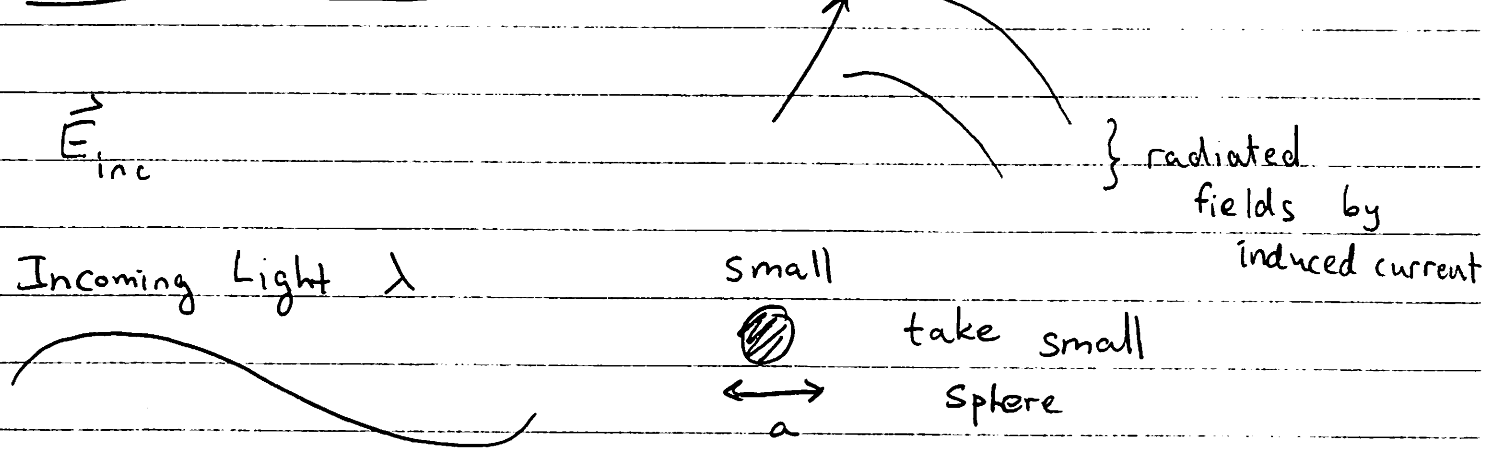
Scattering of Radiation



- Initially study the scattering of radiation by <u>small</u> objects $\lambda \gg a$

- Why does light scatter? The light induces currents in the object. The induced currents radiate.

  So, if you want to know how much light was scattered. You should first compute how the incoming field accelerates the charge, and then compute how the accelerated charges radiate

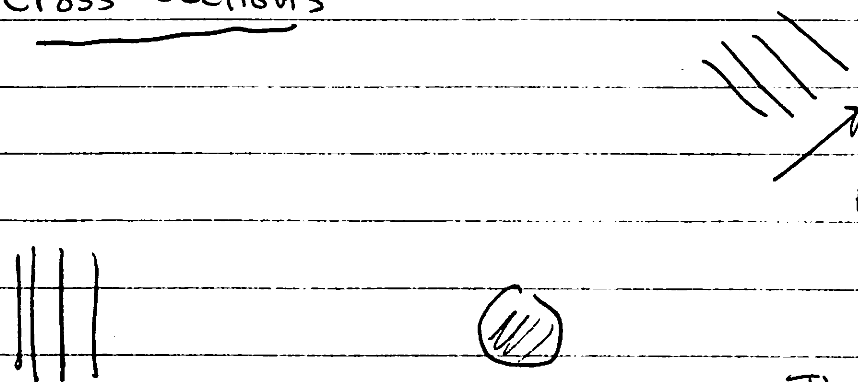
- For an extended object the acceleration in one point can create fields at another which influences the other points leading to a complicated standing wave.
  Treat small objects

Suggests Approximations

① Small objects. $\lambda \gg a$. The field can be considered constant over the extent of object

