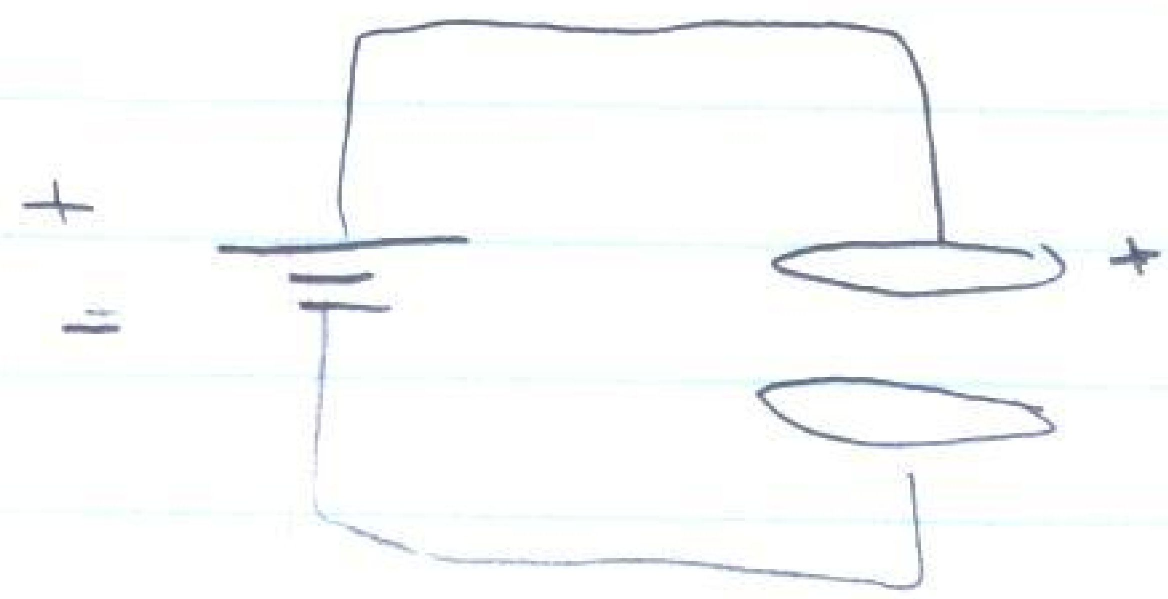
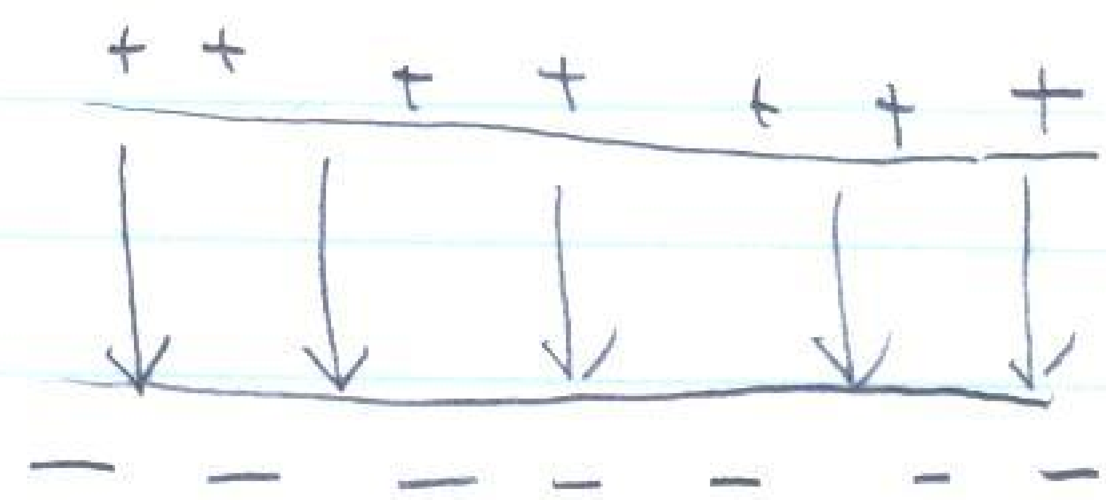
Capacitance:





$Q \propto \Delta V$

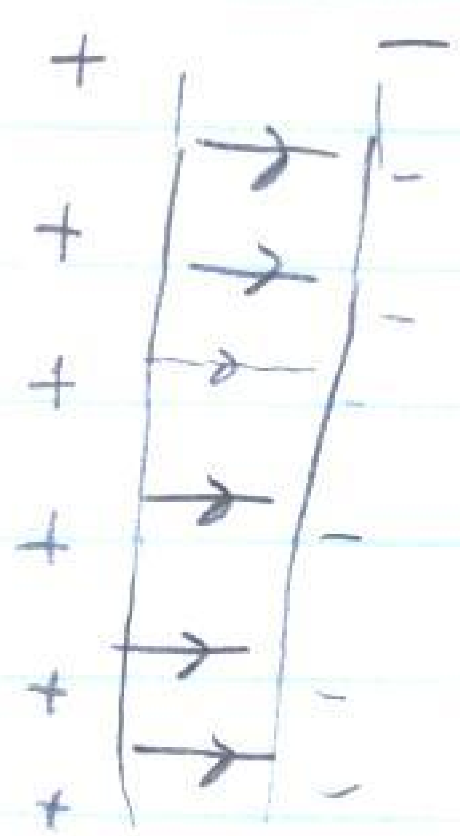
$Q = C\Delta V \Rightarrow \frac{Q}{\Delta V} = C \Rightarrow$ Units $= \frac{\text{Coulomb}}{V}$

