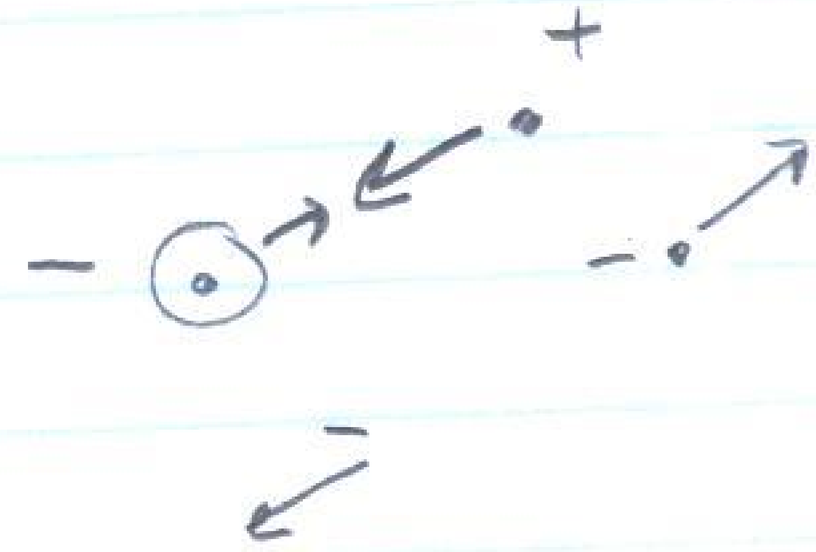
Charges

- Like charges attract unlike repel

$$F = k_e \frac{q_1 q_2}{r^2} \hat{r}$$



- $k_e = 9 \times 10^9 \frac{N m^2}{C^2} = \frac{1}{4\pi \varepsilon_0}$

- Charge is always conserverd!

$k_e =$

<u>Thypical</u> Charges

~ 40 Coulombs ~ lightning bolt!

~ μC ~ typical household sparks.

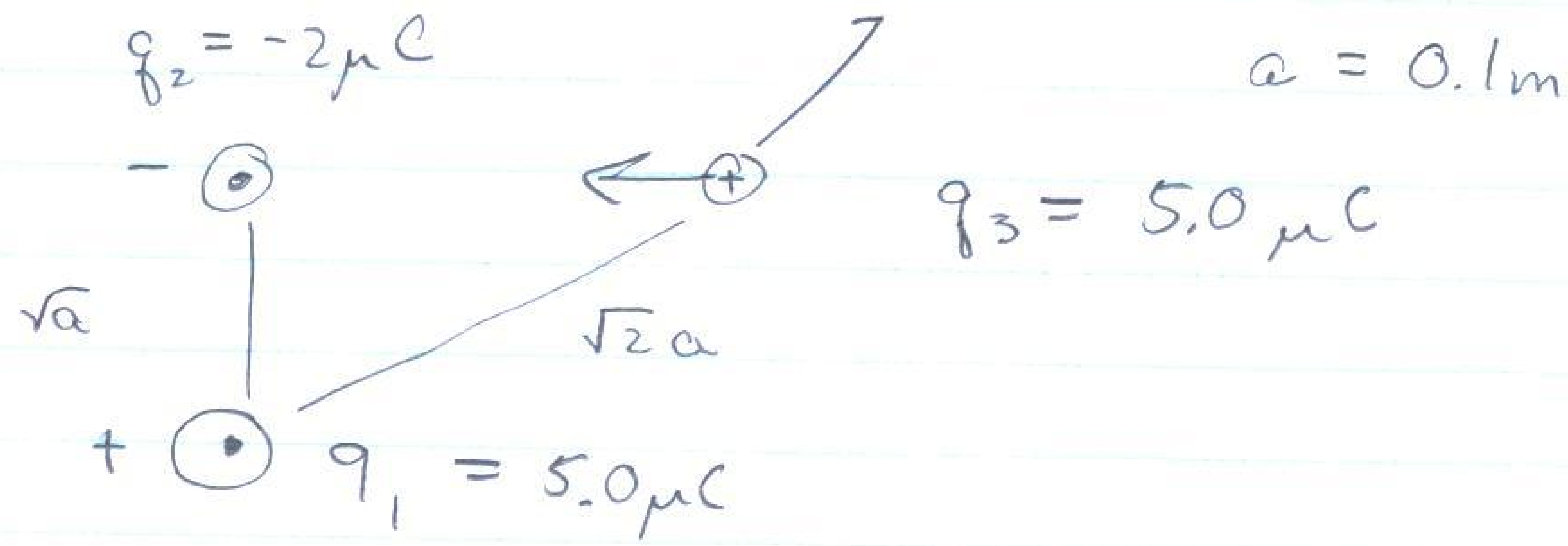
~ 1.602×10^{-19} C ~ charge of electron

1g of Cu ~ has 10^{23} electrons

$$k_e = 90 \; N \frac{cm^2}{(\mu C)^2}$$

- Coulomb is a stupid unit!

