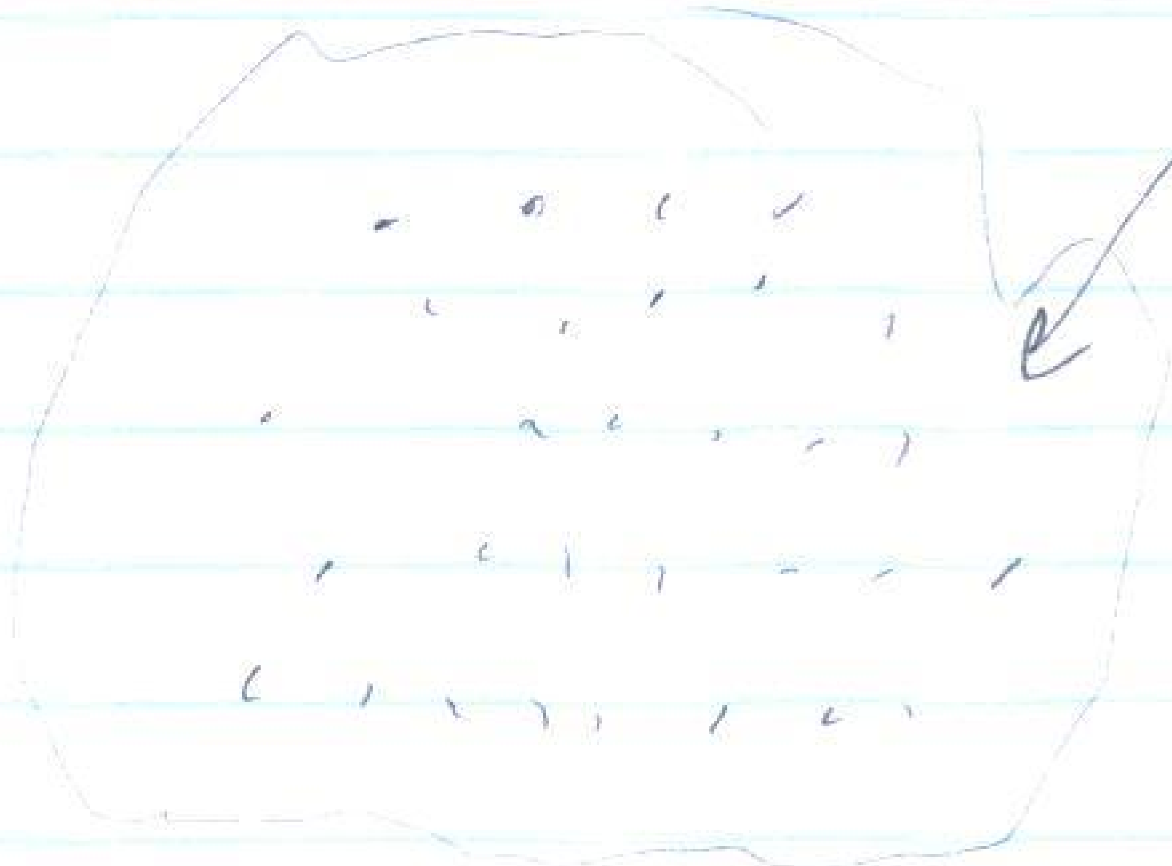
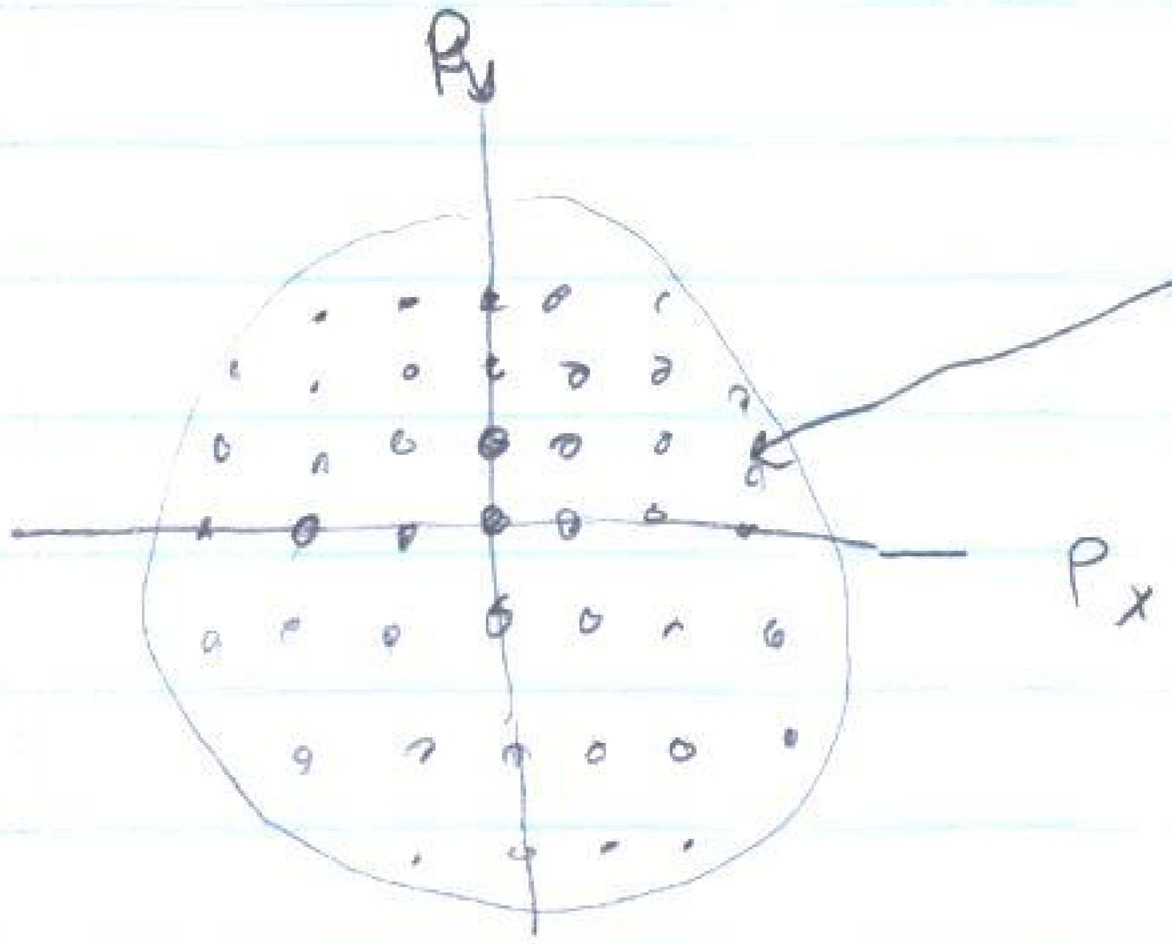
Now consider nuclear matter