$= 1\ F$

enormous

Parallel Plate Cap

No field



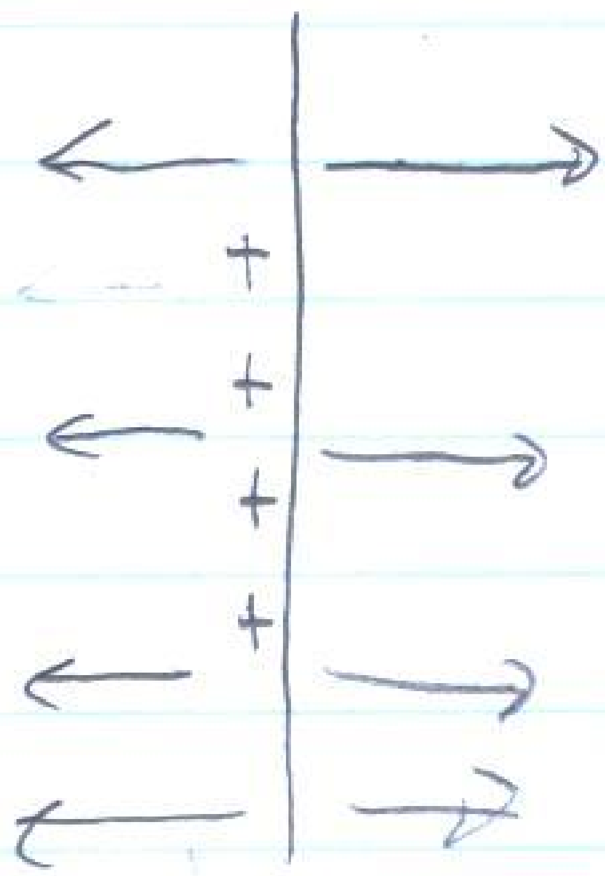
No field

$= 1\ \mu F \approx 1\times 10^{-6}$

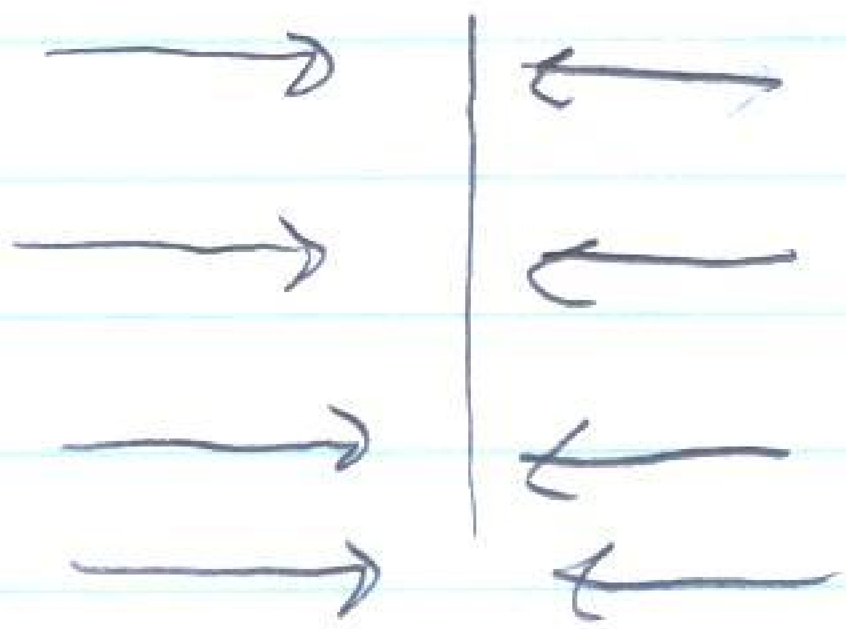
typical

$= 1\ pF \approx 1\times 10^{-12}$

Small



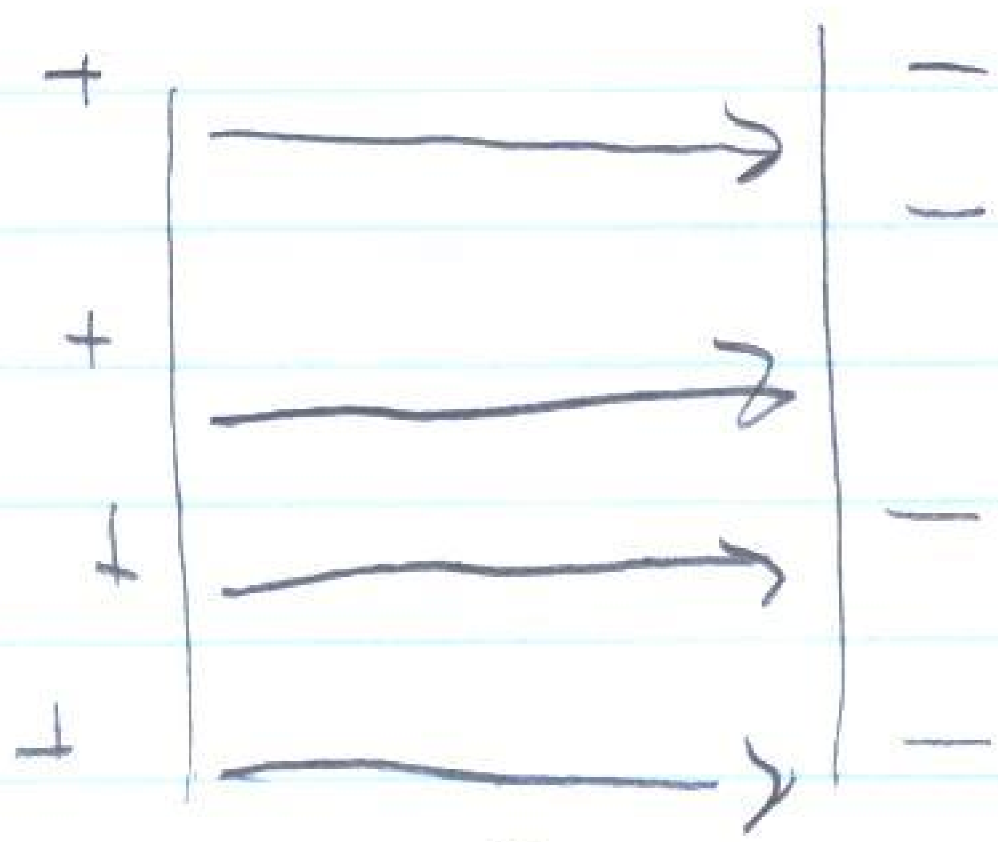
$$E = \frac{\sigma}{2\varepsilon_0}$$



$$E = \frac{\sigma}{2\varepsilon_0}$$

+

=



$$E = \frac{\sigma}{\varepsilon_0}$$

$\Delta V = Ed$ so $C = \frac{Q}{\Delta V} = \frac{\cancel{Q}}{\frac{\cancel{Q}}{A}\frac{d}{\varepsilon_0}}$

$= \frac{\sigma}{\varepsilon_0} d = \frac{Q}{A}\frac{d}{\varepsilon_0}$

$$\boxed{C = \frac{A\varepsilon_0}{d}}$$

- Its a function of Geometry only!

Units

Example

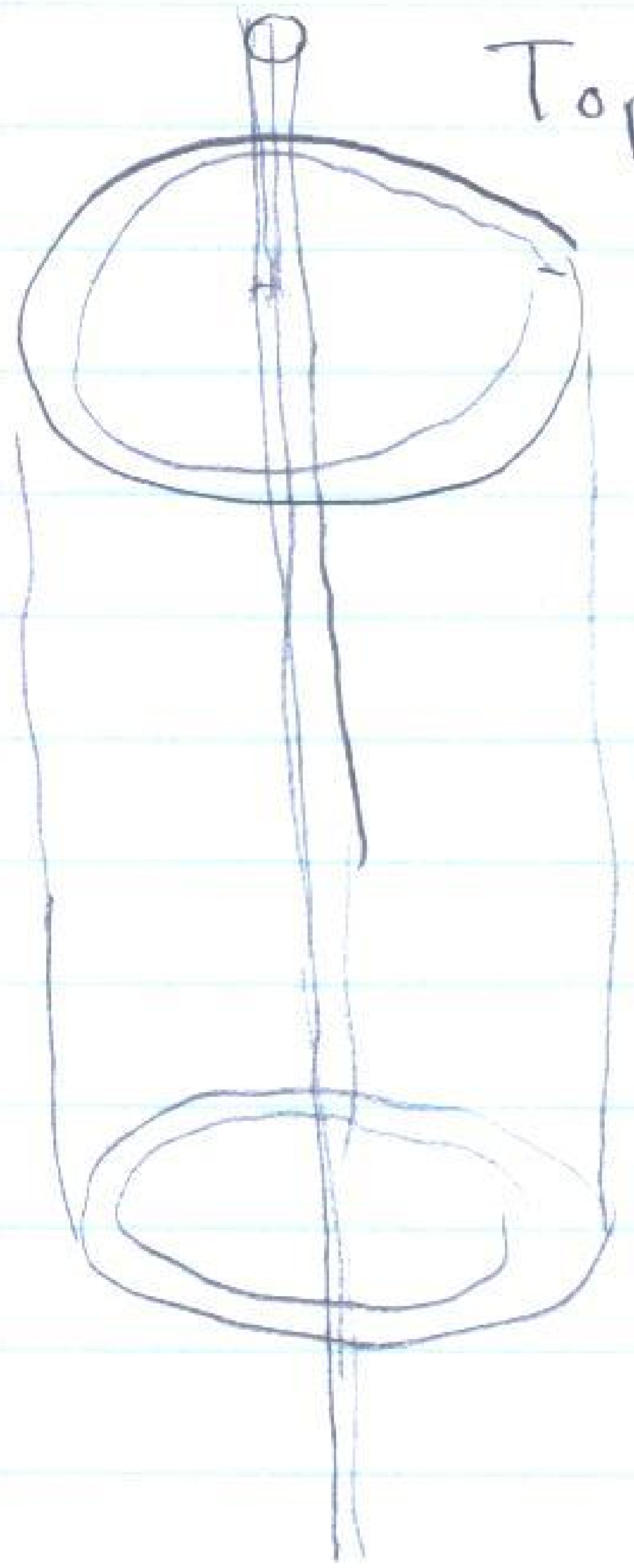
A parallel plate cap has area $2\ cm^2$ and a separation of $1\ mm$. Calculate C

$$C = \frac{\varepsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}\ C^2/Nm^2)(2 \times 10^{-4}\ m^2)}{1 \times 10^{-3}\ m}$$

