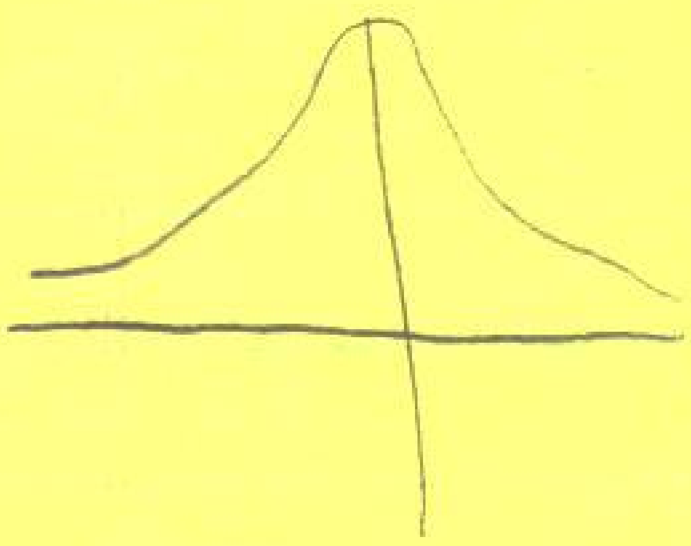
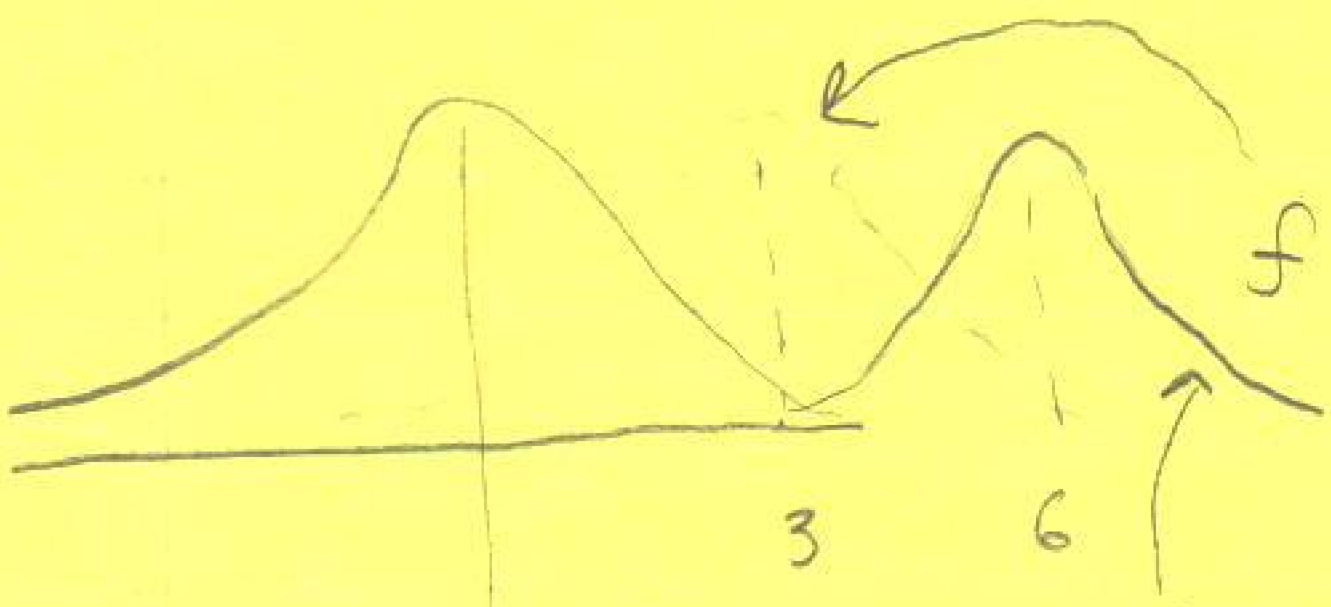
Wave Motion