Problem ~ Example 23.8



• Find the net Force on $\bar{q}_3$

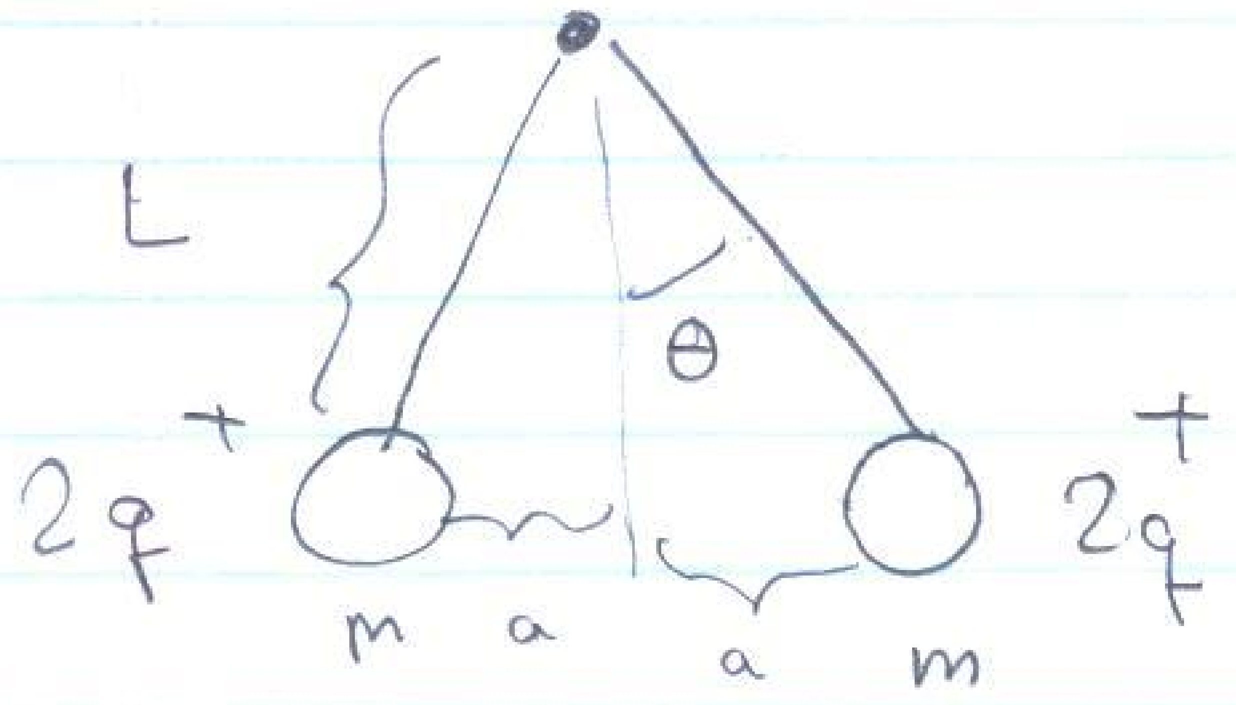
Solution $|F|_{13} = k_e \dfrac{q_1 q_3}{(\sqrt{2}\,a)^2} = 11n$

$$|F_{23}| = k_e \frac{|q_2 q_3|}{a2} = 9N \rightarrow \vec{F}_{23} = -9N\,\hat{i}$$