$$= 1.77 \times 10^{-12}\ F = 1.77\ pF$$

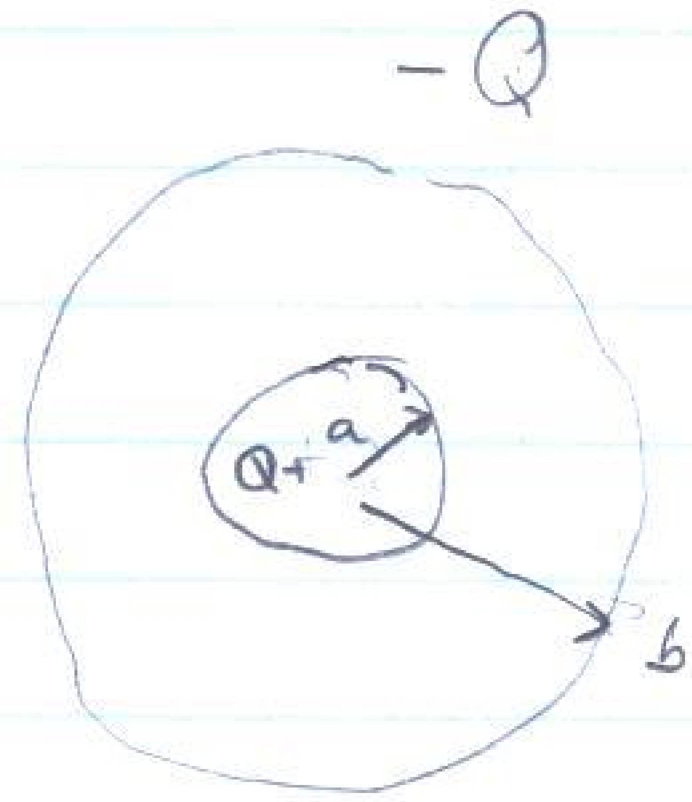
- Typical Construction
  - Polystrene / Paper → add coated, with metal
  - Ceramic / metal

Coaxial Cable



Top

Side :



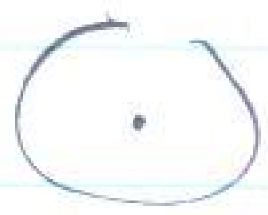
Find the capacitance of the cable:

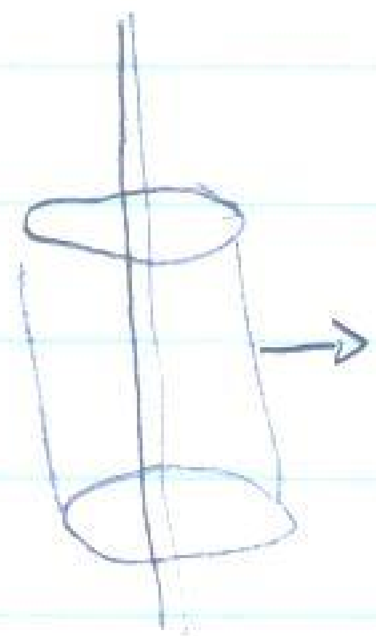
- Calculate the Electric Field / Potential gauss

You will
- Find $V \propto \# Q$ (use $Q = \sigma A$, $Q = \lambda L$ et

$$C = \frac{V}{Q} = \#$$

Solution



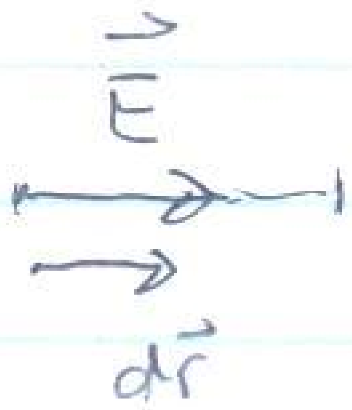


Gauss Law:

$$E \cdot \overbrace{(2\pi r)}^{\text{Area}} L = \overbrace{4\pi}^{\frac{1}{\varepsilon_0}} k_e \left(Q_{enc} \right)$$

$$E = \frac{\cancel{4\pi}^{\,2} k_e Q}{\cancel{2\pi} r L} = \frac{2 k_e Q}{L} \frac{1}{r}$$

$$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{r} = -\int_a^b \frac{2 k_e Q}{L} \frac{1}{r} \, dr = \frac{2 k_e Q}{L} \log r \Big|_a^b$$



$$V_b - V_a = -\frac{2 k_e Q}{L} (\log b - \log a) = -\frac{2 k_e Q}{L} \ln \frac{b}{a}$$

$$\Delta V = +\frac{2 k_e Q}{L} \ln \frac{b}{a}$$

$$C = \frac{Q}{\Delta V} = \frac{\cancel{Q}}{\frac{2 k_e \cancel{Q}}{L} \ln \frac{b}{a}} = \frac{L}{2 k_e \ln \frac{b}{a}}$$

Usually we are more interested in:

$$C/L = \text{Capacitance}/\text{Length} = \frac{1}{2 k_e \ln b/a}$$

Numerically: $b/a \sim 10$

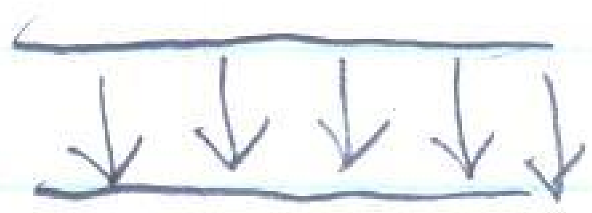
$$\ln 10 = 2.3$$

$$\frac{C}{L} = \frac{1}{2k_e} \cdot \frac{1}{\ln b/a} \simeq 24 \ \frac{pF}{m}$$

~ Amateur radio people say that coaxial cable has

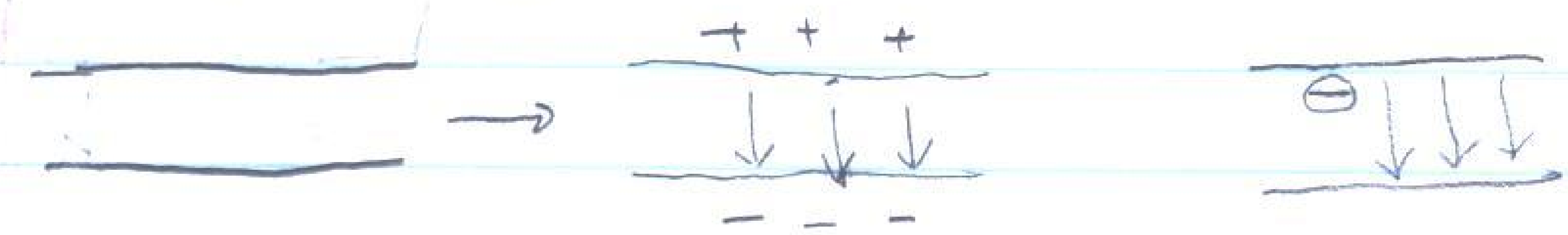
$$\frac{C}{L} \sim 12 pF / \text{foot}$$

Why do we care about capacitance?