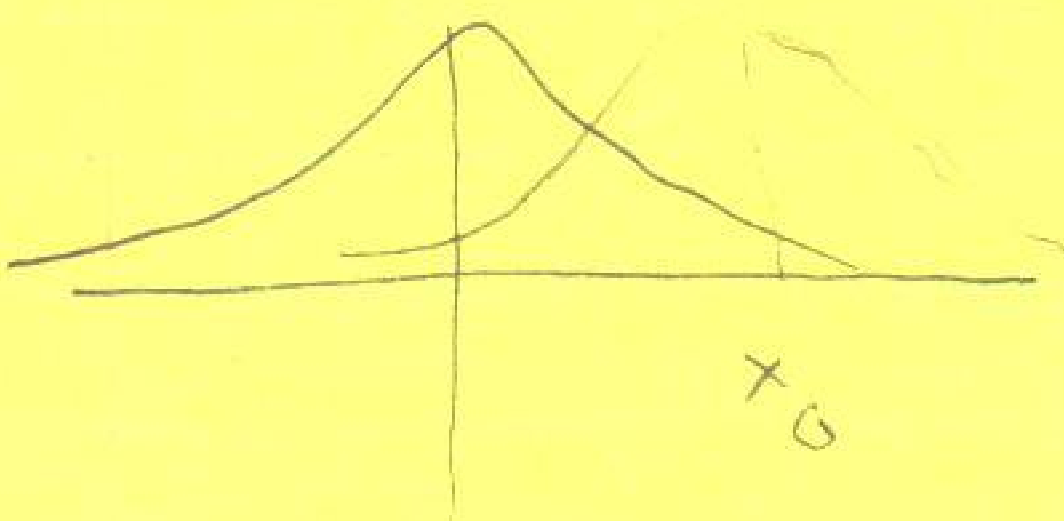


$f(x) \rightarrow \frac{1}{1+x^2}$



$f(x-3) = \frac{1}{1+(x-3)^2}$

$f(x-6)$



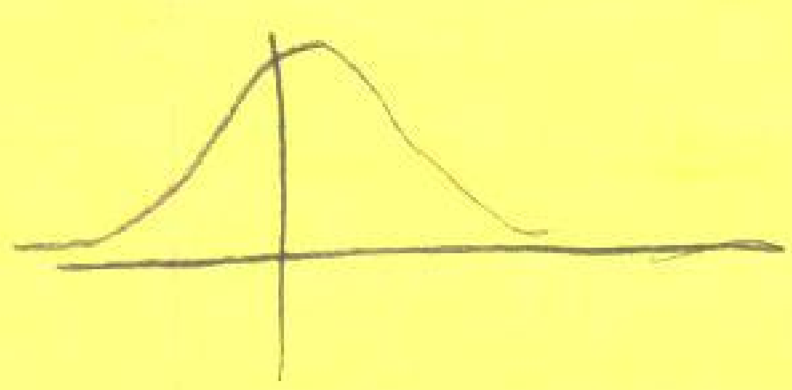
$f(x-x_0)$

Now want that x_0 increases with thime

$x_0 = vt$

So

$f(x-vt) \leftarrow$ moves to the "right" with speed v



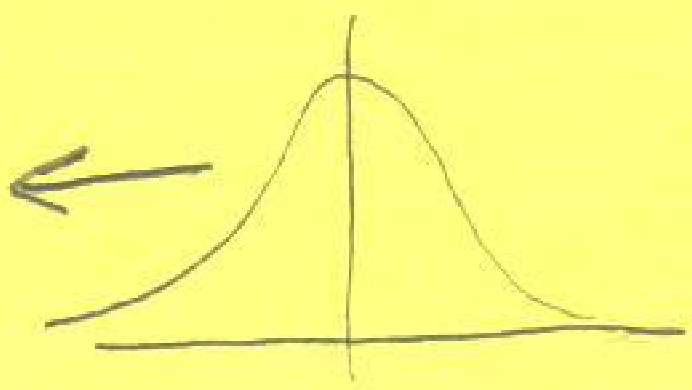
$x_0 = -vt \rightarrow f(x+vt)$

moves to the left with speed

Then problem #3, Chap 16

Consider $y(x,t) = 5e^{-(2x+5t)^2}$

a) draw $y(x, t=0)$



b) Which way is it moving and how fast

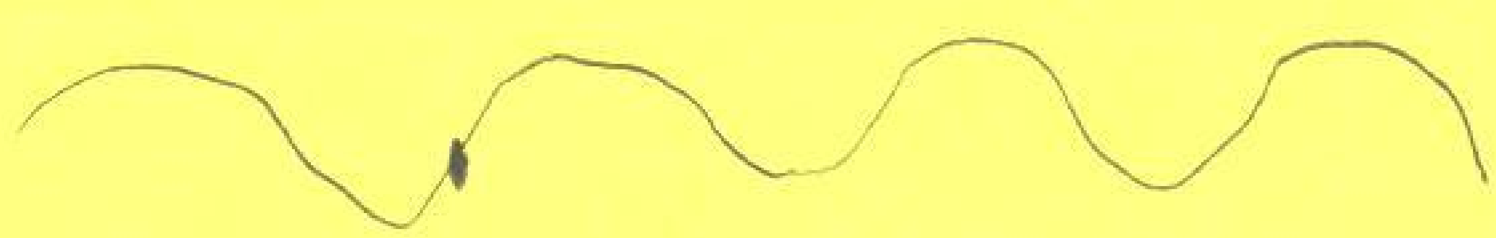
Solution

$y(x,t) = 5e^{-4(x+\frac{5}{2}t)^2}$ Compare $f(x \mp vt)$

$\to$ wave is moving to left with speed $v = \frac{5}{2}$

<u>Sinusoidal Waves</u> $\to$ Take a snapshot

$y \sim A \sin \frac{2\pi}{\lambda} x$



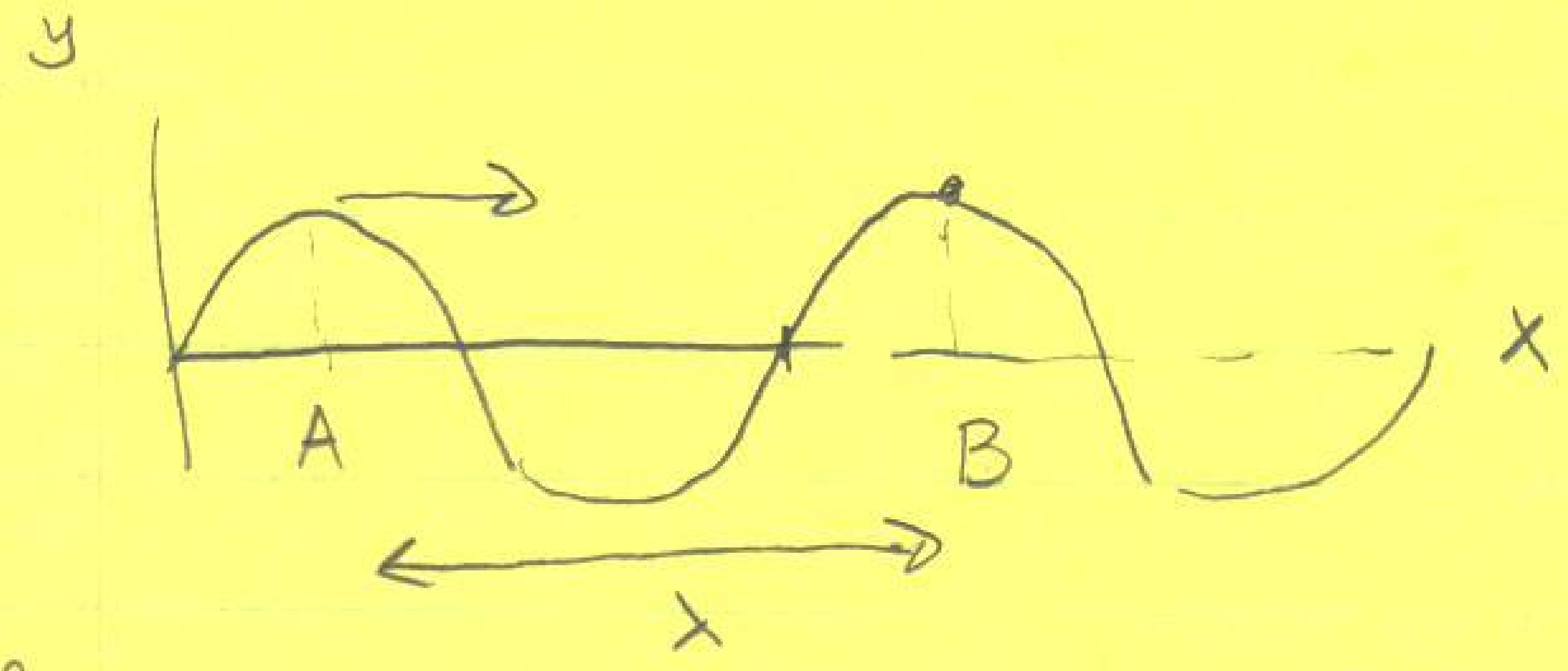
$$\boxed{y(x,t) = A \sin \frac{2\pi}{\lambda}(x - vt) + \phi} \leftarrow \text{general form of sinusoidal wave}$$

right moving

$y(x,t) = A \sin \frac{2\pi}{\lambda}(x + vt) + \phi$

left moving

Consider a wave



① After point A reaches B, then things repeat.

time for one cycle = Period = T

time for A to reach B = $\dfrac{\lambda}{v} = \dfrac{dist}{dist/time} = T$

$\lambda = vT$

Now this is written in terms of f

$f = \dfrac{1}{T}$ = # cycles per sec = Hz

$$\frac{\lambda}{T} = \boxed{\lambda f = v}$$

So

$$y(x,t) = A \sin \frac{2\pi}{\lambda} x - \underbrace{\frac{2\pi}{\lambda} V}\, t$$

$$y(x,t) = A \sin \frac{2\pi}{\lambda} x - \frac{2\pi}{T} t$$

$$= A \sin kx - \omega t$$

$k \equiv \frac{2\pi}{\lambda}$ = wavenumber $\quad\quad$ $\omega =$

$\omega = \frac{2\pi}{T}$ = angular frequency

Then consider one practice

- $\lambda f = v \doteq \frac{\lambda}{2\pi} \cdot 2\pi f = v$

$$\boxed{\frac{\omega}{k} = v}$$

- If you double the frequency of a wave the
  - speed remains const
  - λ is twice shorter

