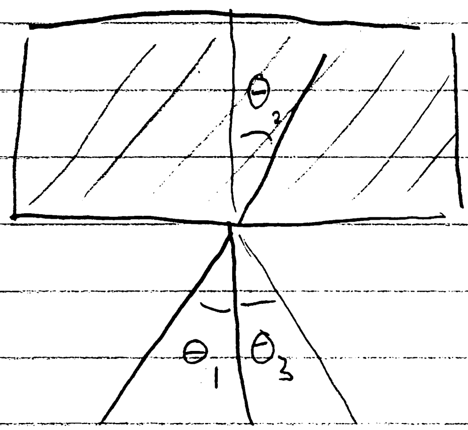
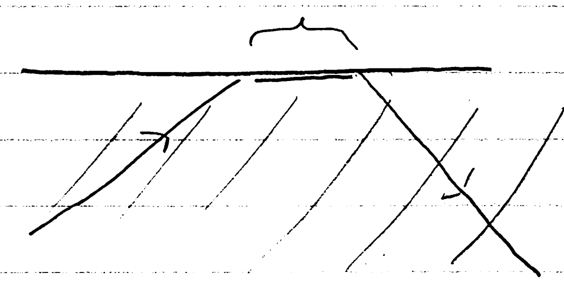
Reflection of Light at Interfaces - Introdution



Points to understand / derive :

- Snell's Law $\quad n_1 \sin\theta_1 = n_2 \sin\theta_2$

- Internal Reflection



This is known as the Goos-Hánchen effect and is similar to tunnelling.

when $n_1/n_2 \sin\theta_1 > 1$

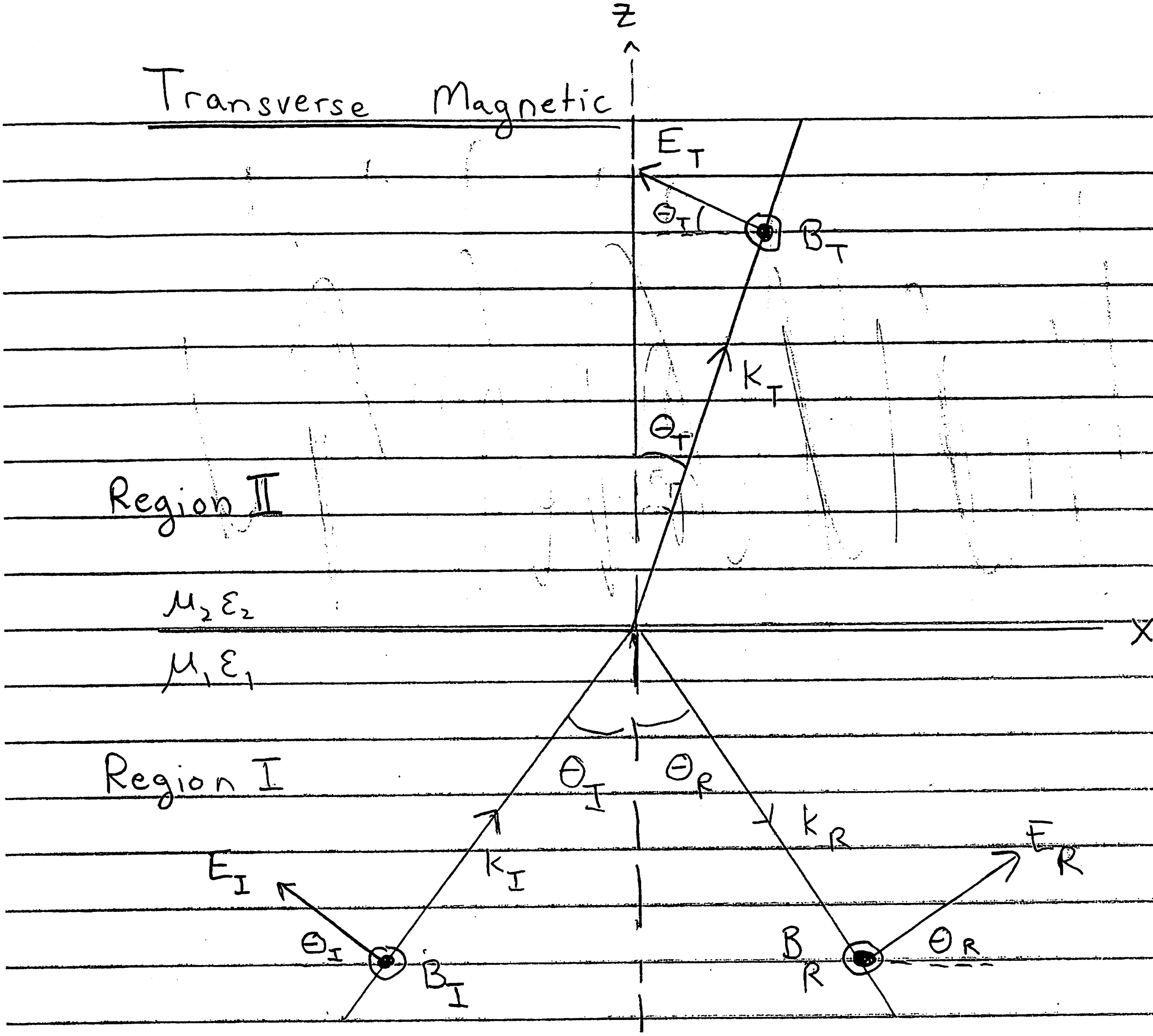
- Evaluate the forces on the interface, by evaluating the stress tensor

