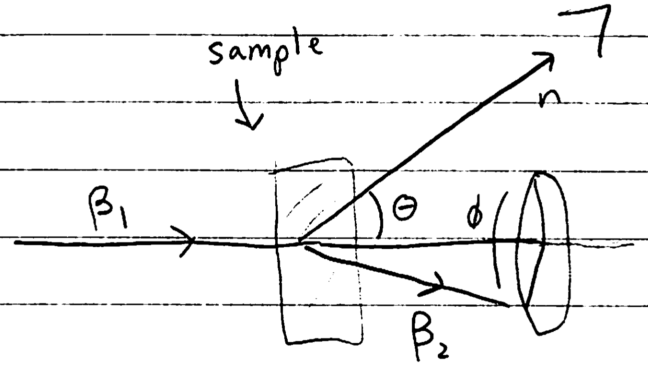
$$N_\gamma = \frac{2\alpha}{\pi}\left[\int_{\omega_{min}}^{\omega_{max}} \frac{d\omega}{\omega}\right] \log \frac{E}{mc^2}$$

For the lower limit, we recognize that there will be a frequency cutoff ω_{cut} on any detector. For the upper limit, eventually the photon has energy comparable to the energy of the particle $\sim E$ and cant be treated classically. Thus we estimate that

$$\boxed{N_\gamma \simeq \frac{2\alpha}{\pi} \log\left(\frac{E}{\hbar\omega_{cut}}\right) \log\left(\frac{E}{mc^2}\right)}$$

Last Time

Discussed Radiation During Collisions

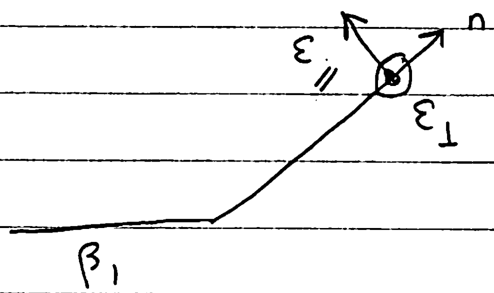


For grazing collisions, $\vec{\beta}_2 \simeq \vec{\beta}_1 + \Delta\vec{\beta}$, and ϕ is often random

Found that

$$\vec{E}(\omega) = \frac{q}{4\pi r c^2} e^{ikr} \left(\frac{n \times n \times \vec{v}_2}{(1 - n \cdot \beta_2)} - \frac{n \times n \times \vec{v}_1}{(1 - n \cdot \beta_1)} \right)$$

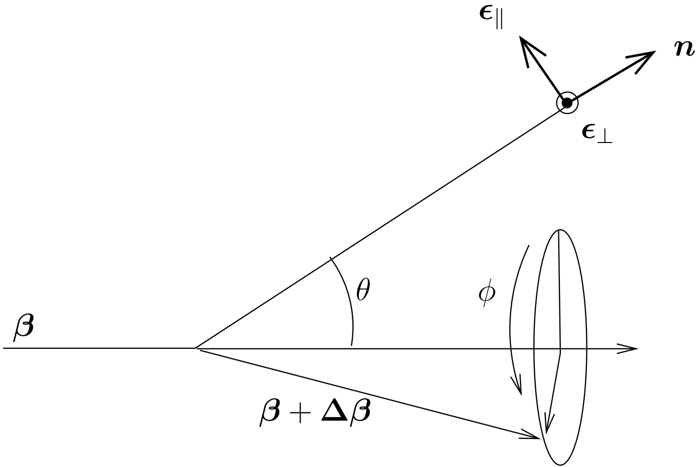
More generally decompose the outgoing light into two polarization vectors



$$\vec{E} = E_{\parallel} \vec{\varepsilon}_{\parallel} + E_{\perp} \cdot \vec{\varepsilon}_{\perp}$$

Then more generally ε's can be complex, for example:

$\varepsilon = (0, 1, i, 0)$ records circularly light



$\epsilon_\parallel$
n
$\epsilon_\perp$
θ
ϕ
β
$\beta + \Delta\beta$

Last time Continued

The properties that we derived from the analys of waves are, $\vec{E} \equiv E_1 \vec{\varepsilon}_1 + E_2 \vec{\varepsilon}_2$

$$\vec{\varepsilon}_a^{\,*} \cdot \vec{\varepsilon}_b = \delta_{ab} \quad \leftarrow \text{orthogonal}$$

$$\vec{n} \cdot \varepsilon_a = 0 \quad \leftarrow \text{transverse to direction}$$

Then we write the energy per frequency per solid angle with polarization $\parallel$ (in $\vec{n}, \vec{\beta}$ plane) and $\perp$ (out of $\vec{n}, \vec{\beta}$ plane) as

$$2\pi \frac{dW_{\parallel}}{d\omega\, d\Omega} = c \left| \vec{\varepsilon}_{\parallel}^{\,*} \cdot E \right|^2 r^2$$

$$2\pi \frac{dW}{d\omega\, \Omega} = c \left| \varepsilon_{\perp}^{*} \cdot E \right|^2 r^2$$

Then since for this example $\vec{n} \times \vec{n} \times \vec{v}$ is already transverse we have

$$E_{\parallel} = \frac{-q\, e^{ikr}}{4\pi r c^2} \left[\frac{\vec{\varepsilon}_{\parallel}^{\,*} \cdot \vec{v}_2}{(1 - n\cdot\beta)_2} - \frac{\varepsilon_{\parallel}^{*} \cdot \vec{v}_1}{(1 - n\cdot\beta_1)} \right]$$

And the frequency spectrum is

$$2\pi \frac{dW_{\parallel}}{d\omega\, d\Omega} = \frac{q}{16\pi c^3} \left| \frac{\varepsilon_{\parallel}^{*} \cdot V_2}{(1 - n\cdot\beta_2)} - \frac{\varepsilon_{\parallel} \cdot V_1}{(1 - n\cdot\beta_1} \right|^2$$

You will need this of homework