## Problems:

Underlines are to be handed in.

## Sources of Magnetic Field

- Currents produce magnetic fields.

$$
\begin{equation*}
d \mathbf{B}=\frac{\mu_{o}}{4 \pi} \frac{I \mathrm{~d} \boldsymbol{\ell} \times \hat{\mathbf{r}}}{r^{2}} \tag{2}
\end{equation*}
$$

with

$$
\mu_{o}=(4 \pi) 1 \times 10^{-7} \mathrm{~T} \cdot \mathrm{~m} / \mathrm{A}
$$

Another unit for $B$ field is gaus 1 gaus $=1 \times 10^{-4} \mathrm{~T}$

- This can be used to determine the B field for some special geometries. Another unit for

1. For a single wire carrying a current $I$ the B-field wraps around the wire as given by the right hand rule

$$
\begin{equation*}
B=\frac{\mu_{o} I}{4 \pi a}\left(\cos \left(\theta_{1}\right)-\cos \left(\theta_{2}\right)\right) \tag{3}
\end{equation*}
$$


2. For an infinitely long wire this becomes

$$
\begin{equation*}
B=\frac{\mu_{o} I}{2 \pi a} \tag{4}
\end{equation*}
$$

You should be able to derive this using Amperes law (see below)

3. For a circular arc of angle $\theta$ in radians the magnetic field at the center of the arc is.

$$
\begin{equation*}
B=\frac{\mu_{o} I}{4 \pi R} \theta \rightarrow \frac{\mu_{o} I}{2 R} \tag{5}
\end{equation*}
$$

The last expression after the arrow is for a closed ring $\theta=2 \pi$. You should be able to derive this from the Biot-Savat Law.


- For two infinite length wires carrying currents $I_{1}$ and $I_{2}$ and separated by a distance $a$ the force per unit length is

$$
\begin{equation*}
\frac{F}{\ell}=\frac{\mu_{o} I_{1} I_{2}}{2 \pi a} \tag{6}
\end{equation*}
$$

If the two currents point in the same direction the force is attractive. It is repulsive if the current points in the opposite direction. You should be able to derive this form the magnetic field and boxed above and the force on a wire in a mgnetic field

$$
\begin{equation*}
\mathbf{F}=I \mathrm{~d} \boldsymbol{\ell} \times \mathbf{B} \tag{7}
\end{equation*}
$$

- Ampere's Law says that the line integral of $\mathbf{B} \cdot \mathrm{d} \boldsymbol{\ell}$ around a closed loop is $\mu_{o} I$

$$
\begin{equation*}
\oint \mathbf{B} \cdot \mathrm{d} \boldsymbol{\ell}=\mu_{o} I+\text { Maxwell Term } \tag{8}
\end{equation*}
$$

When applying this rule you want to choose the direction of the loop according to the right hand rule. That is curl fingers so that thumb points in the direction of current. The maxwell term can be neglected provided everything is static (i.e. that currents and charges do not depend on time) You will need it later in the course.

- You can use Ampere's law to derive the magnetic field in some geometries

1. The infinitely long wire given above
2. The Magnetic field inside a solenoid

$$
\begin{equation*}
B=\mu_{o} \frac{N}{L} I \tag{9}
\end{equation*}
$$

You should be able to derive this from Ampere's law. It is zero outside the solenoid. See page 939 for help. $N$ is the number of turns and $L$ is the length of the coil. The magnetic field is $\propto \#$ turns per length.


- The total magnetic flux through a closed surface is zero

$$
\begin{equation*}
\oint \mathbf{B} \cdot \mathrm{d} \mathbf{A}=0 \tag{10}
\end{equation*}
$$