- How much light is reflected depends on the polarization of the incoming light. Depending on wether the magnetic or electric field points out of the scattering plane, (Transverse Magnetic or Transverse Electric -- see handout) more or less light Is reflected.

Thus unpolarized light will be partially polarized upon reflection

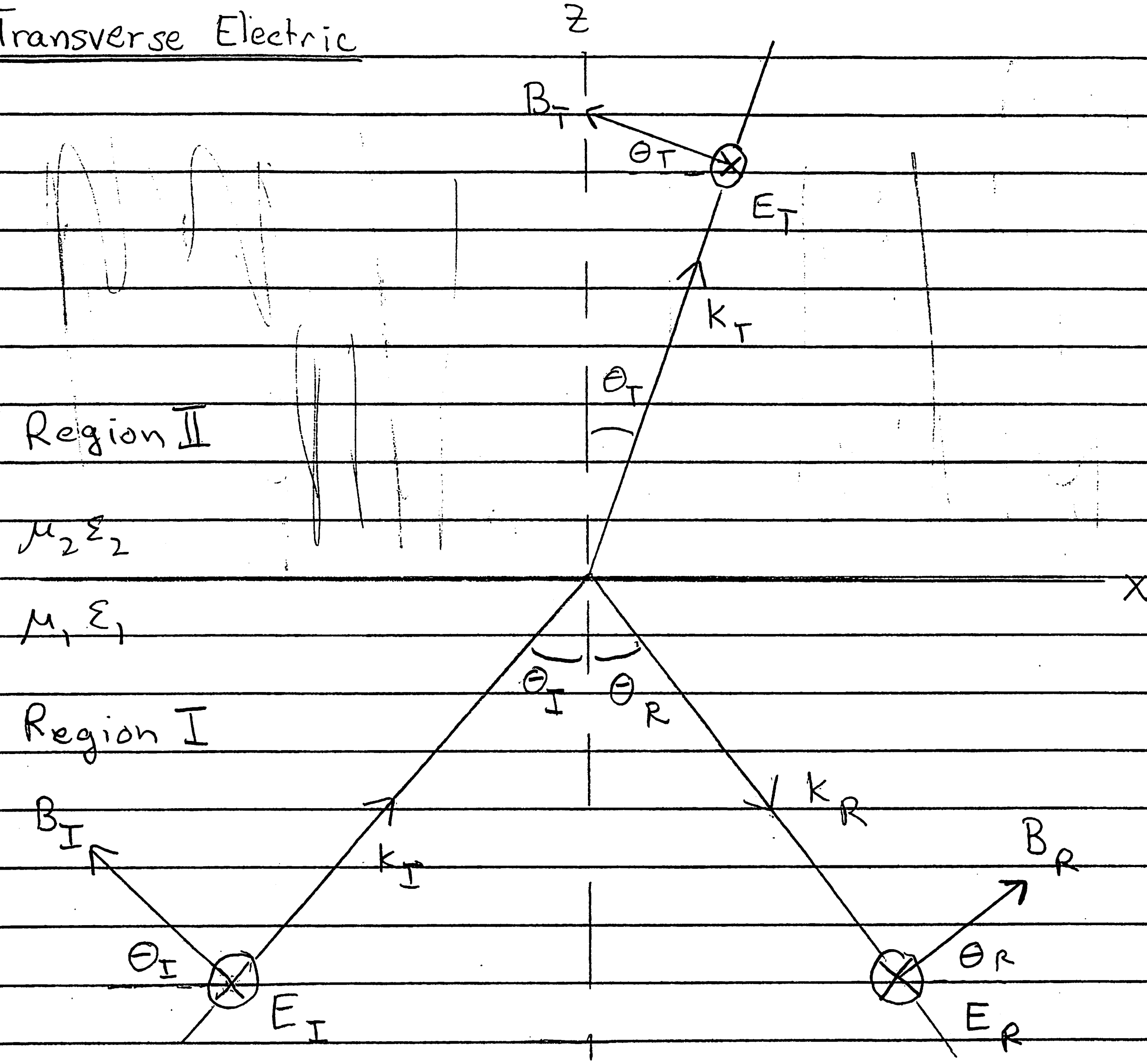
This is used by radio towers to select transmitted light.

