

Problems:

$$27.3, 27.17, 27.65 \quad (1)$$

$$28.9, 28.21, 28.31 \quad (2)$$

$$\underline{29.5}, \underline{29.11}, 29.17, \underline{29.23}, 29.33, 29.37, 29.47 \quad (3)$$

Underlines are to be handed in.

Currents and Circuits

- The current is defined as

$$I = \frac{dQ}{dt} = \frac{\text{Charge passing through surface } A}{\Delta t} \quad (4)$$

Generally it is found that the current is proportional to the electric field which in turn is proportional to voltage

$$V = IR \quad (5)$$

where R is the resistance The power dissipated in the resistor

$$\mathcal{P} = I^2 R \quad (6)$$

- Microscopically it is more useful to deal with the current density which is the current divided by the cross sectional area

$$\mathbf{J} = \frac{I}{A} = nq\mathbf{v}_d \quad (7)$$

n is the density of charge carriers, q is their charge and \mathbf{v}_d is the average/drift velocity of the charge carriers. You should be able to derive Eq. (7) from Eq. (5) in order to understand this formula

The current density is usually proportional to the electric field

$$\mathbf{J} = \sigma\mathbf{E} \quad (8)$$

with σ the conductivity. Finally we define the resistivity ρ

$$\rho = \frac{1}{\sigma} \quad (9)$$

The resistance of a wire with area A and length ℓ is related to the resistivity by

$$R = \rho \frac{\ell}{A} \quad (10)$$

which makes sense – the wider the area the less the resistance, the longer the wire the more resistance.

- Resistors in series

$$R_{eq} = R_1 + R_2 \dots \quad (11)$$

Resistors in ||

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \dots \quad (12)$$

- Kirchoffs Laws allow you to determine the flow of current in a circuit
For a given circuit

1. For each wire indicate a current with an arrow
2. The sum of the currents entering a vertex is zero. Thus in Fig 4. the sum of the vertex

$$I_1 + (-I_2) + (-I_3) = 0 \quad (13)$$

where we have written $(-I_2)$ and $(-I_3)$ because these currents are drawn exiting rather than entering the vertex.

3. For every closed loop, draw a circle and indicate the loop direction. The sum of the potential drops going around the loop is zero.

$$\sum \Delta V = 0 \quad (14)$$

Imagine moving a positive charge in the loop direction and sum every change in voltage for each circuit element. This amounts to the follow rules

- (a) If the current is moving with loop direction (Fig. 5) the voltage drop across the resistor is

$$(\Delta V)_R = -IR \quad (15)$$

If the current and loop direction are opposite get $+IR$

- (b) If the loop direction is the with the battery (Fig. 3) the voltage change is

$$(\Delta V)_\mathcal{E} = +\mathcal{E} \quad (16)$$

If the loop and battery are opposite $-\mathcal{E}$

- (c) For each capacitor if the loop direction is in the same as the current direction (Fig. 1) the voltage drop is

$$(\Delta V)_C = -\frac{q}{C} \quad (17)$$

where q is the charge on the capacitor and C is the capacitance. If current and loop direction are opposite get $+q/C$.

- In an RC circuit shown below when the switch is thrown the charge builds up on the capacitor. The charge and the capacitor and the current in the circuit is

$$q(t) = Q(1 - e^{-\frac{t}{RC}}) \quad (18)$$

$$I(t) = \frac{\mathcal{E}}{R} e^{-\frac{t}{RC}} \quad (19)$$

where R is the resistance, C is the capacitance. We also have

- Q is the final charge on the plate $Q = C\mathcal{E}$.
- The initial current in the wire is \mathcal{E}/R .

You should understand where these last two items come from otherwise you do not understand this stuff. The combination

$$\tau = RC \quad (20)$$

is known as the time constant and is a measure of how fast the circuit can change.

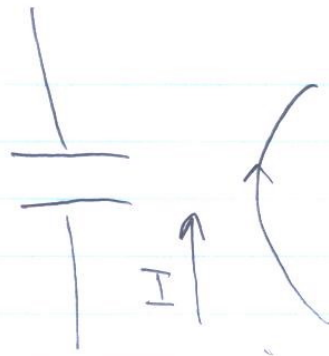
- If a charged capacitor is discharged through the switch shown below then the charge on the plates decreases as

$$q(t) = Qe^{-\frac{t}{RC}} \quad (21)$$

The current is negative (since charge is moving off the plates)

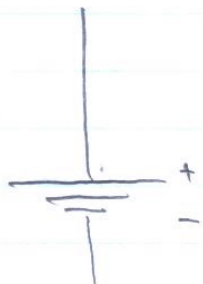
$$I(t) = -I_o e^{-\frac{t}{RC}} \quad (22)$$

and the magnitude decreases. I_o is the magnitude of the initial current $I_o = Q/RC$.