Problem

$$y = (0.120m)\ \overset{\cos}{\sin}\left(\frac{\pi}{8}x + 4\pi t\right)$$

- λ, period, $v = ?$, $A = ?$
- Determine the transverse speed and acceleration at $t = 0.2s$ and $x = 1.6\ m$

$y = A\sin(kx + \omega t + \phi) \longleftarrow$ Left moving

$k = \frac{\pi}{8}$ $\qquad \omega = +4\pi$

So

$k = \frac{2\pi}{\lambda} = \frac{\pi}{8} \Rightarrow \boxed{\lambda = 16}$

$|\omega| = 4\pi = \frac{2\pi}{T}$ $\qquad T = \frac{1}{2}$

$\lambda f = 32 = V$

$\boxed{f = \frac{1}{T} = 2}$

Then Determine V_y the velocity up and down

$$y = A \sin(kx \mp \omega t + \phi)$$

$$\frac{\partial y}{\partial t} = A \cos(kx \mp \omega t + \phi) \mp \omega$$ ← Up down speed "trans spe

$$\frac{\partial^2 y}{\partial t^2} = -A \omega^2 \sin(kx \mp \omega t + \phi)$$

Then

$$V_y = \mp A \omega \cos(kx - \omega t + \phi)$$

$$a_y = -A \omega^2 \sin(kx - \omega t + \phi)$$

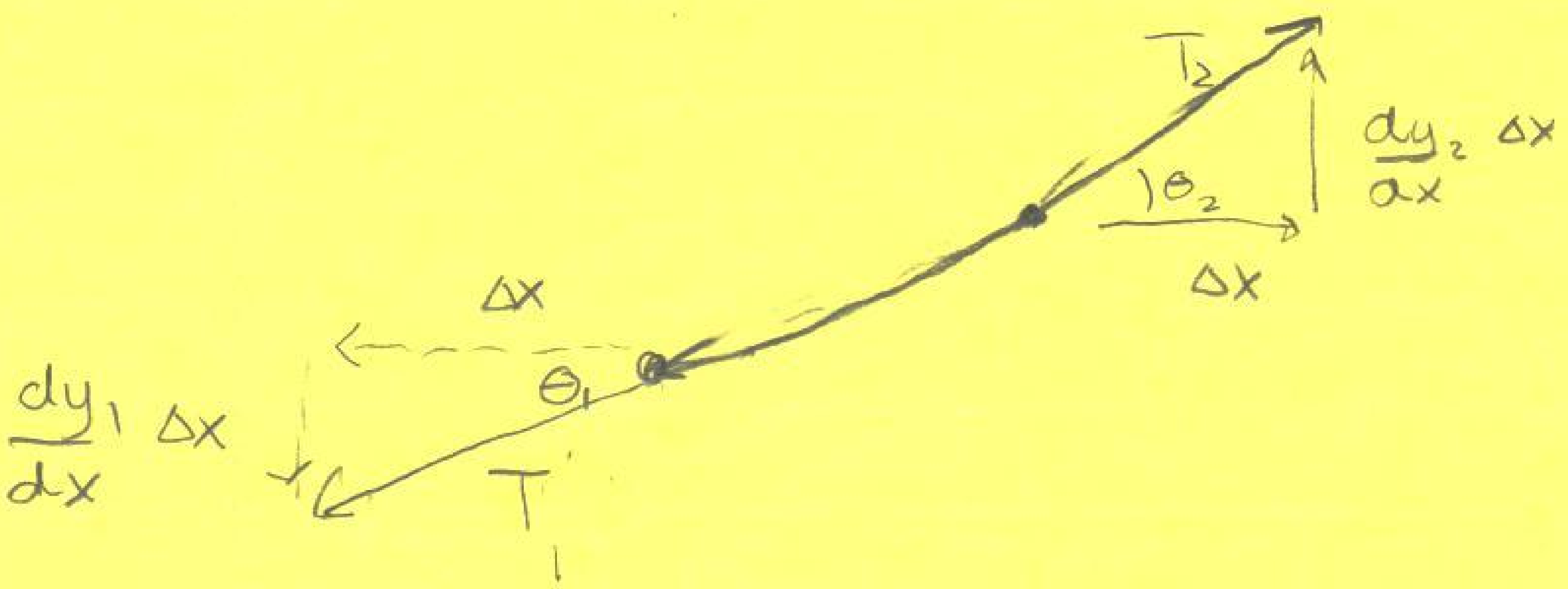
$A = 0.12$, $\omega = 4\pi$, $x = 1.6m$, $t = 0.2m$

$$\text{arg} = \frac{\pi}{8} 1.6m + 4\pi \, 0.2 = \pi$$

$$V_y = 0.120m \cdot 4\pi \cdot \overbrace{\cos \pi}^{-1} = -1.51 \, m/s$$

$$a_y = -A \omega^2 \underbrace{\sin \pi}_{0} = 0$$

Consider a bit of string - Newtons Laws give the Wave equ



$$\sin\theta_2 \simeq \tan\theta_2 \simeq \theta_2 = \frac{dy_2}{dx}$$

$$\sin\theta_1 \simeq \tan\theta_1 \simeq \theta_1 = \frac{dy_1}{dx}$$

So $F^y_{Net} = T\sin\theta_2 - T\sin\theta_1 = m\, a^y$

$$T\left(\frac{dy_2}{dx} - \frac{dy_1}{dx}\right) = \mu\, \Delta x\, \frac{\partial^2 y}{\partial t^2}$$