$$\vec{F}_{13} = |F_{13}| \cos\theta\,\hat{i} + |F_{13}| \cos\theta\,\hat{j}$$

$$= 7.8N\,\hat{i} + 7.8\,N\,\hat{j}$$

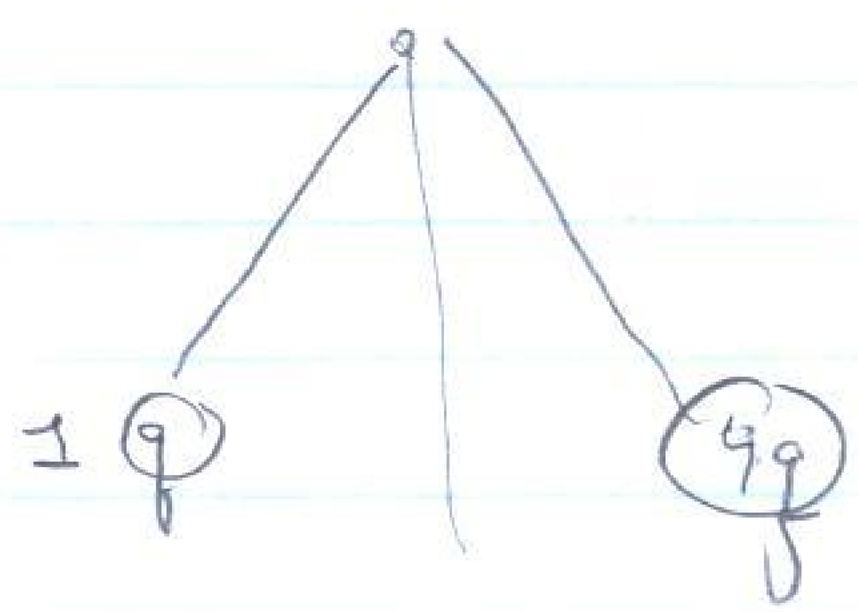
Problem



$\theta = 5^{\circ}$

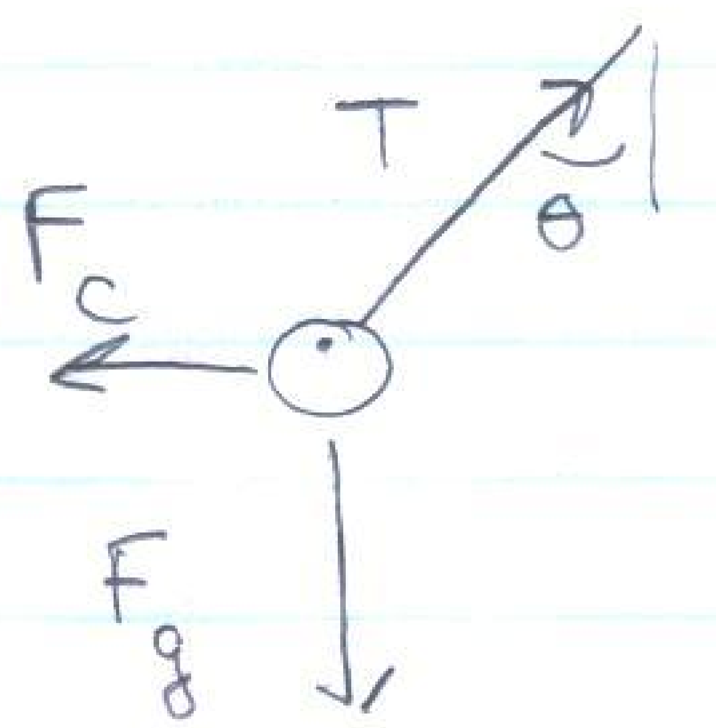
$m = 30\,g$
$L = 15\,cm$

? What if



The problem is the same

– Determine the charge q:



$$T\cos\theta - mg = 0 \quad (1)$$
$$T\sin\theta - F_c = 0 \quad (2)$$

$$F_c = k_e \frac{q_1 q_2}{(2a)^2} = \frac{k_e}{4} \frac{q_1 q_2}{(L\sin\theta)^2}$$

Solving:

(1) $T = \dfrac{mg}{\cos\theta} \rightarrow mg\dfrac{\sin\theta}{\cos\theta} - k_e \dfrac{q_1 \cdot q_2}{(2a)^2} = 0$