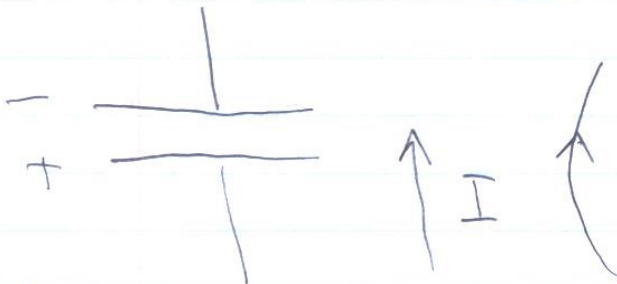
$$\Delta V = -\frac{q}{C}$$

Fig 1



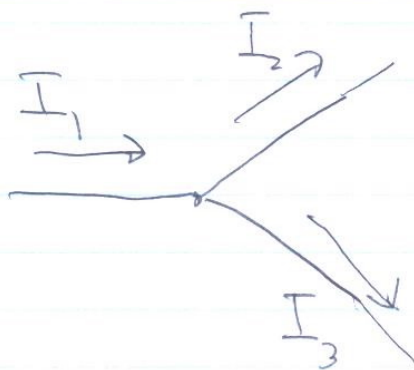
$$\Delta V = \mathcal{E}$$

Fig 2



$$\Delta V = -\frac{q}{C}$$

Fig 3



$$I_1 + (-I_2) + (-I_3) = 0$$

Fig 4



$$(\Delta V) = -IR$$

Fig. 5

Forces and Magnetic Fields

- A charge moving with velocity \mathbf{v} in a magnetic field \mathbf{B} experiences a force

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B} \quad (23)$$

Beware the sign of the charge.

- If the field is uniform the charge will move in a circle The radius of the circular orbit is

$$r = \frac{mv}{qB} \quad (24)$$

You should be able to derive this using the fact that the centripetal acceleration is

$$a_c = \frac{v^2}{r} \quad (25)$$

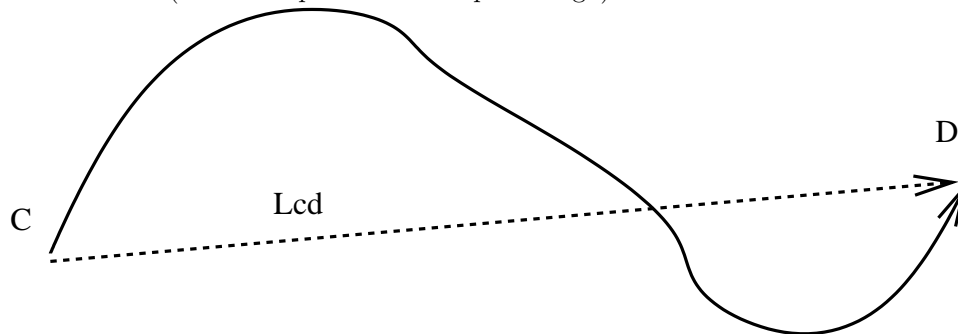
- If a piece of wire of length $d\mathbf{L}$ carries a current the carries a current I

$$d\mathbf{F} = Id\mathbf{L} \times \mathbf{B} \quad (26)$$

- In a uniform magnetic field a wire of arbitrary shape running from point C to point D carrying a current I , experiences a force

$$\mathbf{F} = I\mathbf{L}_{CD} \times \mathbf{B} \quad (27)$$

where \mathbf{L}_{CD} is the line connecting C to D . A corollary is that a closed loop in a magnetic field experiences no net force (it does experience a torque though). You should understand how this comes from Eq. (26).



- For a closed loop with area A carrying a current I the dipole moment is

$$\boldsymbol{\mu} = IA \quad (28)$$

The direction of the dipole moment is found by the right hand screw rule.

- The torque on a current loop in a magnetic field is

$$\boldsymbol{\tau} = \boldsymbol{\mu} \times \mathbf{B} \quad (29)$$

- The potential energy is least when the dipole moment is pointing in the direction of the magnetic field and greatest when pointing in the opposite direction. Check that the following formula expresses this

$$U = -\boldsymbol{\mu} \cdot \mathbf{B} = -\mu B \cos(\theta) \quad (30)$$