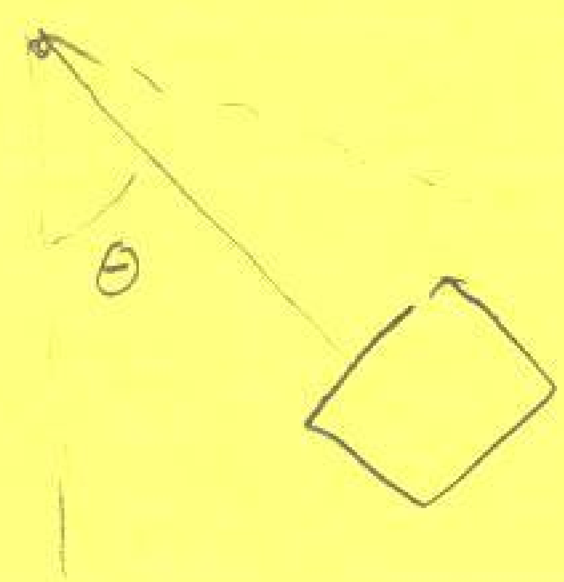
$$\underbrace{\frac{\partial^2 y}{\partial x^2} = \frac{\mu}{T}\frac{\partial^2 y}{\partial t^2}}$$

wave equation, normally

$$\frac{1}{v^2} = \frac{\mu}{T} \Longrightarrow v = \sqrt{\frac{T}{\mu}}$$

See Example 16.4 from book

Then suppose we have a block swinging back and forth



$\mu = 0.050\ kg/m$

$m = 2.0\ kg$

$\theta_{max} = 20^{G}$

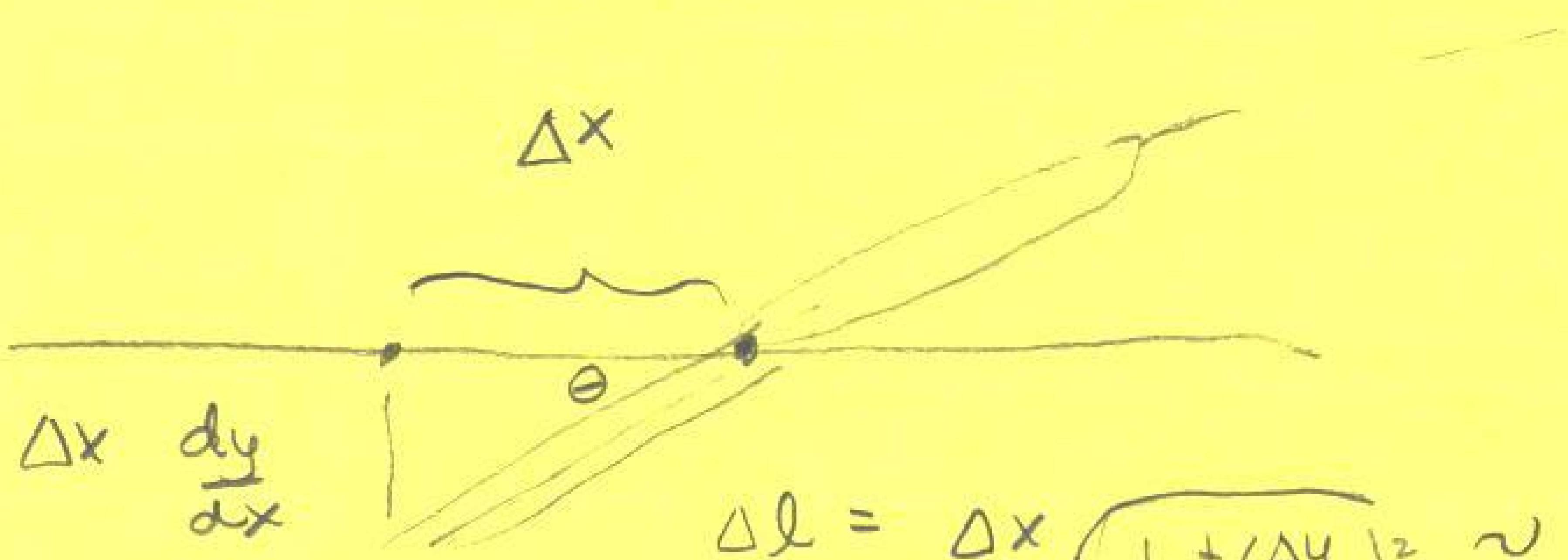
Find the maximum and minimum wave speeds of the pendulium