 Because Energy is stored

Energy Stored in a Capacitor:

Thought Exp. imagine pulling electrons



$C\,\Delta V = q \leftarrow$ potential build up

$dW_{I\,do} = \Delta V\, dq \leftarrow$ next dq charge transfe

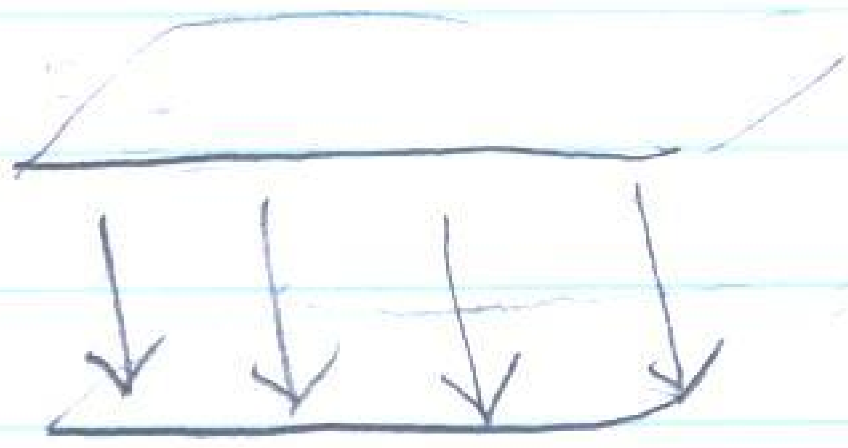
$$dW_{I\,do} = \frac{1}{C}\, q\, dq$$

$$W_{\substack{tot\\ I\,do}} = \int dW = \int_0^Q \frac{1}{C}\, q\, dq = \frac{1}{2}\frac{1}{C} Q^2$$

$$PE = W_{\substack{I\,do\\ \text{on Cap}}} = \frac{1}{2}\frac{1}{C}(C\,\Delta V)^2 = \frac{1}{2} C\,\Delta V^2$$

$$\boxed{PE = \frac{1}{2} C(\Delta V)^2 = \frac{1}{2}\frac{Q^2}{C}}$$

Then, the volume is

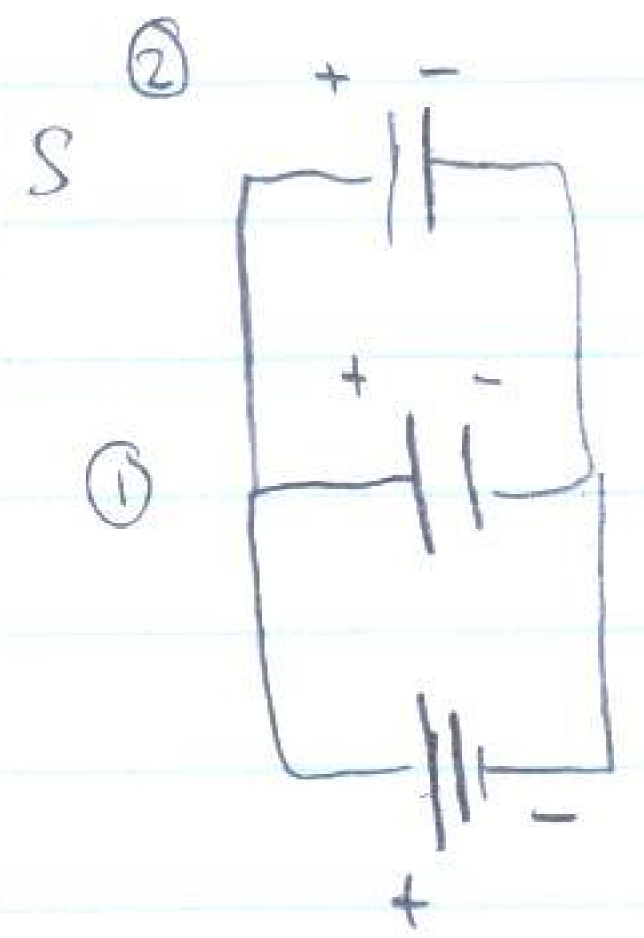


$$\frac{PE}{Vol} = \frac{1}{2}\frac{C(\Delta V)^2}{A\cdot d} = \frac{1}{2}\overbrace{\left(\varepsilon_0 \frac{A}{d}\right)}^{C}\frac{\overbrace{(E\cdot d)^2}^{\Delta V^2}}{A\cdot d}$$

$$\boxed{\frac{PE}{Val} = \frac{1}{2}\varepsilon_0 E^2}$$

Doesn't Depend on geometry Etc only Electric Field

Capacitors in Parallel and Series



Parallel

We see that

$\Delta V = \Delta V_1 = \Delta V_2$ $\qquad$ $Q_{TOT} = Q_1 + Q_2$

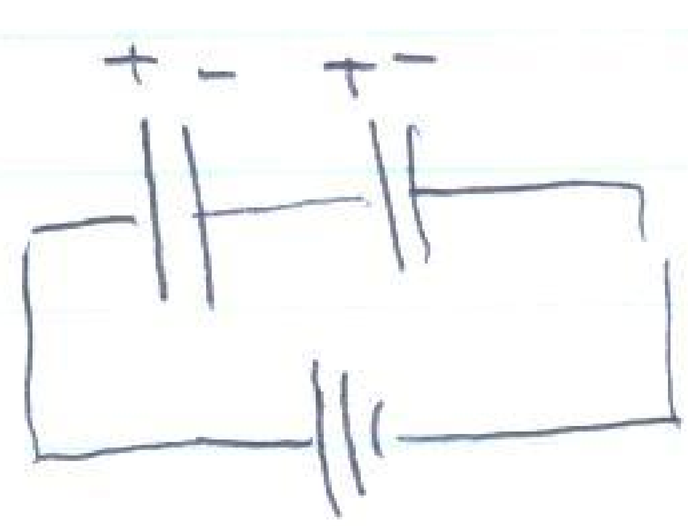
So $\quad C_{eff} = \dfrac{Q_{TOT}}{\Delta V} = \dfrac{Q_1 + Q_2}{\Delta V} = \dfrac{Q_1}{\Delta V} + \dfrac{Q_2}{\Delta V}$

$$C_{eff} = C_1 + C_2$$

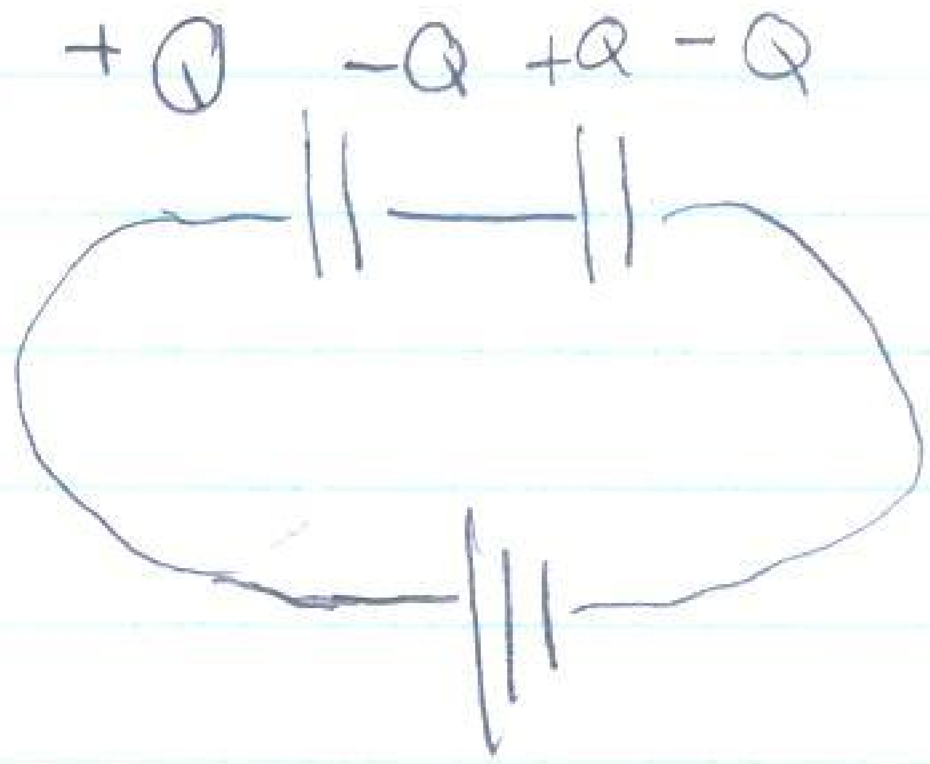
General Case; $\quad C = C_1 + C_2 + C_3 \ldots$

Also note $\quad \dfrac{Q_1}{C_1} = \dfrac{Q_2}{C_2} \Rightarrow \dfrac{Q_1}{Q_2} = \dfrac{C_1}{C_2}$