$$k_e q_1 q_2 = 4mg L^2 \sin^2\theta \tan\theta$$

$$q_1 q_2 = \frac{4mg}{k_e} L^2 \sin^2\theta \tan\theta$$

$k_e \sim N$

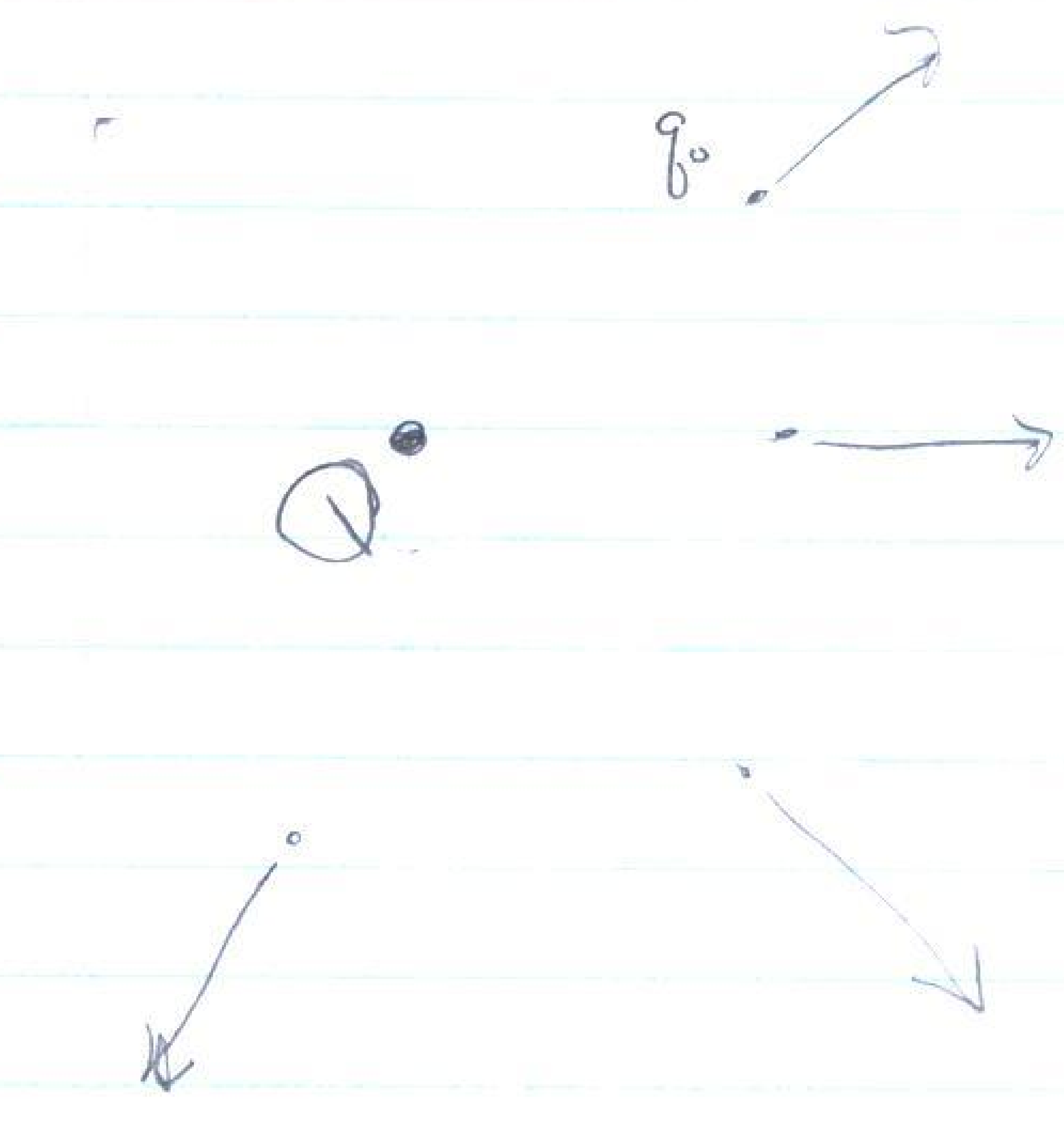
Units

$$q_1 q_2 \sim \frac{N L^2}{\frac{N m^2}{C^2}} \sim C^2 \checkmark$$

$$q_1 q_2 = 4 \cdot (0.03\,kg)(9.8\,m/s^2) \ldots\ldots$$

$$\sqrt{q_1 q_2} = 4.4 \times 10^{-8} = 4.4 \times 10^{-2}\,\mu C$$

Electric Field



Place a positive test charge

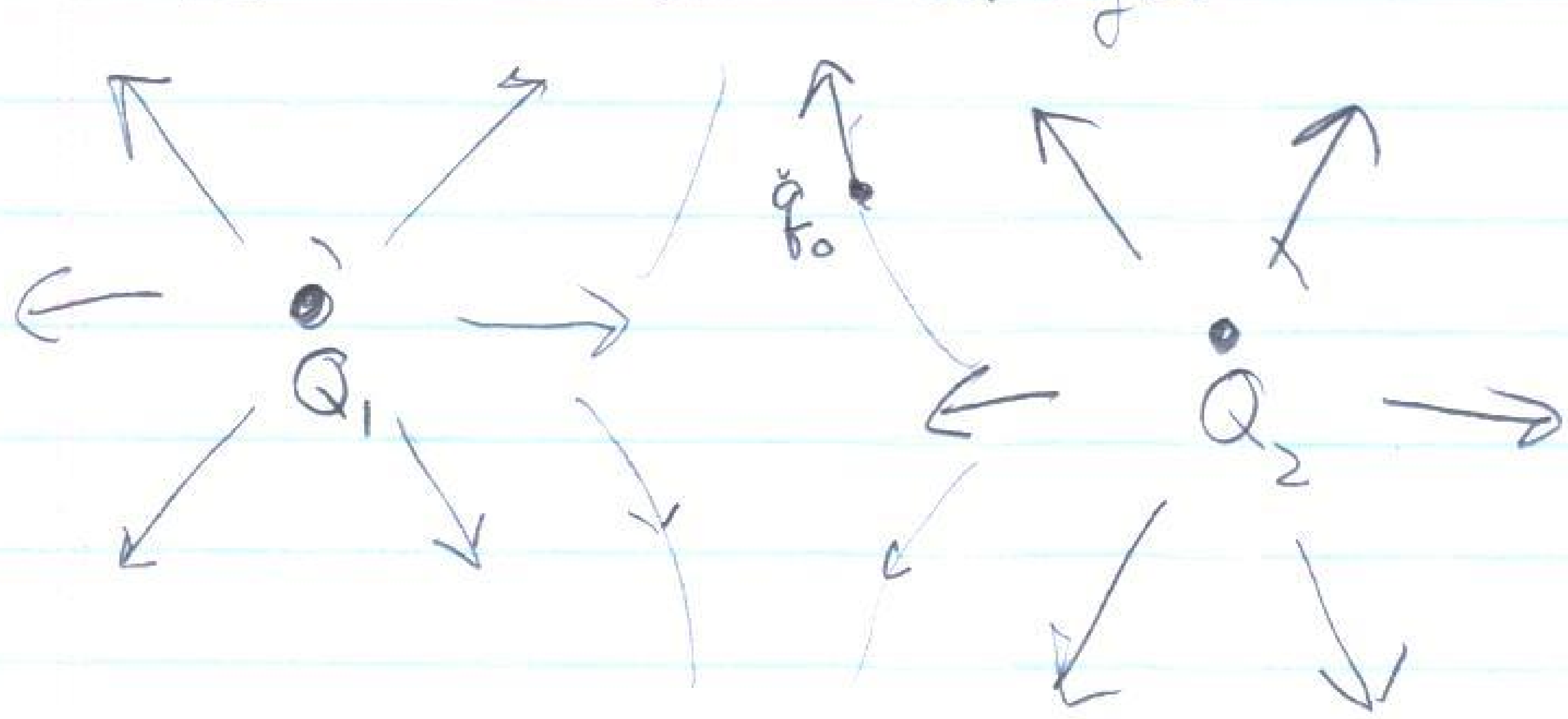
$$\vec{F} = q_0 \left(\frac{Q}{r^2}\right) \hat{r}$$