$V_{max} = 21.0\ m/s$

$v_{min} = 19.2 m/s$

Waves on String

- Competition between PE due to stretching and KE of moving up and down



$$\Delta l = \Delta x \sqrt{1 + \left(\frac{\Delta y}{\Delta x}\right)^2} \approx \Delta x \left(1 + \frac{1}{2}\left(\frac{dy}{dx}\right)^2\right)$$

$$\Delta l = \Delta x + \underbrace{\frac{1}{2} \Delta x \left(\frac{\partial y}{\partial x}\right)^2}_{\text{amount stretched}}$$

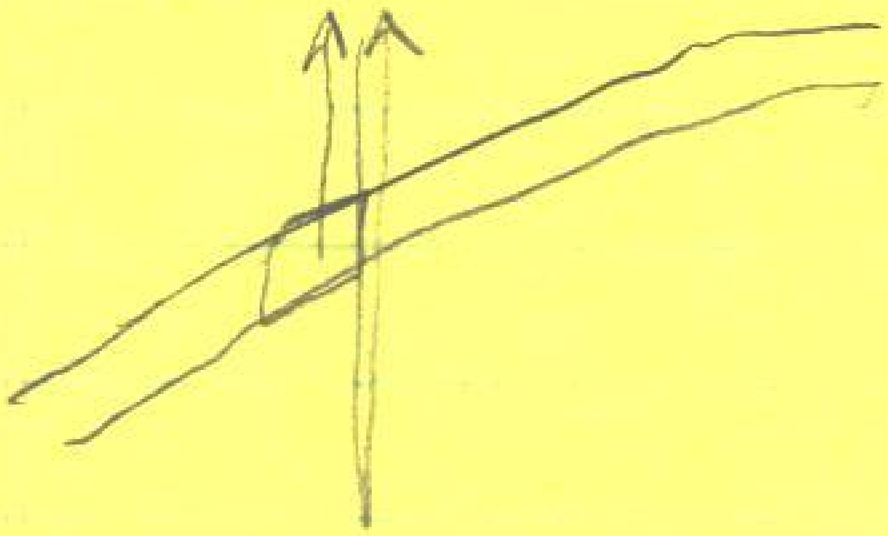
(1) Work done on rope

$$\Delta PE = F \cdot \text{distance} = \frac{T}{2} \Delta x \left(\frac{\partial y}{\partial x}\right)^2$$

$$\boxed{\frac{\Delta PE}{\Delta x} = \frac{T}{2}\left(\frac{\partial y}{\partial x}\right)^2} \leftarrow \text{potential energy per unit length}$$

(2)

Then consider KE

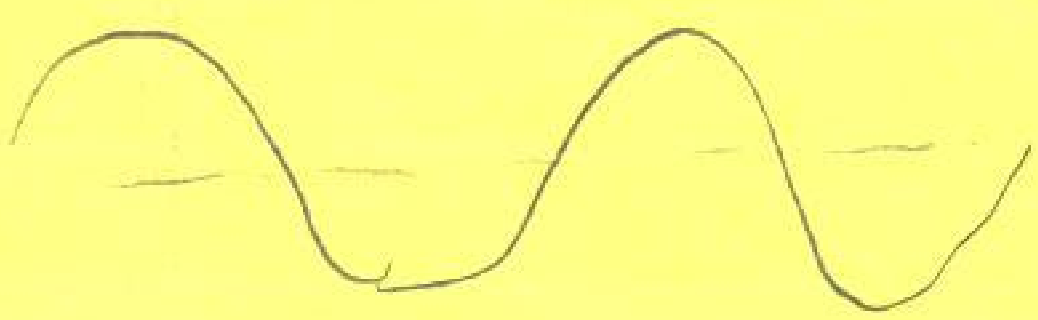


$$KE = \frac{1}{2} m v^2 = \frac{1}{2} \mu \, \Delta x \left(\frac{\partial y}{\partial t} \right)^2$$

So

$$\boxed{\frac{\Delta KE}{\Delta x} = \frac{\mu}{2} \left(\frac{\partial y}{\partial t} \right)^2}$$

For a sinusoidal wave consider the average KE



$$y(x,t) = A \sin(kx - \omega t + \phi)$$

$$\frac{\partial y}{\partial t} = - A\omega \sin (kx - \omega t + \phi)$$

$$\frac{dKE}{dx} = \frac{1}{2} \mu \left(\frac{\partial y}{\partial t} \right)^2 = \frac{1}{2} A^2 \omega^2 \sin^2 (kx - \omega t)$$

$$\left\langle \frac{dKE}{dx} \right\rangle = \frac{1}{2} A^2 \omega^2 \left\langle \sin^2(kx - \omega t) \right\rangle$$