Next



Capacitors in Series



$$\Delta V = \Delta V_1 + \Delta V_2$$

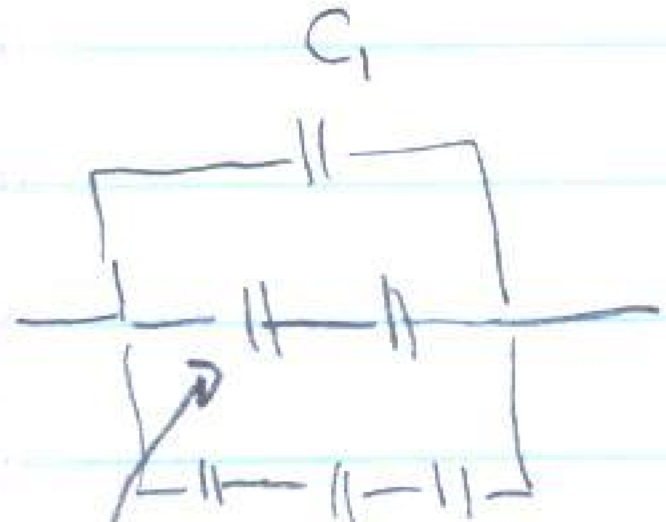
$$\frac{\Delta V}{Q} = \frac{\Delta V_1}{Q} + \frac{\Delta V_2}{Q}$$

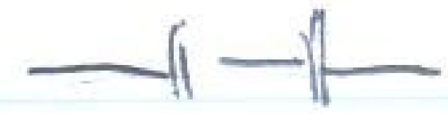
$Q = Q_1 = Q_2$

$$\frac{1}{C} = \frac{\Delta V}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

Problem #18



$C_2 =$ 

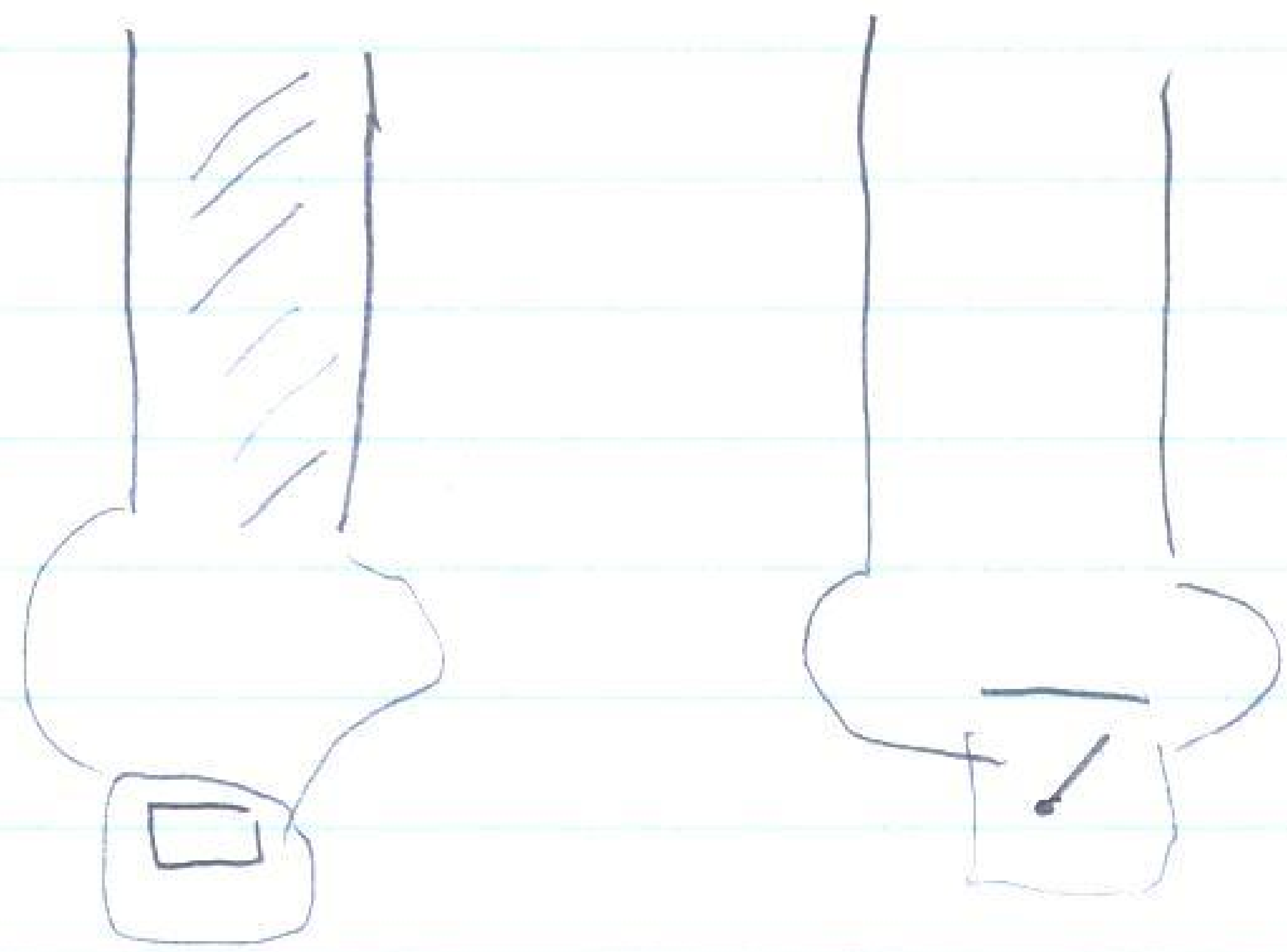
$$\frac{1}{C_2} = \frac{1}{C} + \frac{1}{C}$$

$$\frac{1}{C_2} = \frac{2}{C} \Rightarrow C_2 = \frac{C}{2}$$

$$\frac{1}{C_3} = \frac{1}{C} + \frac{1}{C} + \frac{1}{C} \Longrightarrow C_3 = \frac{C}{3}$$

$$C_{TOT} = C_1 + C_2 + C_3 = C + \frac{C}{2} + \frac{C}{3} = \frac{11}{6}C$$

Media



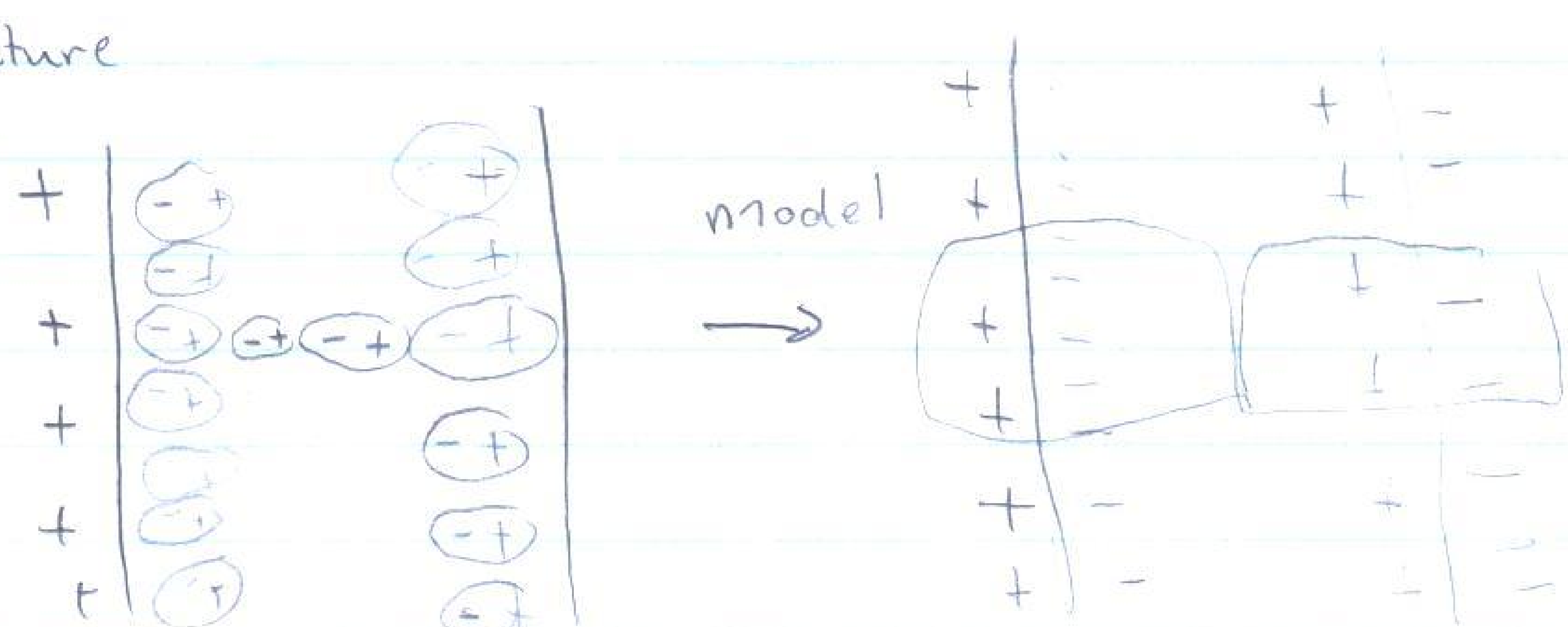
$$\Delta V_{\substack{\text{with}\\\text{Stuff}}} = \frac{\Delta V_0}{\kappa}$$

$$C = \frac{Q_0}{\Delta V} = \frac{Q_0}{\Delta V_0/\kappa} = \kappa \frac{Q_0}{\Delta V_0} = \kappa C_0$$