Size V, # N



fill; all states up to p_f

$$N = \sum_{p}^{p_f} = \int \frac{V\, d^3p}{(2\pi\hbar)^3} = \frac{V}{(2\pi)^3} \frac{4}{3}\pi\, p_f^3$$

$$\frac{N}{V} = \frac{1}{2\pi^2} \frac{p_f^3}{3\hbar^3}$$

Estimate of Fermi momentum for nuclear matter

$$\rho_0 = \frac{N}{V} = \frac{1}{6}\,\frac{\text{nucleons}}{fm^3} = \frac{1}{6\pi^2}\left(\frac{p_f}{\hbar}\right)^3$$

$$\frac{p_f}{\hbar} = \left(6\pi^2 \rho_0\right)^{1/3} = 2.14\,\frac{1}{fm}$$

$$c p_f = 2.14\,\frac{1}{fm}\cdot \overbrace{\hbar c}^{200\ \text{MeV}\,fm}$$

$$c p_f \approx 400\ \text{MeV}$$

quite large

doesn't incl
spin-iso-s

$$\frac{BE}{A} \simeq 8\ \frac{\text{MeV}}{A}$$

with this
$\simeq 200$ MeV

Problem Compute The average Energy

$$\langle E \rangle = \sum_{p}^{p_f} \frac{p^2}{2m} = \int^{p_f} \frac{V\,d^3p}{(2\pi)^3\hbar^3}\,\frac{p^2}{2m}$$

$$= \frac{V}{2\pi^2}\int^{p} p^2\,dp\,\frac{p^2}{2m} = \frac{V}{2\pi^2}\,\frac{1}{2m}\,\frac{1}{\hbar^3}\,\frac{p_f^5}{5}$$