Two ways to do it:

$$\underbrace{\left\langle \sin^2(kx - \omega t) \right\rangle = \frac{1}{\lambda} \int_0^\lambda \sin^2 kx \, dx = \ldots\ldots = \frac{1}{2}}$$

See book pg. 502
hard way

Better way:

$\langle \sin^2 \rangle = \langle \cos^2 \rangle$ and $\langle \sin^2 + \cos^2 \rangle = 1$

$$\langle \sin^2 \rangle = \frac{1}{2}$$

$$\boxed{\left\langle \frac{dKE}{dx} \right\rangle = \frac{\mu}{4} A^2 \omega^2}$$

Then $\left\langle \frac{dPE}{dx} \right\rangle = \frac{T}{2}\left(\frac{\partial y}{\partial x}\right)^2 = \frac{T}{2} A^2 k^2 \langle \sin^2 kx - \omega t \rangle$

$$= \frac{T}{4} A^2 k^2 \qquad V = \frac{\omega}{k}$$

$$= \frac{1}{4} \overbrace{\frac{T}{\mu}}^{v^2} k^2 A^2 \qquad V^2 = \frac{\omega^2}{k^2}$$

So

$$\boxed{\left\langle \frac{dPE}{dx} \right\rangle = \frac{1}{4}\mu A^2 \omega^2 \leftarrow \text{same as KE}}$$

$\langle$Potential$\rangle$ + $\langle$KE$\rangle$ are equal

- Proportional To ω^2 and A^2

Then consider one hump of wave moving through the wall

Then, in a time of one period the energy transported is

$$\lambda \quad \frac{dKE}{dx} + \frac{dPE}{dx} = E \text{ in one period}$$

Dividing by the time

$$\text{Power} = \frac{E}{T} = \overbrace{\frac{\lambda}{T}}^{v} \left(\frac{1}{4}\mu A^2 \omega^2 + \frac{1}{4}\mu A^2 \omega^2 \right)$$

$$\boxed{\text{Power} = \frac{1}{2}\mu A^2 \omega^2 v}$$

Example

Sinusoidal 5.0 cm in amplitude on string $\mu = 4.0 \times 10^{-2}$ kg/m, Source delivers a maximum power of 300 W, with tension 100 N, What is the highest frequency the source can operate

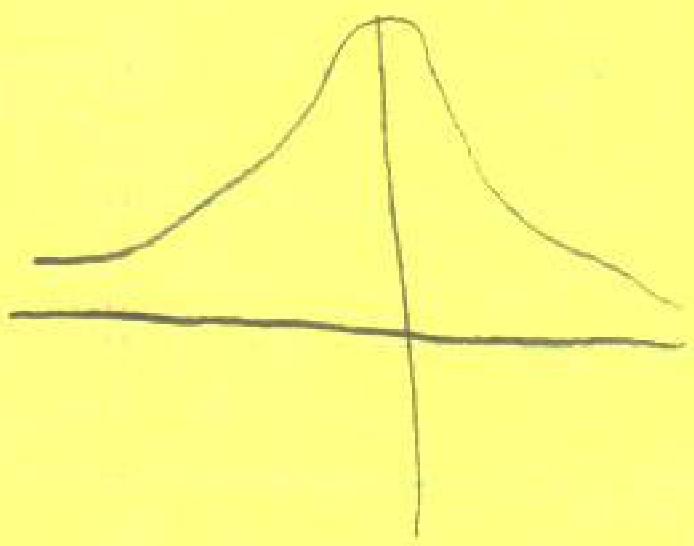
$$P_{max} = \frac{1}{2} \mu A^2 \omega_{max}^2 \cdot \underbrace{\sqrt{\frac{T}{\mu}}}_{v}$$