② Weak scattering. The induced fields, $\vec{E}_{scatt}$, are small compared to $\vec{E}_{inc}$

Cross Sections



$$E_{scatt} = \frac{\text{Const}\, e^{i\vec{k}\cdot\vec{r} - i\omega t}}{r}$$

$$\vec{E}_{inc} = \varepsilon_0 \vec{E}_0 \, e^{ikz - i\omega t}$$

The const is a vector which depends on direction and is $\propto E_0$

$$E_{scatt} = E_0 \, \vec{f}(\vec{k}) \, \frac{e^{i\vec{k}\cdot\vec{r} - i\omega t}}{r}$$

Then

"Scattering amplitude"

$$\vec{E} = \vec{E}_{inc}(r) + \vec{E}_{scatt}(r)$$

units meters

The incoming energy is:

time ave

$$\overline{\vec{S}\cdot\hat{z}} = \frac{1}{2} c |E_{inc}|^2 \leftarrow$$ time averaged energy per area per time

The outgoing energy per solid angle

$$\overline{\frac{dP(\varepsilon)}{d\Omega}} = r^2 \, \overline{\vec{S}_\varepsilon \cdot \vec{n}} = \frac{1}{2} c \left| \vec{r}\; \varepsilon^{*} \cdot \vec{E}_{scatt} \right|^2$$

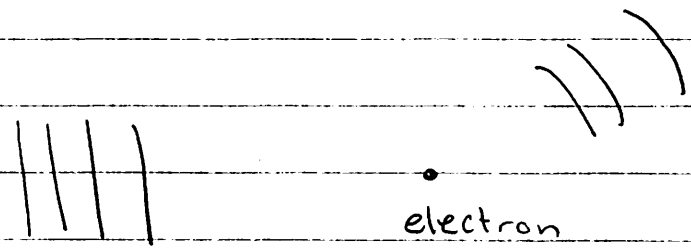
So define the cross section

$$\frac{d\sigma}{d\Omega}(\varepsilon;\varepsilon_0) = \frac{\frac{1}{2} c \left| r \vec{\varepsilon}^{*} \cdot E_{scatt} \right|^2}{\frac{1}{2} c |E_0|^2} = \left| \vec{\varepsilon}^{*} \cdot \vec{f}(k) \right|^2 \propto (\text{meters})^2$$

energy scattered (per solid angle + polarization ε) per incoming flux.

Thomson Cross Section

- Light electron scattering



$$\sigma = \frac{\langle \text{Power radiated} \rangle}{\frac{1}{2} c |E_{inc}|^2} = \frac{P}{\frac{1}{2} c |E_{inc}|^2}$$

For an non-rel electron experiencing acceleration a

$$\overline{P} = \frac{q^2}{4\pi} \frac{2}{3c^3} \overline{a^2}$$

The force / accel is determined by the incoming Light

$$\vec{a} = \frac{q \vec{E}_{inc}}{m} e^{-i\omega t}$$

Then

$$\overline{a^2} = \frac{1}{2} |a|^2 = \frac{q^2}{m^2} \frac{1}{2} |E_{inc}|^2$$

Thus

$$\sigma = \frac{\frac{q^2}{4\pi}\frac{2}{3c^3}\frac{q^2}{m^2}\frac{1}{2}|E_{inc}|^2}{\frac{1}{2}c\,|E_{inc}|^2}$$

$$= \frac{8\pi}{3}\underbrace{\left(\frac{q^2}{4\pi mc^2}\right)}^2 \equiv \frac{8\pi}{3}r_e^2$$

this is known as the classical electron radius

$$r_e = \frac{q^2}{4\pi mc^2}$$

Note

$$r_e = \left(\frac{q^2}{4\pi\hbar c}\right)\left(\frac{\hbar}{mc}\right) = \alpha\,\lambda\!\!\!^{-}_c$$

α ← 1/137

$\lambda\!\!\!^{-}_c$ ← compton wavelength (by 2π)