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{Q}{r^2} \hat{r}$$

Field due a point charge

$$E = \frac{Q}{r^2} \hat{r}$$

Then consider two charges



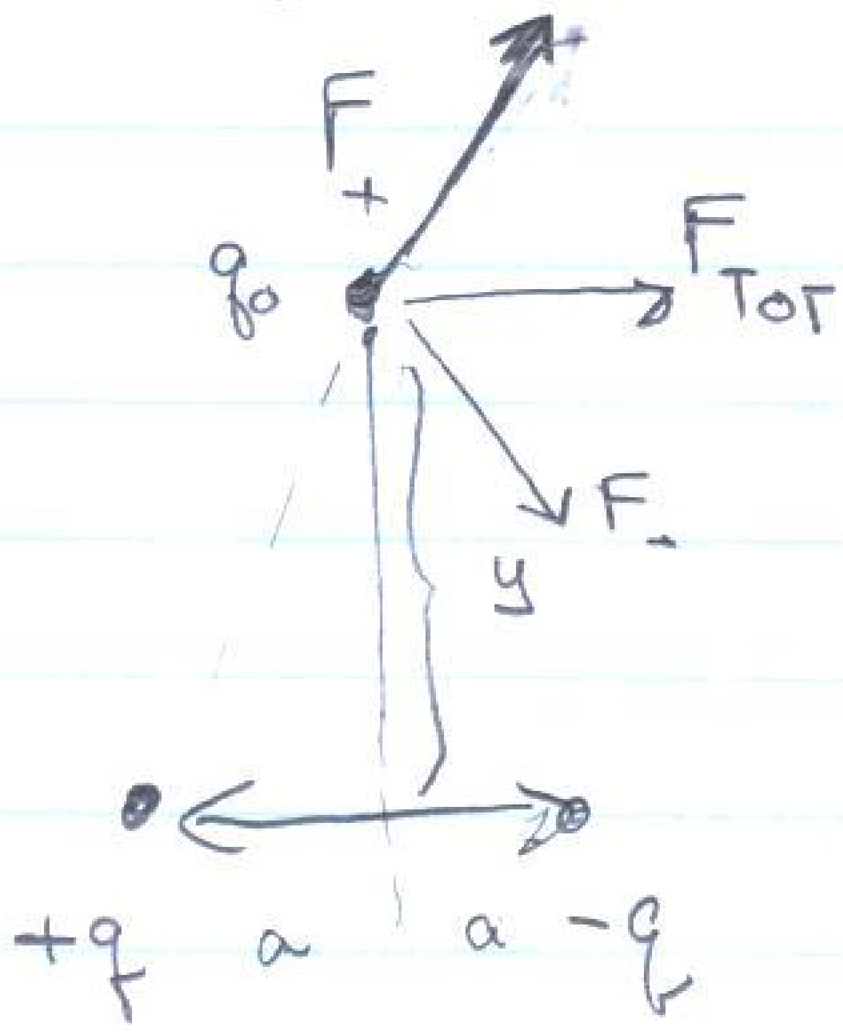
http://qbx.ltu.edu/s-schneider/physlets/main/efield.sht

- One charge
- Two charges

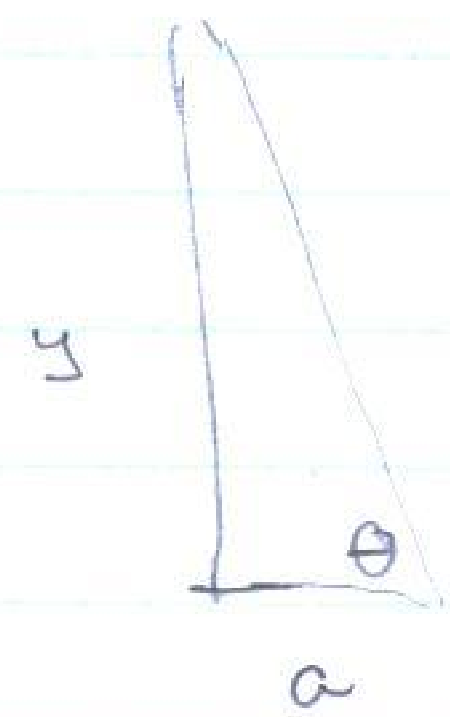
$$\boxed{\vec{F} = q\,\vec{E}(x)}$$

Problem

Consider a charges $+q$ and $-q$



Determine the electric field at point y.