$$\frac{\langle E \rangle}{\langle N \rangle} = \frac{3}{5}\,\frac{p_f^2}{2m}$$

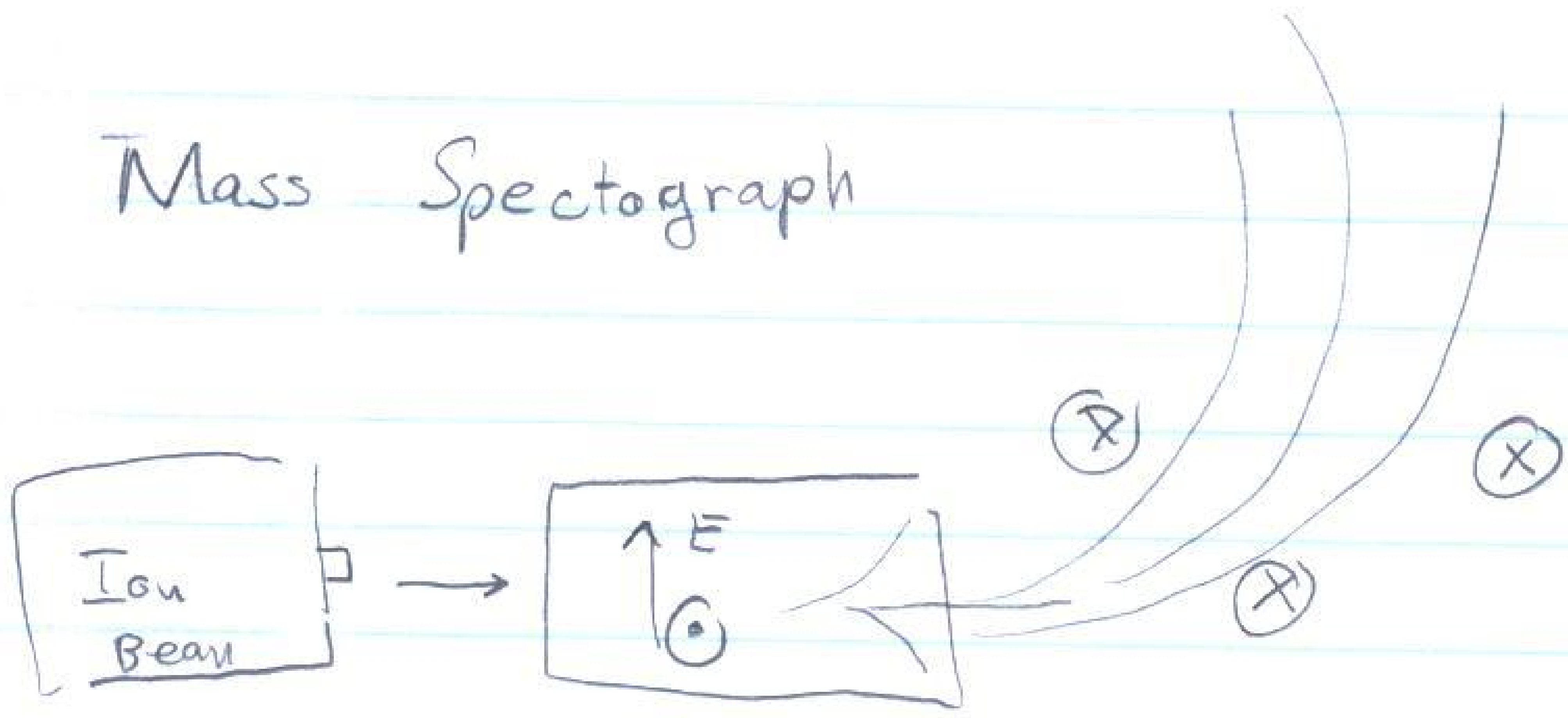
$$\frac{\langle E \rangle}{\langle N \rangle} \simeq 51.1 \text{ MeV}$$

much more compatible with spin iso-spin

$$BE \simeq 8 \frac{\text{MeV}}{A}$$

$\simeq 25$ MeV

Mass Spectograph



$F = qE$ up $\qquad F = qvB$ down

Straight Line:

$$\cancel{q}E = \cancel{q}vB$$

$$\frac{E}{B} = v$$

Now we enter a magnetic Field:

$$\frac{mv^2}{R} = qBv \qquad R = \frac{mv}{qB}$$

$$\boxed{m = \frac{qRB^2}{E}}$$

Doublet method

Compare Two species @ same mass

C_9H_{20} and $C_{10}H_8$

$$\Delta = [m(C_9H_{20}) - m(C_{10}H_8)]$$

$$\Delta = 12\,m(H) - m(^{12}C)$$ ← measured precisely

$$m(H) = \frac{1}{12}[m(^{12}C) + \Delta]$$

$$m(H) = 1.00782503 \pm .00000001$$

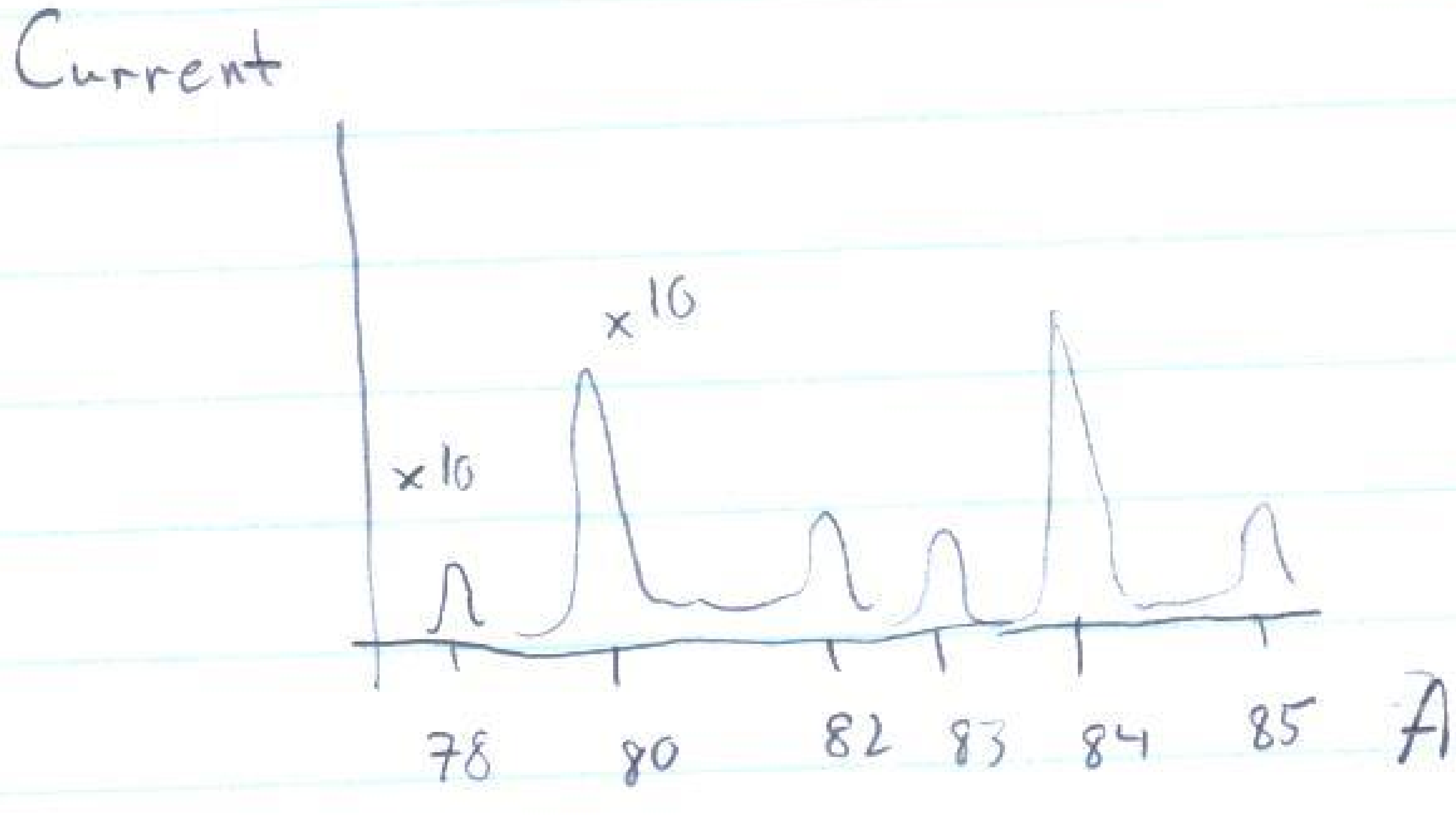
Then to determine say ^{14}N look at C_2H_4 and N_2

$$\Delta' = m(C_2H_4) - 2m(^{14}N)$$ ← measured precisely

$$m(^{14}N) = m(^{12}C) + 2m(^1H) - \frac{1}{2}\Delta$$

$$= 14.00307396 \pm 0.00\ .0000002$$

The spectro-graph can also be used to measure the relative abundance



$^{78}Kr = 0.356\%$

$^{80}Kr = 2.27\%$

$^{82}Kr = 11.6\%$

$^{83}Kr = 11.5\%$

$^{84}Kr = 57\%$

$^{86}Kr = 17.3\%$

Determining masses of Unstable Nuclei

$$x + X \rightarrow y + Y$$

$$Q = \left[m(x) + M(X) - m(y) + M(Y) \right] c^2$$

Measure the kinetic energy of outgoing constituents, and incoming energies

Example:

$$({}^1H) + {}^{14}N \rightarrow {}^{12}N + {}^3H$$

Measure the kinetic energies $Q = -22$ MeV

$$m({}^{12}N) = m({}^1H) + m({}^{14}N) - m({}^3H) - Q$$

Now we can fit the masses

$$B(A, Z) = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_{sym} A \left(1 - \frac{2Z}{A}\right)^2$$

Volume $\frac{B}{A}$ Const   0.72 MeV   34 MeV

Then there is a coulomb repulsion:

$$V = \frac{k_e Q}{R^2}\left(r - \frac{1}{2}\frac{r^2}{R}\right)$$

$$U_{\substack{electro\\stat}} = \int 4\pi r^2 \, dr \, \rho \, \frac{k_e q(r)}{r}$$

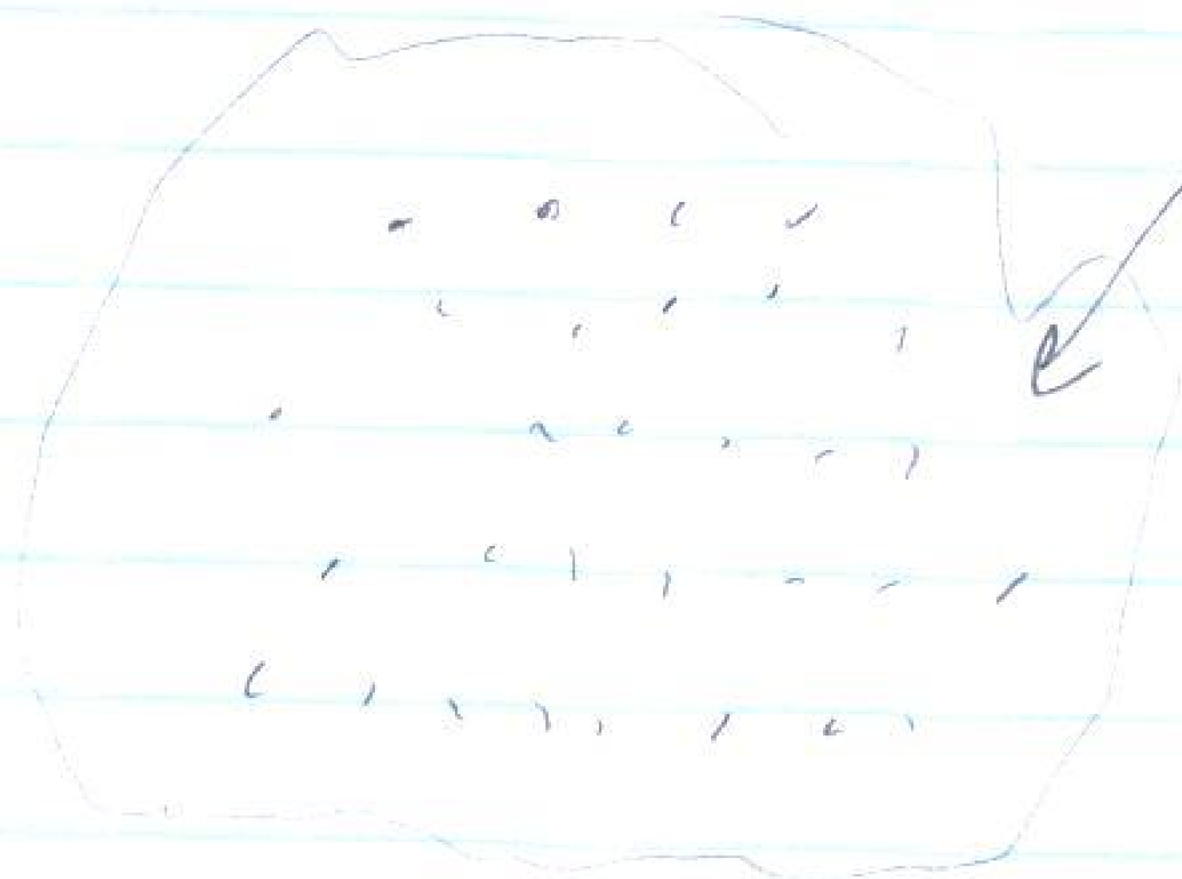
$$= 4\pi k_e \rho \int_0^R \frac{4}{3}\pi r^4 \rho \, dr$$