Capacatance increases by a factor kappa

~ Polystyrene ~ 3,5 = κ

Picture



$$E = \left(\frac{\sigma_0}{2\varepsilon_0}\right) + \left(\frac{\sigma_0}{2\varepsilon_0}\right) - \left(\frac{\sigma_{ind}}{2\varepsilon_0} + \frac{\sigma_{ind}}{2\varepsilon_0}\right)$$

$$E = \frac{1}{\varepsilon_0}\left(\sigma_0 - \sigma_{ind}\right) = \frac{E_0}{k}$$

$$\frac{1}{\cancel{\varepsilon_0}}\left(\sigma_0 - \sigma_{ind}\right) = \frac{\sigma_0}{k\cancel{\varepsilon_0}}$$

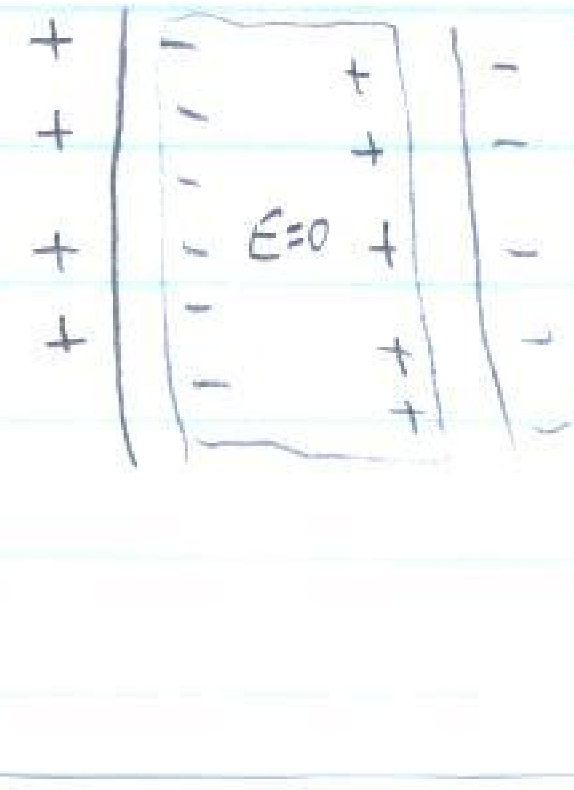
$$\sigma_0 - \frac{\sigma_0}{k} = \sigma_{ind}$$

$$\boxed{\left(1 - \frac{1}{k}\right)\sigma_0 = \sigma_{ind}}$$

k = metal $= \infty$

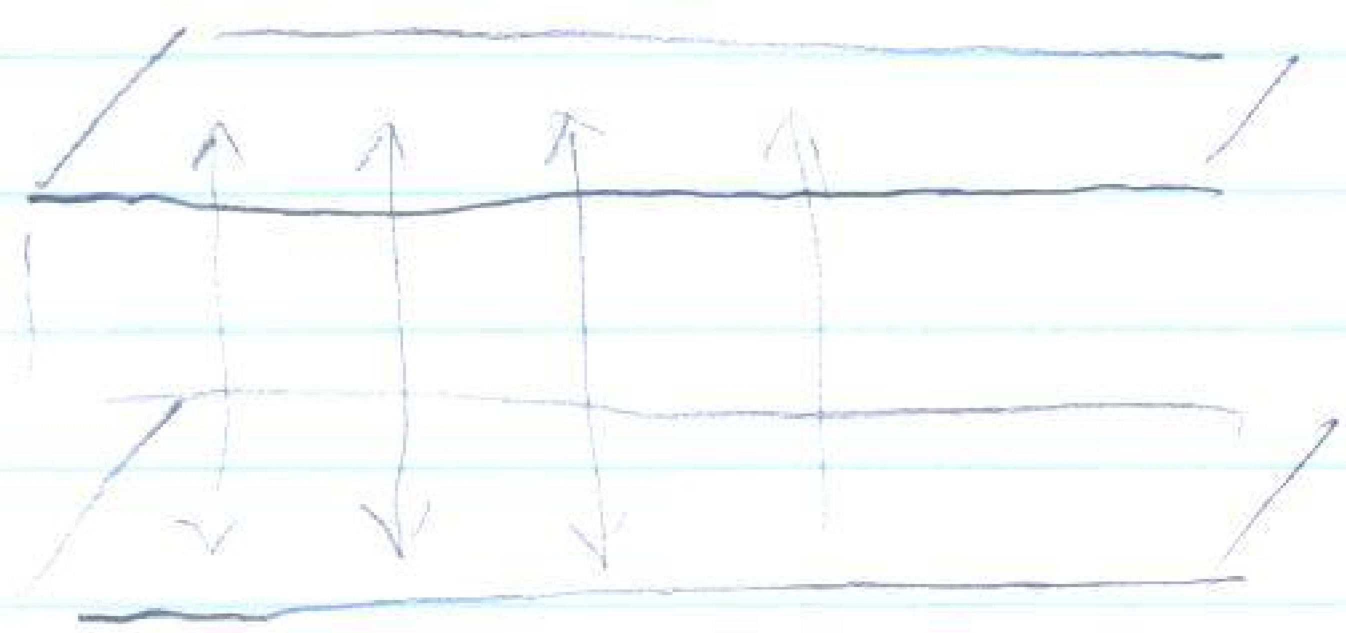
$$\sigma_{ind} = \left(1 - \frac{1}{\infty}\right)\sigma_0 = \sigma_0$$

So For a metal

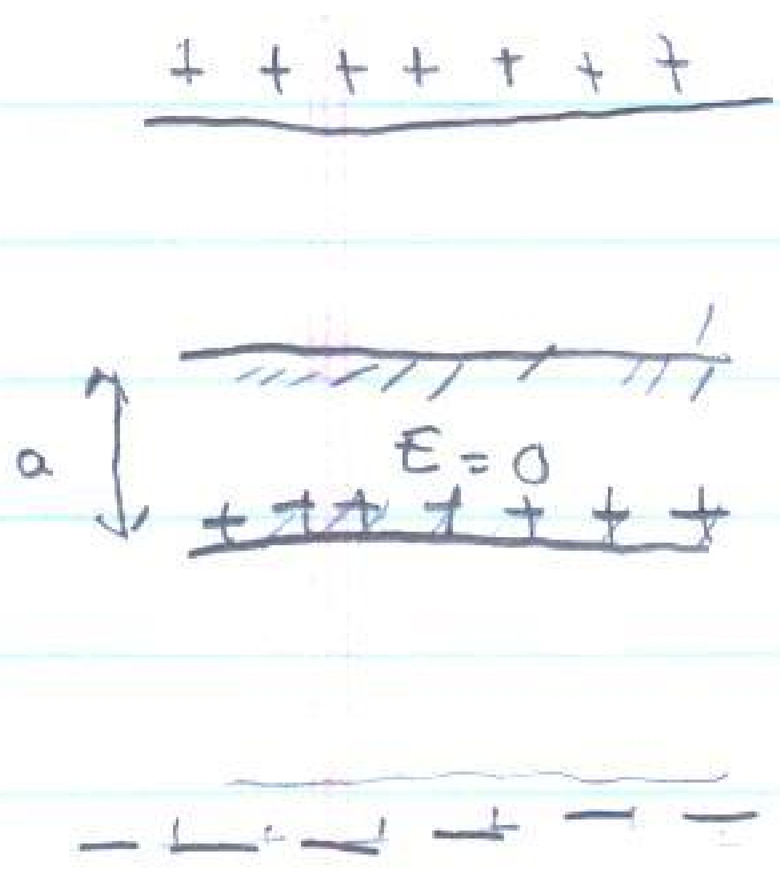


Problem

A // plate cap has separation, d, and area A. An uncharged metallic slab of thickness a is inserted midway between the plates



