Problem 1. A particle in a magnetic field

(a) Write down the Lagrangian and Hamiltonian for a particle in a magnetic field, $\boldsymbol{B}(\boldsymbol{r})$. Compute the Poisson brackets of velocity:

$$\{v_i, v_j\}$$

(b) Prove that the value of any function $f(q(t), p(t))$ of coordinates and momenta of a system at a time t can be expressed in terms of the values of p and q at $t = 0$ as follows:

$$f = f_0 + \frac{t}{1!}\{f_0, H\} + \frac{t^2}{2!}\{\{f_0, H\}, H\} + \dots\,, \tag{1}$$

where $f_0 = f(p(0), q(0))$. Apply this formula to evaluate $p^2(t)$ for a harmonic oscillator.

(c) Evaluate $\boldsymbol{v}(t)$ for a particle in a constant magnetic field $\boldsymbol{B}_0$ using the results of this problem.

Problem 2. Canonical transformations and Poisson Brackets

Consider an infinitesimal change of coordinates, which is not necessarily canonical:

$$q \to Q = q + \lambda \frac{dQ(q,p)}{d\lambda}\,, \tag{2}$$

$$p \to P = p + \lambda \frac{dP(q,p)}{d\lambda}\,. \tag{3}$$

Show that if the Poisson bracket is to remain fixed under the transformation, i.e.

$$\{Q, P\} = 1\,, \tag{4}$$

$$\{P, P\} = 0\,, \tag{5}$$

$$\{Q, Q\} = 0\,, \tag{6}$$

then there must exist a $G(q, p)$ which generates the transformation. (Hint recall the following theorem: if a vector field is curl free, $\nabla \times \boldsymbol{v} = 0$ it may be written as a gradient of a scalar function, $\boldsymbol{v} = -\nabla\phi$.)