$$\cos\theta = \frac{a}{\sqrt{a^2+y^2}}$$

- Use Applets

$$F^x_{TOT} = F_+ \cos\theta + F_- \cos\theta$$

$$F^x_{TOT} = k_c q_0 \frac{|q| \cos\theta}{r^2} + q \frac{|q| \cos\theta}{r^2}$$

$$F^x_{TOT} = k_c q_0 \left(2|q| \underbrace{\left(\frac{a}{\sqrt{y^2+a^2}}\right)}_{\cos\theta} \underbrace{\frac{1}{y^2+a^2}}_{r^2} \right)$$

$$E^x = \frac{F^x_{TOT}}{q_0} = \left(\frac{k_c 2|q| a}{(y^2+a^2)^{3/2}} \right)$$

$$E = k_e \frac{2|q|a}{(y^2+a^2)^{3/2}}$$

for $y \gg a$

$$E \rightarrow 2|q| \frac{a}{y^{3}} k_e$$ Goes down faster than Coulomb

Compare a point charge

$$E \propto \frac{q}{y^{2}} k_e$$

So consider

$$\vec{E}_{tot} = k_e \frac{q}{y^2} \hat{j} + 2q \frac{a}{(y^2+a^2)^{3/2}} k_e \hat{i}$$

point charge contribution

field from before

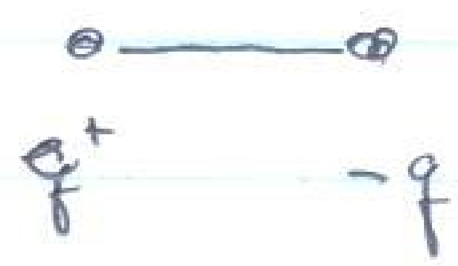
$$\propto \frac{1}{y^2} + \frac{d}{y^3}$$

Long Distances $\propto \dfrac{\text{size of system}}{r}$

$$E = k_e \frac{Q_{net}}{y^2}\left(1 + \overbrace{\text{Correction}}\right)$$

If no net charge : as

$$E \propto \frac{a}{y^3}$$

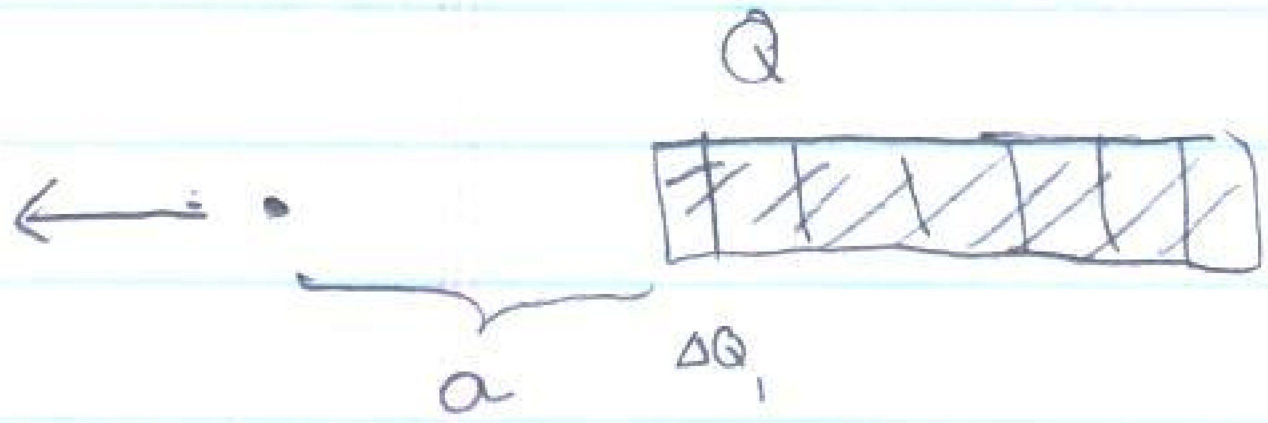


$E = \dfrac{2qa}{y^3}$ ← dipole moment = Charge × separ

$\boxed{p = 2qa}$ ←

dipole moment

Electric Field Due to a rod



λ = charge per length

$= \frac{Q}{L}$ = linear charge density

$$E_{Tot} = \sum_i E_1 + E_2 + \ldots\ldots$$

$$= \sum_i k_e \frac{dQ_i}{r_i^2} \qquad dQ = \lambda\, dx$$

$$E_{TOT} = \int_0^L k_e \frac{\lambda\, dx}{(x+a)^2} = -k_e \lambda \frac{1}{(x+a)}\Big|_0^L$$