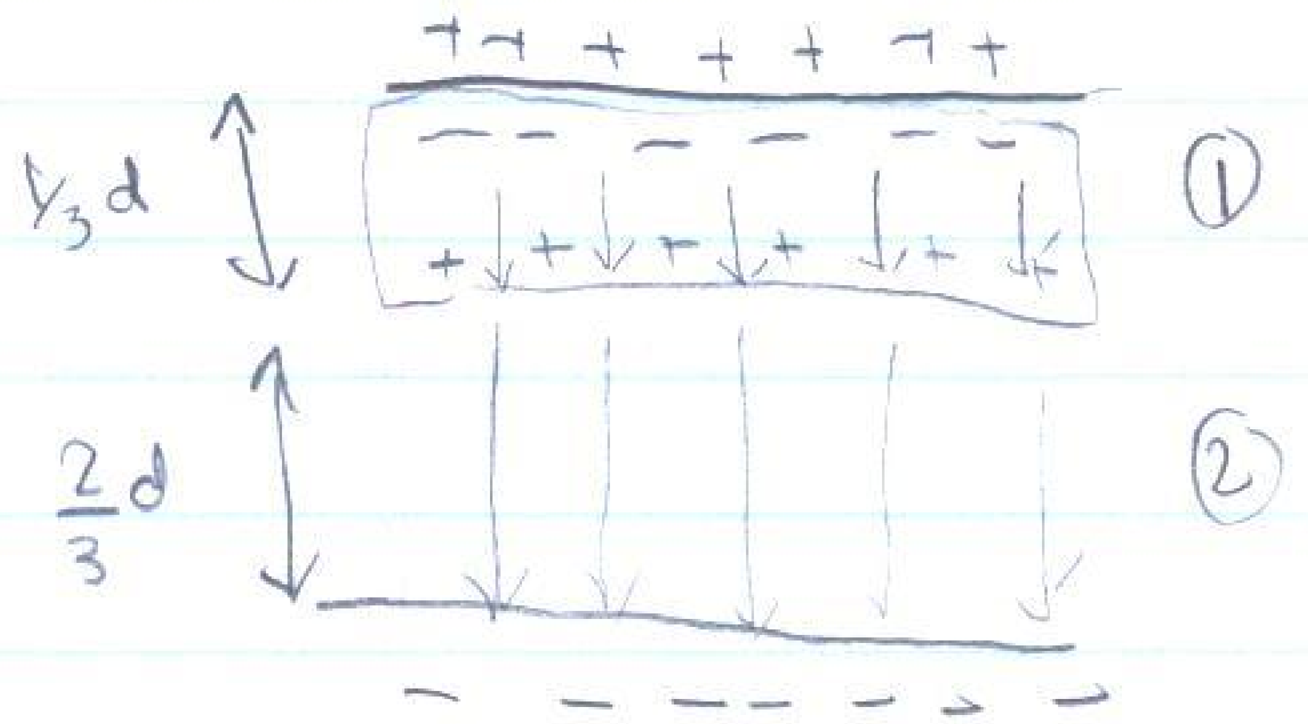
Solution



$$\Delta V = \frac{\sigma}{\varepsilon_0} \cdot \frac{(d-a)}{2} + \frac{\sigma}{\varepsilon} \frac{(d-a)}{2}$$

$$C = \frac{Q}{\Delta V} = \frac{Q}{\frac{\sigma}{\varepsilon_0} \, d-a}$$

$$C = \frac{\varepsilon_0}{\frac{Q}{A}} \frac{Q}{d-a_0} = \frac{\varepsilon_0 A}{d-a}$$



Determine the capacitance

Solution

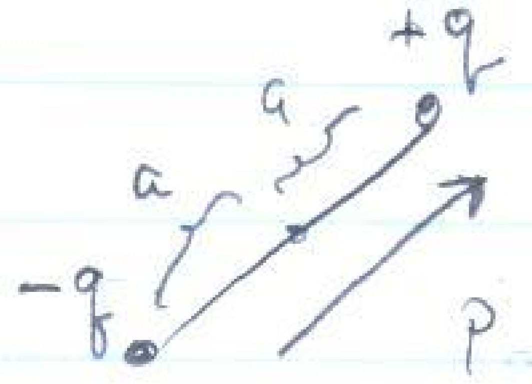
$$E_{①} = \frac{E_0}{K} \qquad E_2 = E_0$$

$$\Delta V = \frac{E_0}{K}\frac{d}{3} + E_0 \cdot \frac{2d}{3} \qquad E_0 = \frac{\sigma_0}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$$

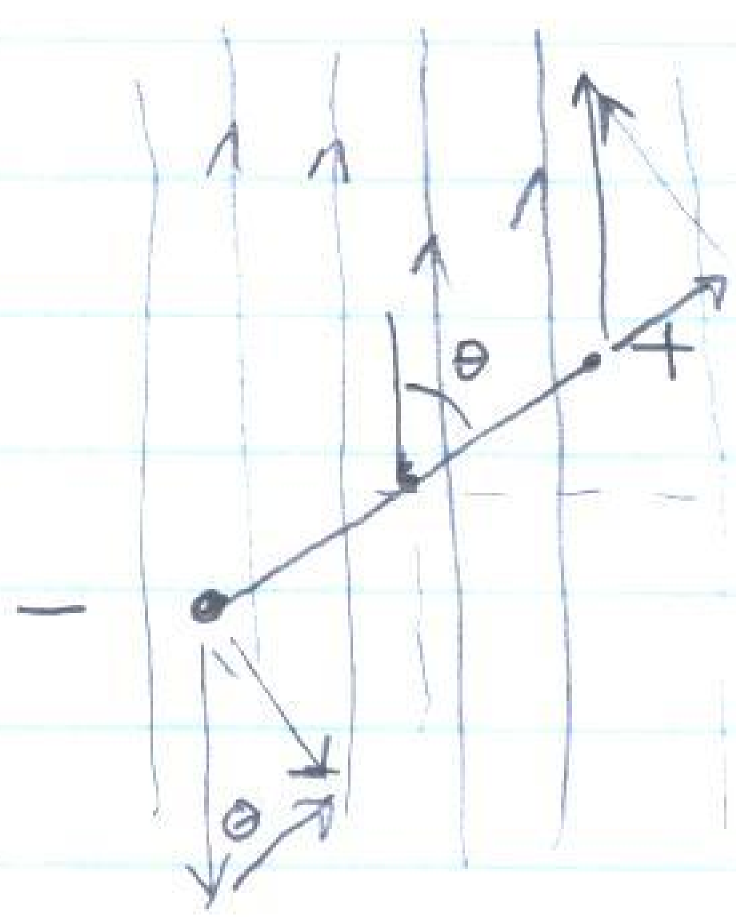
$$C = \frac{Q}{\Delta V} = \frac{Q}{\frac{E_0}{K}\frac{d}{2} + E_0\frac{2d}{3}} = \frac{Q}{\frac{1}{K}\left(\frac{Q}{A\varepsilon_0}\right)\frac{d}{3} + \left(\frac{Q}{A\varepsilon_0}\right)\frac{2d}{3}}$$

