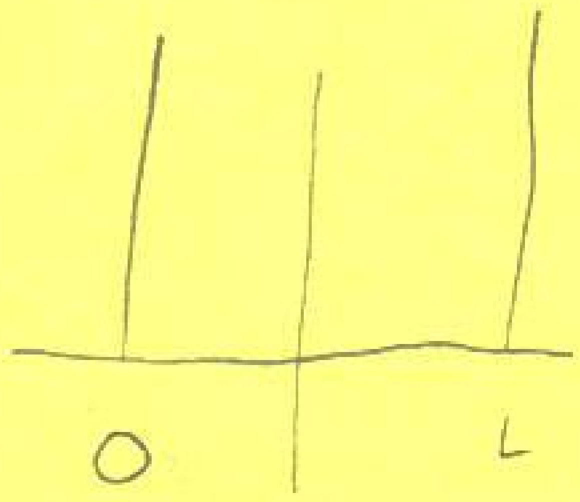
Eigenfunctions of hamiltonian



infinite square well

$$H\,\phi_E = E\,\phi_E$$

$$\left[-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \cancel{V(x)}\right]\phi_E = E\,\phi_E$$

$$\phi_E = \begin{cases} A\cos kx \\ A\sin kx \end{cases}$$

$$\frac{\hbar^2 k^2}{2m} = E$$

We also need boundary Conditions

$$\phi_E(0) = 0 = \phi_E(L)$$

- sin only
- $k = \frac{\pi}{L} n \qquad n = 1, 2, \ldots$

norm

$$\int \phi^*_{E_1} \phi_{E_1} \, dx = 1$$

$$\int_0^L A \sin^2 kx = \frac{A^2 L}{2} = 1 \Rightarrow A = \sqrt{\frac{2}{L}}$$

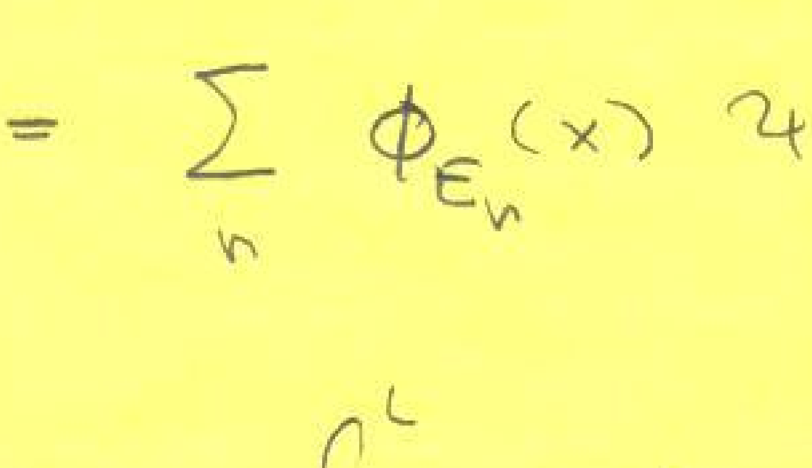
$$\phi_{E_n}(x) = \sqrt{\frac{2}{L}} \sin kx \qquad k = \frac{\pi n}{L} \qquad n = 1, 2, \ldots$$

$$E_n = \frac{\hbar^2 k^2}{2m}$$

Consider a wave function:

norm

$$\psi(x, t_0) = \begin{cases} \sin \frac{2\pi x}{L} \cdot \sqrt{\frac{4}{L}} & x < L/2 \\ 0 & x > L/2 \end{cases}$$



$$\psi(x, t_0) = \sum_n \phi_{E_n}(x) \, \psi(E_n, t_0)$$

$$\psi(E_n, t_0) = \int_0^L dx \, \phi^*_{E_n}(x) \, \psi(x, t) = \int_0^{L/2} dx \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L} \sin \frac{2\pi}{L} \cdot \sqrt{}$$

$$= \sqrt{8}/2\pi \quad \frac{\sin(n\pi/2)}{1 - (n/2)^2}$$

Then why do we care about expansion in Eigen funcs?

Answer: suppose at $t = t_0$ we have

$$\psi(x, t_0) = \sum_n \phi_{E_n}(x)\, \psi(E_n, t_0)$$

Then Later we have

$$\psi(x, t) = \sum_n \Phi_{E_n}(x)\, e^{-\frac{iE_n}{\hbar}(t - t_0)}\, \psi(E_n, t_0)$$

Proof:

$$\hat{H}\psi = +i\hbar \frac{\partial}{\partial t}\psi(x,t)$$

$$\hat{H} \sum_n \Phi_{E_n}(x)\, \psi(E_n, t) = +i\hbar\, \partial_t \sum_n \Phi_{E_n}(x)\, \psi(E_n, t)$$

$$\sum_n E_n \Phi_{E_n}(x)\, \psi(E_n, t) = +i\hbar \sum_n \phi_{E_n}(x)\, \partial_t \psi(E_n, t)$$

Multiply by $\phi^*_{E_{n'}}(x)$ and integrate $\int_x$,

$$E_{n'}\, \psi(E_{n'}, t) = i\hbar\, \partial_t \psi(E_{n'}, t)$$

solution $\longrightarrow$ $$\psi(E_n, t) = e^{-iE_n(t - t_0)/\hbar}\, \psi(E_n, t_0)$$

$$\boxed{\psi(E_n, t) = e^{-iE_n(t-t_0)/\hbar}\, \psi(E_n, t_0)}$$

$$\boxed{\psi(x,t) = \sum_n \phi_n(x)\, e^{-iE_n(t-t_0)/\hbar}\, \psi(E_n, t_0)}$$

Our example

$$\psi(x,t) = \sum_n \overbrace{\sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right)}^{\phi_n(x)} \cdot \overbrace{e^{-\frac{i\hbar^2\pi^2 n^2 t}{2mL^2}}} \cdot \frac{\sqrt{8}}{2\pi} \frac{\sin(n\pi/2)}{1-(n/2)^2}$$

See Links on web page for movie.

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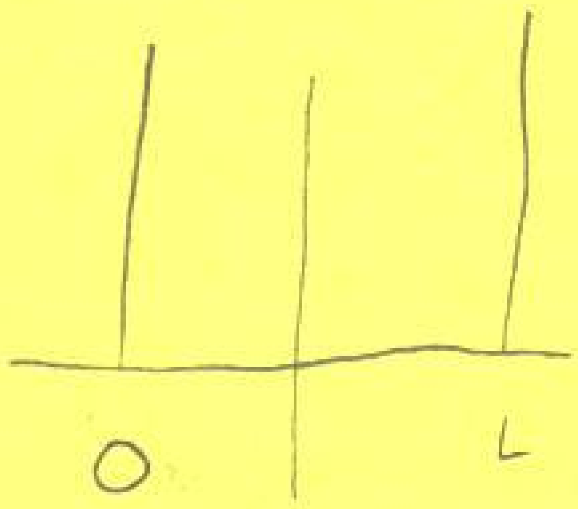

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