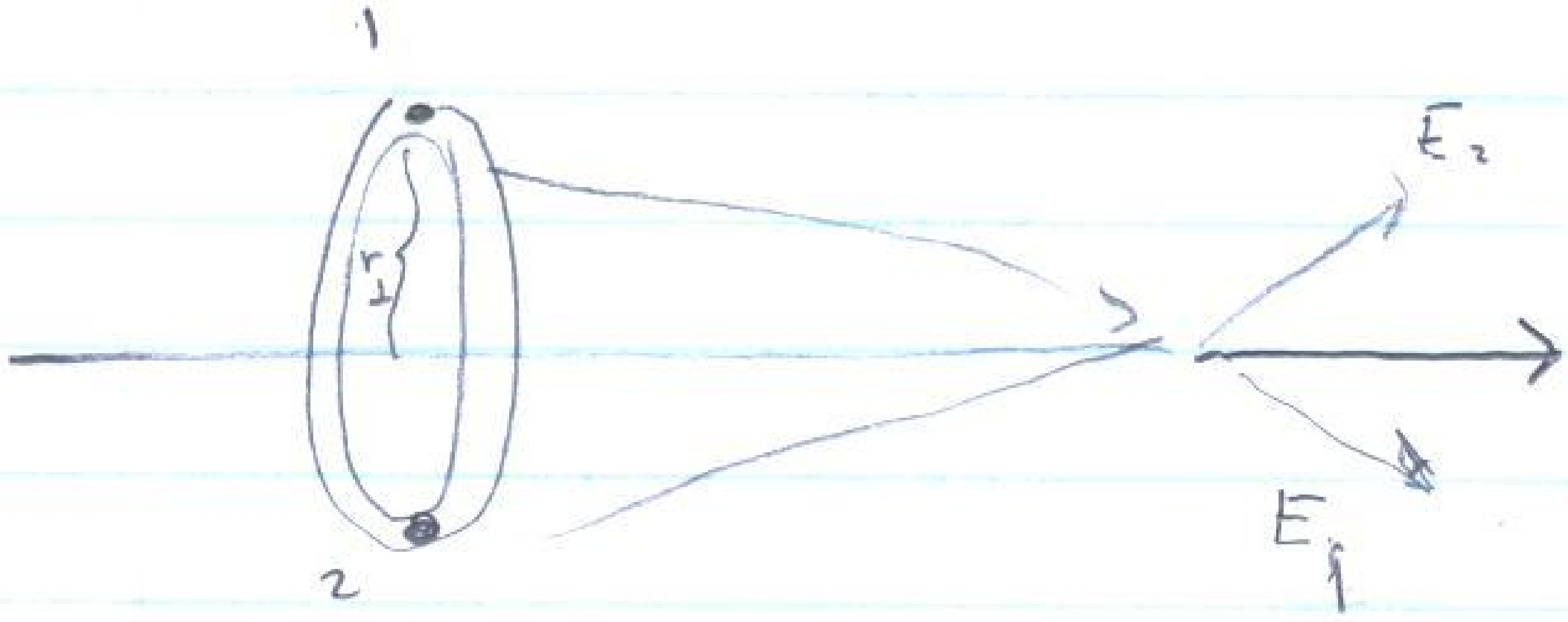
$$= +k_e \lambda \left(\frac{-1}{L+a} + \frac{1}{a} \right)$$

$$E_{TOT} = \frac{k_e \lambda L}{a(L+a)} = \frac{k_e Q}{a(L+a)}$$

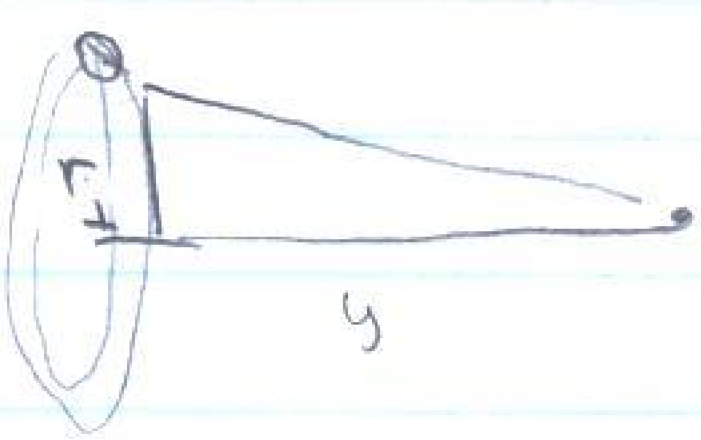
What happens when

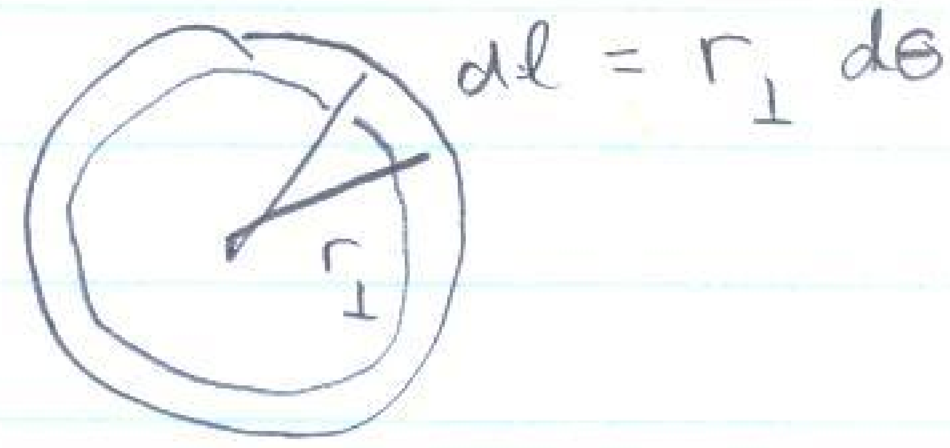
$a \gg L$ answer $E \propto k \frac{Q_e}{a^2}$ ← as we expect

Uniform Ring



- A uniform ring of charge Q calculate the electric field





$$E^x = \sum_i k_c \frac{dQ}{r_i^2} \cos\theta \qquad dQ = \lambda \, dl = \lambda \, a \, d\theta$$

$$E = \int_0^{2\pi} \frac{k_e \lambda r_\perp \, d\theta}{(y^2 + R_\perp)^2} \cdot \frac{y}{(y^2 + R_\perp^2)^{1/2}}$$

$$E = k_e \overbrace{\lambda R_\perp 2\pi}^{Q} \frac{y}{(y^2 + R_\perp^2)^{3/2}} = k_e Q \frac{y}{(y^2 + R_\perp^2)^{3/2}}$$