$A = 5.0\,cm$, $P_{max} = 300\,W$, $\mu = 0.04\,\frac{kg}{m}$

$\omega_{max} = 55.1\,Hz$

answer

Wave Motion



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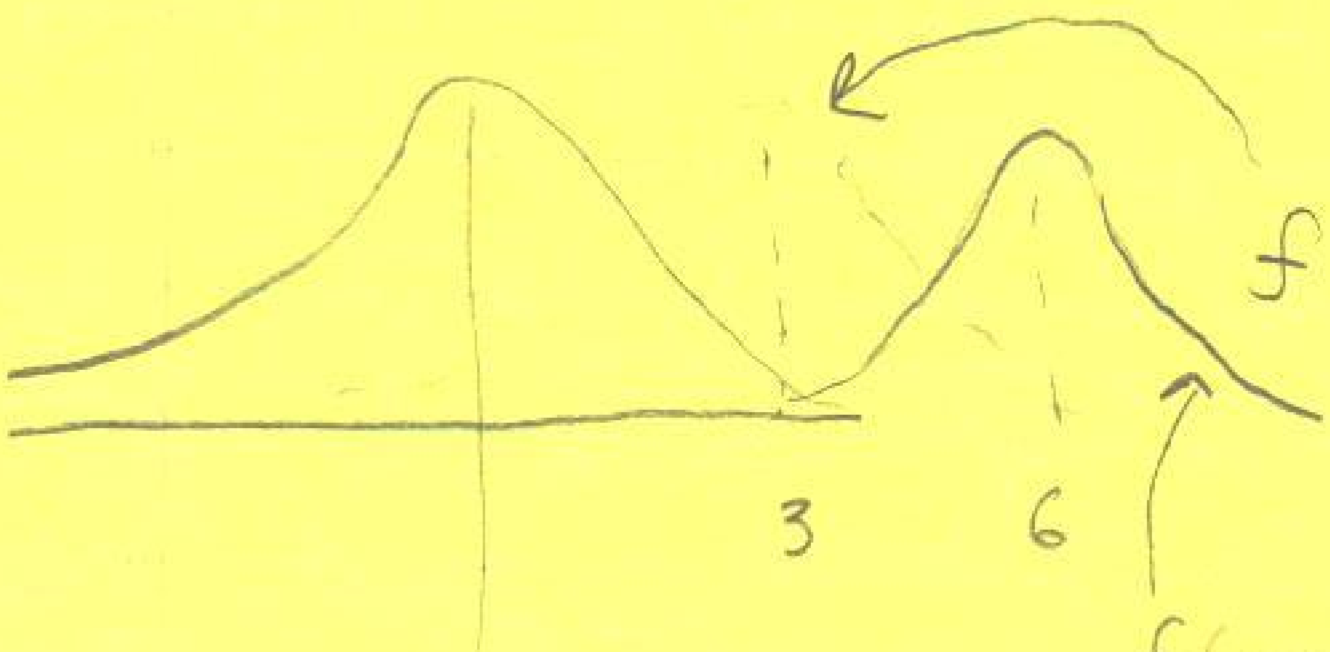

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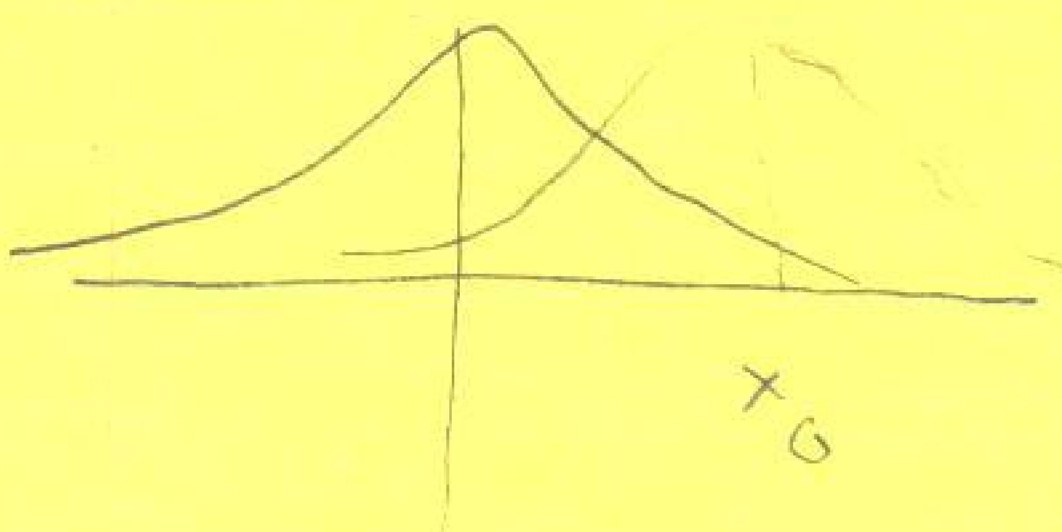

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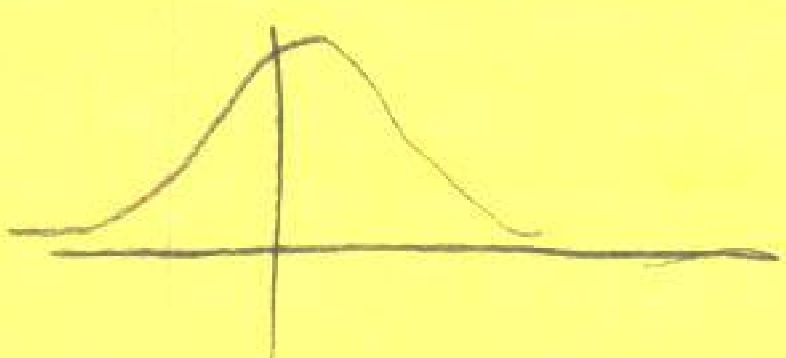

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moves to the left with speed v