$$C = \frac{A\varepsilon_0}{d}\left(\frac{3K}{1+2K}\right)$$

Dynamics of Dipoles



$|\vec{p}| = +2aq$ { Good model for many molecules



- net force on dipole = 0 , There is no net charge

$\tau_1 = rF_\perp = aF\sin\theta = aqE\sin\theta\ \hat{k}$ ← RH Rule

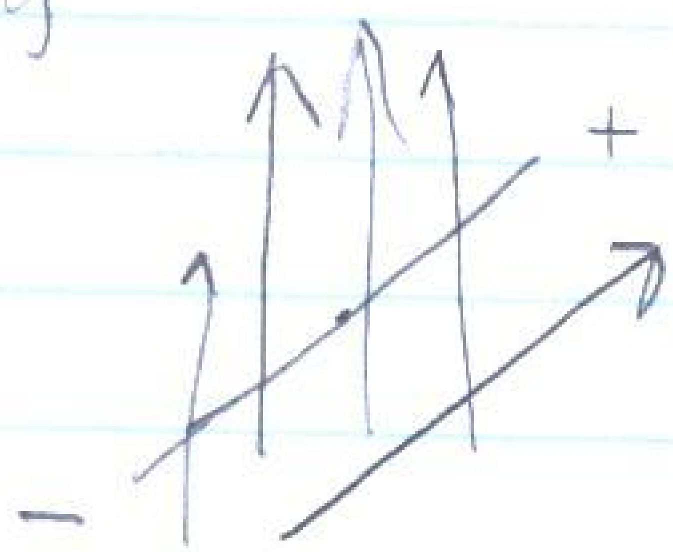
$\tau_2 = rF_\perp = aF\sin\theta = a|q|E\sin\theta\ \hat{k}$

$\tau = \underbrace{2aq}_{p}\, E\sin\theta\ \hat{k} = pE\sin\theta$

p ← dipole moment

$$\boxed{\tau = \vec{p} \times \vec{E}}$$

Energy



$$W_{\substack{by\\you}} = \int \vec{F} \cdot d\vec{x} = \int_0^{\theta_0} \tau \, d\theta$$

$$= \int pE\sin\theta \, d\theta$$

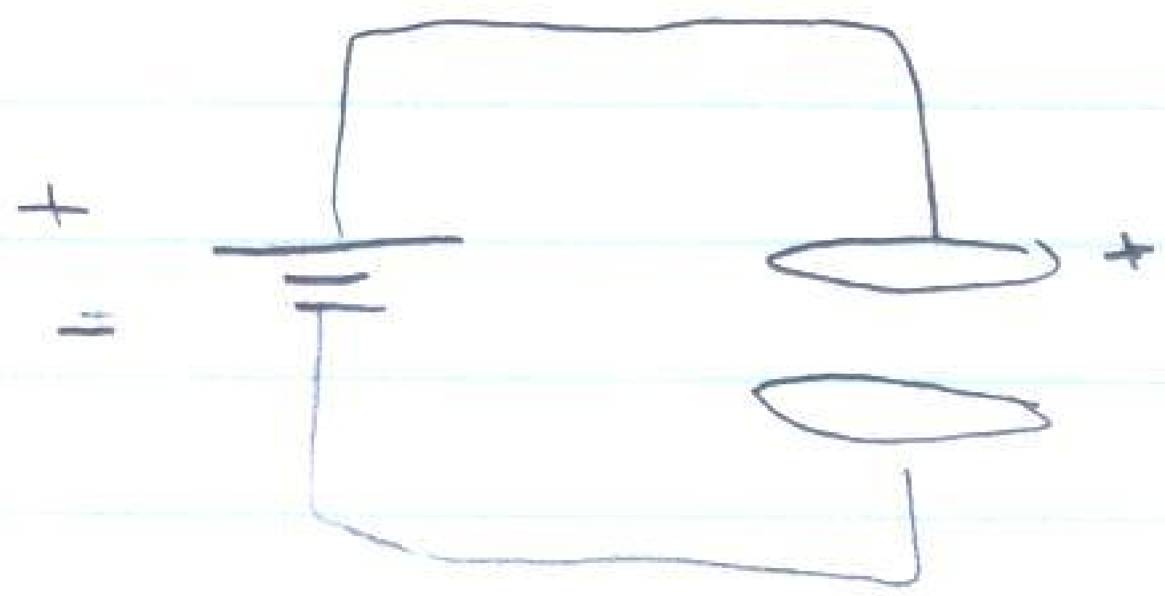
$$= -pE\cos\theta \Big|_0^{\theta_0}$$

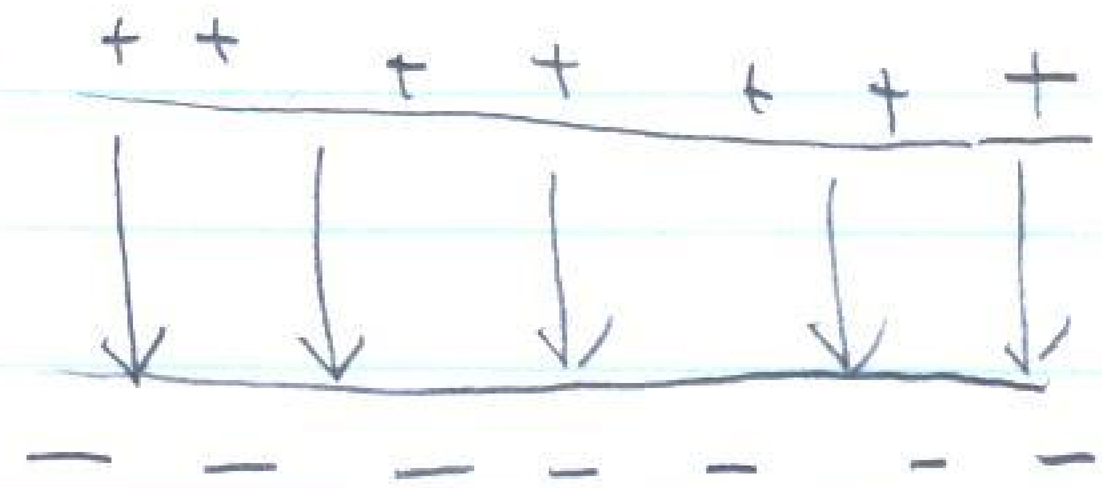
$$W_{\substack{by\\you}} = -pE\cos\theta_0 + pE$$

$$PE = -pE\cos\theta_0 + \text{Const}$$

$$\boxed{PE_{\text{dipole}} = -\vec{p} \cdot \vec{E}}$$

Capacitance :





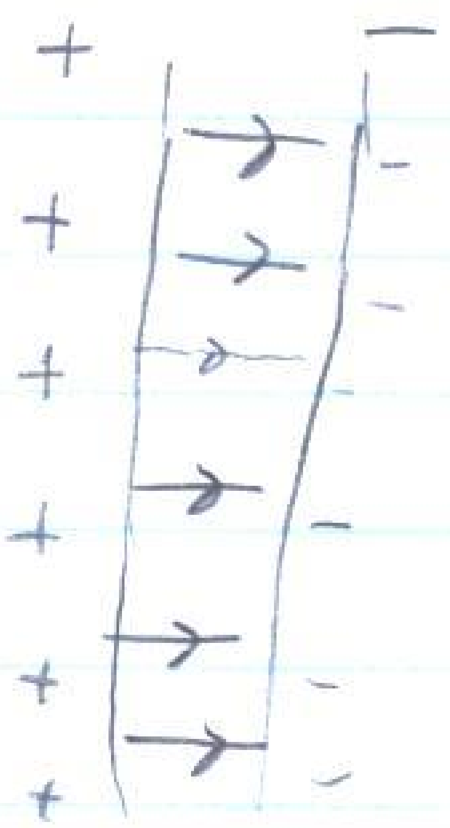
$Q \propto \Delta V$

$Q = C\Delta V \Rightarrow \dfrac{Q}{\Delta V} = C \Rightarrow$ Units $= \dfrac{\text{Coulomb}}{V}$

$= 1\ F$ ← enormous

Parallel Plate Cap

No field



No field

$= 1\ \mu F \approx 1 \times 10^{-6}$ ← typical

$= 1\ pF \approx 1 \times 10^{-12}$ small