Again we see that $y \to \infty$

$$E = \frac{k_e Q}{y^2}$$

? What would happen if a pos. charge were placed at the center

? What would happen if a negative charge were placed

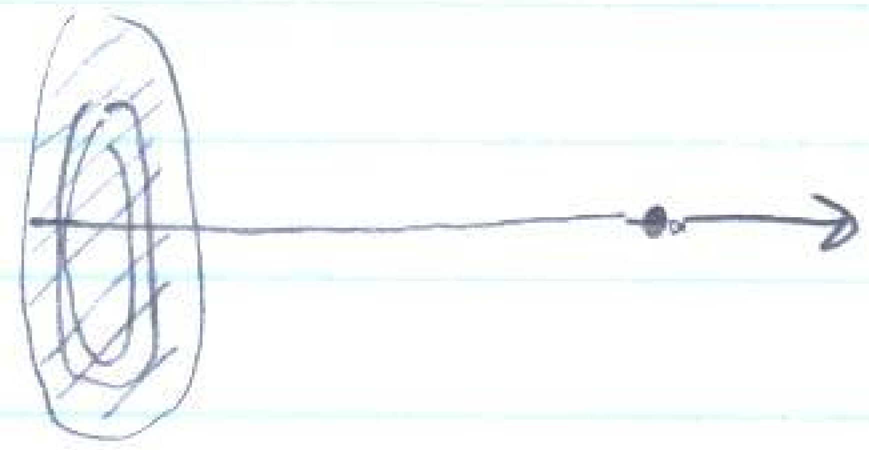
For $y << R_1$

$$E = k_e Q y$$

$$F = -\underbrace{k_e |q| Q}_{k_{spring}}\, y \leftarrow \text{restoring Force}$$

$$\boxed{\omega = \sqrt{\frac{k_{spring}}{m}}}$$

Uniform Disc



$\sigma = \dfrac{\text{Charge}}{\text{Area}}$

$$E = \sum k_e \frac{dQ}{r^2} = \sum_1 k_e \, \sigma \, \overbrace{2\pi R_\perp \, dR_\perp}^{\text{area of ring}} \cdot \frac{1}{(R_\perp^2 + y^2)} \frac{y}{(R_\perp^2+y^2)^{1/2}}$$

$$= \int_0^R dR_\perp \; k_e \, \sigma \, 2\pi \; R_\perp \, dR_\perp \; \frac{y}{(R_\perp^2 + y^2)^{3/2}}$$

$$= k_e \sigma \pi y \int_0^R \frac{2 R_\perp \, dR_\perp}{(R_\perp^2 + y^2)^{3/2}}$$

$$= k_e \sigma \pi y \; -2 \; \frac{1}{(R_\perp^2 + y^2)^{1/2}} \Bigg|_0^R$$

$$= 2\pi k_e \sigma \left(1 - \frac{y}{(y^2 + R^2)^{1/2}} \right)$$

- When $y \ll R$ find

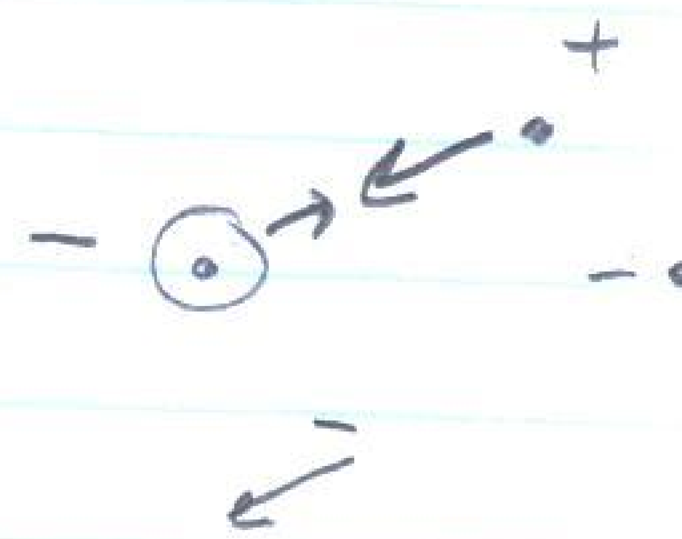
$E = 2\pi k_e \sigma$ Constant

$$\boxed{E = \frac{\sigma}{2\epsilon_0} \qquad k_e = \frac{1}{4\pi\epsilon_0}}$$

Charges

- Like charges attract unlike repel

$$F = k_e \frac{q_1 q_2}{r^2} \hat{r}$$



- $k_e = 9 \times 10^9 \frac{Nm^2}{C^2} = \frac{1}{4\pi\varepsilon_0}$

- Charge is always conserverd!

$k_e =$

Thypical Charges

~ 40 Coulombs ~ lightning bolt!

~ μC ~ typical household spaks.

~ 1.602×10^{-19} C ~ charge of electron

1g of Cu ~ has 10^{23} electrons

$$k_e = 90 \frac{N\, cm^2}{(\mu C)^2}$$

- Coulomb is a stupid unit!