Transverse Magnetic



In this case the magnetic field points out of the scattering plane.

Transverse Electric



The transverse electric case is the magnetic dual of the transverse magnetic case

$$E \to B$$

$$B \to -E$$

The E-field points into page ($\perp$ to scattering plane)

We will study the Transverse Magnetic Case:

- Basic idea write the solution in region II and region I as plane waves (sum of), and use boundary conditions to relate the two regions

Region II

polarization vector

$$\vec{E} = \vec{E}_T \, e^{i\vec{k}_T \cdot \vec{r} - i\omega t}$$

$$\vec{H} = H_T \, e^{i\vec{k}_T \cdot \vec{r} - i\omega t} (-\hat{y}) \leftarrow \text{out of page}$$

transverse magnetic case

see figure!

Region I

$$\vec{E}_1(t, \vec{r}) = \vec{E}_I \, e^{i\vec{k}_I \cdot \vec{r} - i\omega t} + \vec{E}_R \, e^{i\vec{k}_R \cdot \vec{r} - i\omega t}$$

$$\vec{H}_1(t, r) = \left(H_I \, e^{i\vec{k}_I \cdot \vec{r} - i\omega t} + H_R \, e^{i\vec{k}_R \cdot \vec{r} - i\omega t} \right)(-\hat{y})$$

Boundary Conditions

$$n \cdot (\vec{D}_2 - \vec{D}_1) = 0$$

$$n \times (\vec{H}_2 - \vec{H}_1) = 0 \quad \text{(Parallel components of } H \text{ continuous)}$$

$$n \cdot (\vec{B}_2 - \vec{B}_1) = 0$$

$$n \times (\vec{E}_2 - \vec{E}_1) = 0 \quad \text{(Parallel components of } E \text{ continuous)}$$

Solving the B.C:

- Has to hold at all times and for every point on the interface

$$i\vec{k}_I \cdot \vec{r} - i\omega t \Big|_{z=0} = i\vec{k}_R \cdot \vec{r} - i\omega t \Big|_{z=0} = i\vec{k}_T \cdot \vec{r} - i\omega t \Big|_{z=0}$$

- Frequencies have to be the same, So:

$$k_I = |\vec{k}_I| = |\vec{k}_R| = \frac{\omega n_1}{c}$$

$$|\vec{k}_T| = \frac{\omega n_2}{c}$$

Thus the wavelengths are related $k_T = \frac{n_2}{n_1} k_I$

- At $z=0$, $\vec{k} \cdot \vec{r} \Big|_{z=0} = k_x x = k \sin\theta \, x$, so must have:

$$k_I \sin\theta_1 = k_R \sin\theta_3 = k_T \sin\theta_2$$

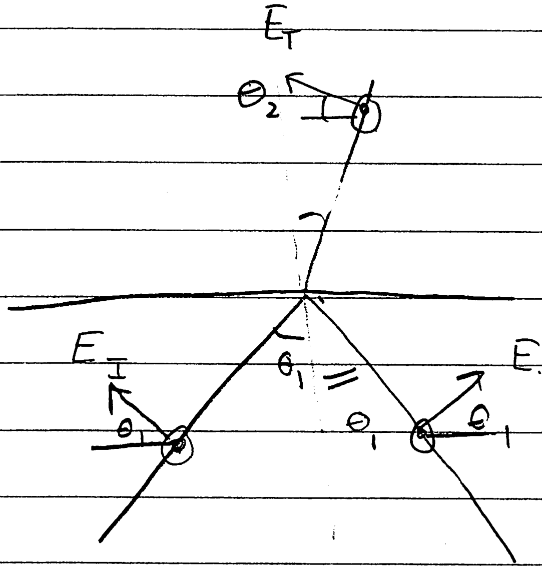
<u>Or</u>

$$\theta_1 = \theta_3 \quad \text{incident} = \text{reflected}$$

$$\sin\theta_1 = \frac{k_T}{k_I} \sin\theta_2$$

$$\sin\theta_1 = \frac{n_2}{n_1} \sin\theta_2 \quad \text{(snells law)}$$

Now $E_{//}$ continious and $H_{//}$ continuous



So the x-components of E are continuous

$$-E_T\cos\theta_2 - (-E_I\cos\theta_1 + E_R\cos\theta_1) = 0$$

And from continuity of H

$$H_T - (H_I + H_R) = 0$$

So H is related to E, $H = E/Z$

$$+\frac{E_T}{Z_2} - (E_I + E_R)/Z_1 = 0$$

Solving for the reflected + Transmitted amplitudes:

Find

$$\frac{E_R}{E_I} = \frac{Z_1 \cos\theta_1 - Z_2 \cos\theta_2}{Z_1 \cos\theta_1 + Z_2 \cos\theta_2} \xRightarrow[\text{on}]{\text{head}} \frac{Z_1 - Z_2}{Z_1 + Z_2}$$

$$\frac{E_T}{E_I} = \frac{2 Z_2 \cos\theta_1}{Z_1 \cos\theta_1 + Z_2 \cos\theta_2} \xRightarrow[\text{on}]{\text{head}} \frac{2 Z_2}{Z_1 + Z_2}$$

Now we want to analyze this:

- Energy Transport

$$\vec{S} = \frac{1}{2} c \, E \times H^* = \frac{c}{2} \frac{1}{Z} |E|^2 \hat{k}$$

time averaged poynting flux

So the transmitted power, relative to the input power:

$$T_p = \frac{\vec{S}_T \cdot \vec{n}}{\vec{S}_I \cdot \vec{n}} = \frac{\cos\theta_2}{\cos\theta_1} \frac{Z_1}{Z_2} \frac{|E_T|^2}{|E_I|^2}$$

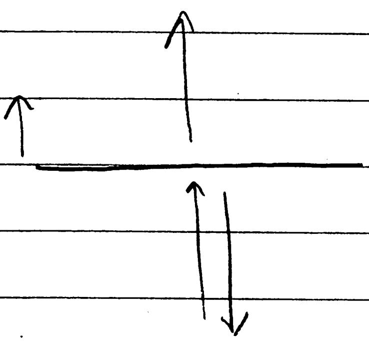
$$\xRightarrow[\text{on}]{\text{head}} \frac{4 Z_1 Z_2}{(Z_1 + Z_2)^2}$$

$$R_p = \frac{\vec{S}_R \cdot (-\vec{n})}{\vec{S}_I \cdot \vec{n}} = \frac{\cos\theta_1}{\cos\theta_1} \frac{Z_1}{Z_1} \frac{|E_R|^2}{|E_I|^2} \xRightarrow[\text{on}]{\text{head}} \frac{(Z_1 - Z_2)^2}{(Z_1 + Z_2)^2}$$

Find generally

$$R_p + T_p = 1$$

Momentum Transport:



$n_2 = \text{something}$, $\mu_2 = 1$, $Z_2 = \frac{1}{n_2} \equiv \frac{1}{n}$

$n_1 = 1$, $\mu_1 = 1$, $Z_1 = 1$

$$\frac{\text{Force}}{\text{Area}} = -\left(T^{zz}_{out} - T^{zz}_{in}\right)$$

$H = E/\underset{\sim}{Z}$

$$T^{zz} = \frac{\varepsilon}{2}\left(\cancel{-E^z E^{z*}} + \frac{1}{2} E\cdot E^* \delta^{zz}\right) + \frac{\mu}{2}\left(\cancel{-H^z H^{z*}} + \frac{1}{2}\vec{H}\cdot\vec{H}^* \delta^{zz}\right)$$

time ave

$$= \frac{\varepsilon}{4}|E|^2 + \frac{\mu}{4}\frac{1}{\mu/\varepsilon}\vec{E}\cdot\vec{E} = \frac{\varepsilon}{2}\vec{E}\cdot\vec{E}^*$$

So

$$T^{zz}_{out} = \frac{\varepsilon_2}{2} E_T^2 = \frac{\varepsilon_1 E_I^2}{2}\left(\frac{\varepsilon_2}{\varepsilon_1}\left|\frac{E_T}{E_1}\right|^2\right)$$

$$= \langle u_I \rangle \, (n T_p)$$

index of refraction

incident energy density

transmission coefficient

Similarly

$$T^{zz}_{in} = \frac{\varepsilon}{2} (E_I + E_R) \cdot (E_I + E_R)^*$$

$$T^{zz}_{in} = \underbrace{\frac{1}{2} \varepsilon_I E_I^2}_{\langle u_I \rangle} (1 + R_p)$$

Find

$$\frac{\langle \text{Force} \rangle}{\text{Area}} = \langle u_I \rangle \left[1 + R_p - n T_p \right]$$

So for $\mu \simeq 1$ and $n = 1/z$

$$\langle \text{Force}/\text{Area} \rangle = \langle u_I \rangle \; \frac{2(n-1)}{n+1} \xrightarrow[n \to \infty]{} 2 \langle u_I \rangle$$

Total reflection

Note:

$$\langle u_I \rangle = c \langle g_{em} \rangle = \langle S \rangle / c$$

energy density $\simeq \frac{m}{s}$ $\frac{\text{momentum}}{\text{vol}}$ - $\frac{\text{Energy}}{\text{Area} \cdot \text{time}} \cdot \frac{1}{m/s}$