$$= 4\pi k_e \frac{Q^2}{\left(\frac{4}{3}\pi R^3\right)^2} \frac{4\pi}{3} \frac{R^5}{5} = \boxed{\frac{3}{5} k_e \frac{Q^2}{R} = \text{Potential Energy of a charged Sphere of radius } R}$$

$$U_{\substack{elect\\stat}} = \frac{3}{5} \frac{k_e e^2}{\hbar c} \cdot \frac{\hbar c}{R_A}$$

$$= \frac{3}{5} \underbrace{\alpha}_{\frac{1}{137}} \cdot \frac{197 \text{ MeV fm}}{1.2 \text{fm } A^{1/3}} = 0.72 \text{ MeV}$$

Now consider nuclear matter



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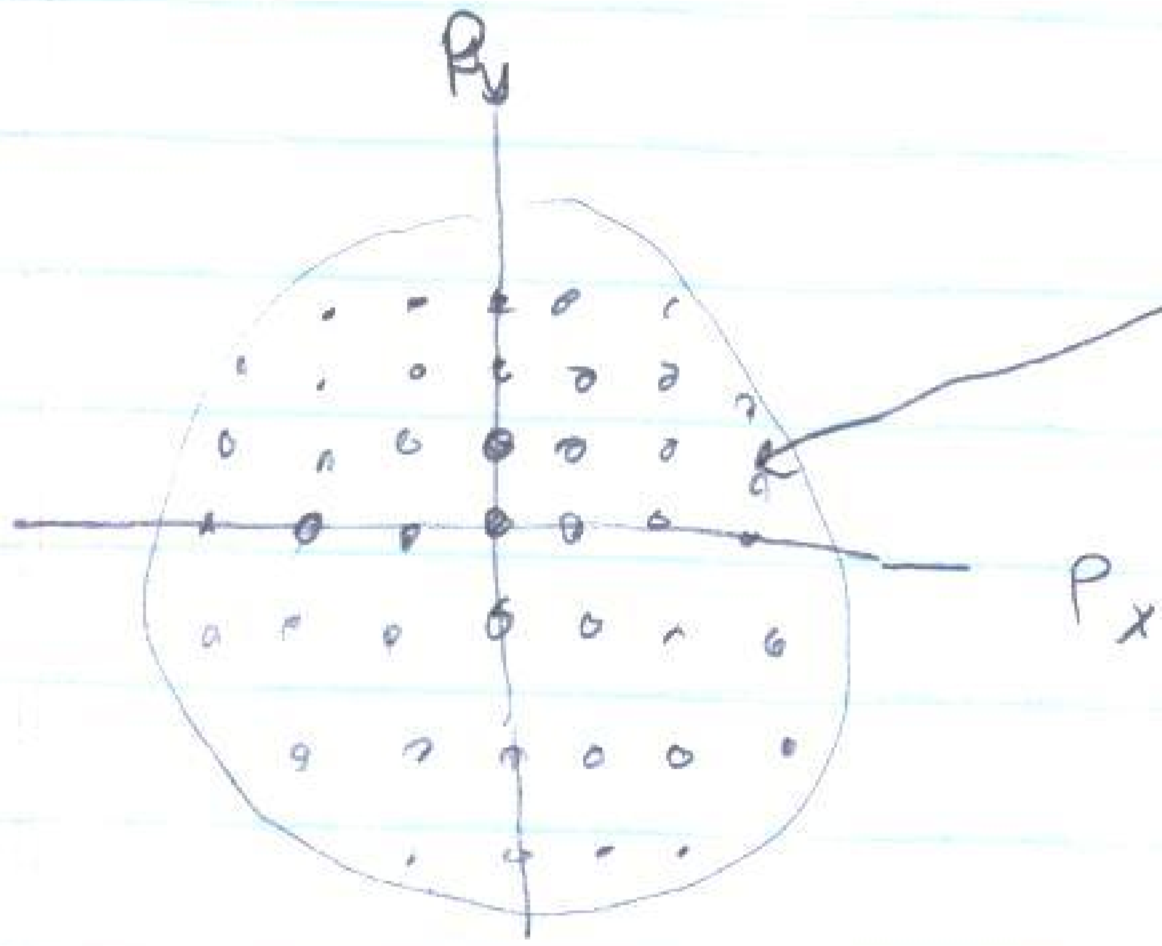

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