A Heavside Lorenz (HL) Units

A.1 MKS to HL Units

- The HL Maxwell Equations follow from the MKS maxwell equations by defining

$$\boldsymbol{E}_{HL} = \sqrt{\epsilon_o}\boldsymbol{E}_{MKS} \qquad \boldsymbol{B}_{HL} = \frac{\boldsymbol{B}_{MKS}}{\sqrt{\mu_o}} \tag{A.1}$$

$$\rho_{HL} = \frac{\rho_{MKS}}{\sqrt{\epsilon}} \qquad \frac{\boldsymbol{j}_{HL}}{c} = \sqrt{\mu_o}\boldsymbol{j}_{MKS} \tag{A.2}$$

and using $c = 1/\sqrt{\epsilon_o \mu_o}$

- To convert from MKS to HL set $\epsilon_o = 1$ (and thus $\mu_o = 1/c^2$, $\sqrt{\mu_o} = 1/c$) and use this table

Quantity	$\epsilon_o = 1$ relation
B-field	$c\boldsymbol{B}_{MKS} = B_{HL}$
A-field	$c\boldsymbol{A}_{MKS} = \boldsymbol{A}_{HL}$
magnetic dipole moment	$\frac{m_{MKS}}{c} = m_{HL}$
magnetization	$\frac{M_{MKS}}{c} = M_{HL}$
induction	$\frac{H_{MKS}}{c} = H_{HL}$
permeability	$\mu_{MKS}/\mu_o = \mu_{HL}$
permitivity	$\epsilon_{MKS}/\epsilon_o = \epsilon_{HL}$

In each of these examples the $\Longrightarrow$ indicates that I have set $\epsilon_o = 1$, so $\mu_o = 1/c^2$ when $\epsilon_o = 1$.

Example: the magnetic potential energy

$$U_B = \frac{1}{2}\frac{B^2_{MKS}}{\mu_o} \Longrightarrow \frac{1}{2}(cB_{MKS})^2 = \frac{1}{2}B^2_{HL} \tag{A.3}$$

Example: The poynting vector

$$S = \frac{1}{\mu_o}\boldsymbol{E}_{MKS} \times \boldsymbol{B}_{MKS} \Longrightarrow c\boldsymbol{E}_{MKS} \times (c\boldsymbol{B}_{MKS}) = c\boldsymbol{E}_{HL} \times \boldsymbol{B}_{HL} \tag{A.4}$$

Example: The force law

$$\boldsymbol{F} = q_{MKS}(\boldsymbol{E}_{MKS} + \boldsymbol{v} \times \boldsymbol{B}_{MKS}) \Longrightarrow q_{MKS}(\boldsymbol{E}_{MKS} + \frac{\boldsymbol{v}}{c} \times c\boldsymbol{B}_{MKS}) = q_{HL}(\boldsymbol{E}_{HL} + \frac{\boldsymbol{v}}{c} \times \boldsymbol{B}_{HL}) \tag{A.5}$$

Example: The magnetic energy of a dipole

$$U = -\boldsymbol{m}_{MKS} \cdot \boldsymbol{B}_{MKS} \Longrightarrow -\frac{\boldsymbol{m}_{MKS}}{c}(c\boldsymbol{B}_{MKS}) = -\boldsymbol{m}_{HL} \cdot \boldsymbol{B}_{HL} \tag{A.6}$$

Example: The Magnetic energy in matter

$$U = \frac{1}{2}\boldsymbol{B}_{MKS} \cdot \boldsymbol{H}_{MKS} \Longrightarrow \frac{1}{2}c\boldsymbol{B}_{MKS}\frac{\boldsymbol{H}_{MKS}}{c} = \frac{1}{2}\boldsymbol{B}_{HL} \cdot \boldsymbol{H}_{HL} \tag{A.7}$$

Example: Consistency of definition of $\boldsymbol{H}$

$$H_{MKS} = \frac{1}{\mu_o} B_{MKS} - M_{MKS} \Longrightarrow H_{MKS} = c^2 B_{MKS} - M_{MKS} \quad \text{or} \quad H_{HL} = B_{HL} - M_{HL} \tag{A.8}$$

The last step follows by dividing both sides by c.

A.2 HL to MKS

- The relation between charges and and currents in the HL and MKS units are

$$Q_{HL} = \frac{Q_{MKS}}{\sqrt{\epsilon}} \qquad \rightarrow \qquad \frac{1}{\sqrt{\epsilon_o}}\,(1\,\mu\text{C}) = 0.336\,\sqrt{N \cdot m^2} \tag{A.9}$$

$$\frac{I_{HL}}{c} = \frac{I_{MKS}}{\sqrt{\epsilon} c} = \sqrt{\mu_o} I_{MKS} \qquad \rightarrow \qquad \sqrt{\mu_o}(1\,\text{amp}) = 0.00112\,\sqrt{N \cdot m^2} \tag{A.10}$$

- The relation between Field strengths and is

$$E_{HL} = \sqrt{\epsilon_o}\, E_{MKS} \qquad \rightarrow \qquad \sqrt{\epsilon_o}\,(1\,\text{kV/cm}) = 0.2975\,\sqrt{N/m^2} \tag{A.11}$$

$$B_{HL} = \sqrt{\epsilon_o}\,(c B_{MKS}) = \frac{1}{\sqrt{\mu_o}} B_{MKS} \qquad \rightarrow \qquad \frac{1}{\sqrt{\mu_o}}(1\,\text{Tesla}) = 892.062\,\sqrt{N/m^2} \tag{A.12}$$