$\lambda_c = \frac{h}{mc}$ $\lambda\!\!\!^{-}_c = \frac{\hbar}{mc}$

Numerically

$$\lambda\!\!\!^{-}_c = \frac{\hbar c}{m_e c^2} = \frac{197\,\text{eV nm}}{0.5\,\text{MeV}}$$

Then

$$r_e = 2.8\,\text{fm} = 2.8 \text{ femptometers} = 2.8 \text{ fermi}$$

The cross section is

$$\sigma = \frac{8\pi}{3} r_e^2 = 66 \text{ fm}^2$$

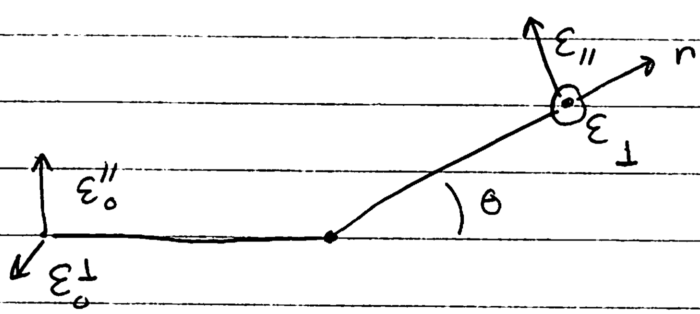
$1 \text{ fm}^2 = 10$ millibarn

$$= 660 \text{ millibarn}$$

$1 \text{ barn} = 10^{-24} \text{ cm}^2$

$$= 0.66 \text{ barns}$$ $\leftarrow$ units of cross section

Polarization



Want to show that the polarized cross section is

$$\frac{d\sigma}{d\Omega}(\vec{\varepsilon}; \vec{\varepsilon}_0) = r_e^2 \left| \varepsilon^* \cdot \varepsilon_0 \right|^2$$

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First Recall

$$E_{rad} = n \times n \times \frac{1}{c}\frac{\partial A_{rad}}{\partial t} = \frac{q}{4\pi r c^2}\, n \times n \times \vec{a}(t_e)$$

Lets Rederive this result, by approximating Lienard-Wiechert

$$A_{rad} = \frac{q}{4\pi r}\frac{V(\tau)/c}{(1 - n \cdot V(\tau)/c)} \qquad \tau = t - \frac{r}{c} + \frac{n \cdot r_*(\tau))}{c}$$

$$\simeq t - \frac{r}{c} \equiv t_e$$

The non-rel approximation replaces $\tau \simeq t_e = t - \frac{r}{c}$, and expands $V/c \ll 1$,

$$A_{rad} \simeq \frac{q}{4\pi r}\vec{V}(t_e)/c$$

And so

$$E_{rad} = \frac{q}{4\pi r c^2}\, n \times n \times \vec{a}(t_e)$$

The acceleration is along $\vec{\varepsilon}_0$

$$\vec{a} = \vec{\varepsilon}_0 \frac{q}{m} E_0 e^{-i\omega t}$$

Then we want to compute

$$|\varepsilon^* \cdot E_{rad}|^2 \propto |\varepsilon^* \cdot (n \times n \times \vec{a})|^2$$

final polarization

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Using $b(ac) - (ab)c$

$$\vec{\varepsilon}^* \cdot (n \times n \times a) = \varepsilon^* \cdot (-\vec{a} + \vec{n}(\vec{n} \cdot \vec{a}))$$

$$= -\vec{\varepsilon}^* \cdot \vec{a}$$ (since ε^* is transverse to $\vec{n}$ it projectso out the longitudinal pieces)

Then

$$\overline{|\varepsilon_0 \cdot n \times n \times a|^2} = \frac{q^2}{m^2} \overset{\text{time ave}}{\frac{1}{2}} |E_0|^2 \; |\vec{\varepsilon}^* \cdot \vec{\varepsilon}_0|^2$$

So we compute:

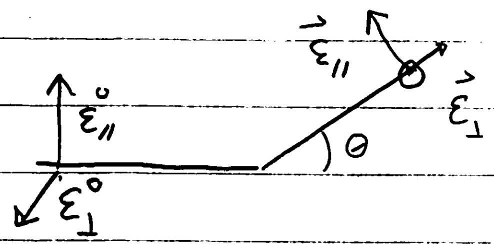
$$\frac{d\sigma(\vec{\varepsilon};\vec{\varepsilon}_0)}{d\Omega} = \frac{\overline{dP/d\Omega}}{\frac{c}{2}|E_0|^2} = \frac{\frac{1}{2} r^2 c |\varepsilon^* \cdot E_{rad}|^2}{\frac{1}{2} c |E_0|^2}$$

$$= \underbrace{\left(\frac{q^2}{4\pi m c^2}\right)^2}_{\equiv r_e^2} |\varepsilon^* \cdot \varepsilon_0|^2$$

$$= r_e^2 \; |\varepsilon^* \cdot \varepsilon_0|^2$$

Transverse and Parallel polarized Cross section

There are four cases here



① $\varepsilon^{\circ *}_{\parallel} \cdot \varepsilon_{\parallel} = \cos\theta$ (see figure)

② $\varepsilon^{\circ *}_{\perp} \cdot \varepsilon_{\perp} = 1$

③ $\varepsilon^{\circ *}_{\parallel} \cdot \varepsilon_{\perp} = 0$

④ $\varepsilon^{\circ *}_{\perp} \cdot \varepsilon_{\parallel} = 0$

So the cross section for initially unpolarized light (i.e. $\vec{\varepsilon}^{\circ}$ is 50% of time $\parallel$ & 50% of time $\perp$) to produce light polarized in the $\parallel$ or $\perp$ direction

$$\frac{d\sigma_{\parallel}}{d\Omega} = \frac{1}{2}\left[\frac{d\sigma}{d\Omega}(\varepsilon_{\parallel};\varepsilon^{\circ}_{\parallel}) + \cancel{\frac{d\sigma}{d\Omega}(\varepsilon_{\parallel};\varepsilon^{\circ}_{\perp})}^{\,0}\right]$$

$$= \frac{1}{2} r_e^2 \cos^2\theta$$

$$\frac{d\sigma_{\perp}}{d\Omega} = \frac{1}{2}\left[\cancel{\frac{d\sigma}{d\Omega}(\varepsilon_{\perp};\varepsilon^{\circ}_{\parallel})}^{\,0} + \frac{1}{2}\frac{d\sigma}{d\Omega}(\varepsilon_{\perp},\varepsilon^{\circ}_{\perp})\right]$$

$$= \frac{1}{2} r_e^2$$

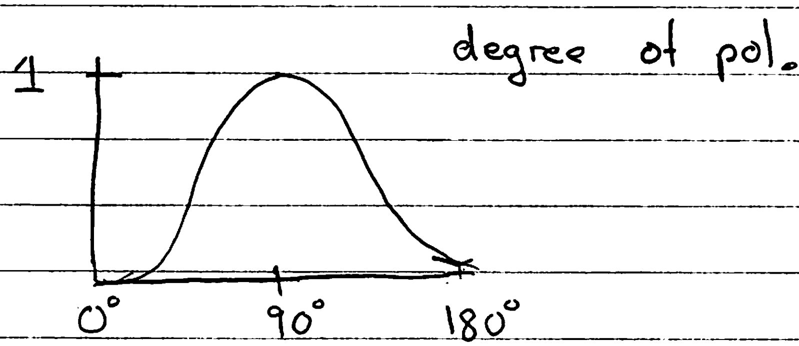
The cross section to produce light of <u>any</u> polarization by initially unpolarized light is

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma_{/\!/}}{d\Omega} + \frac{d\sigma_{\perp}}{d\Omega}$$

$$= r_e^2 \left(\frac{1+\cos^2\theta}{2} \right)$$

The degree of polarization depends on the angle

$$\text{degree of pol} = \frac{d\sigma_{\perp} - d\sigma_{/\!/}}{d\sigma_{\perp} + d\sigma_{/\!/}} = \frac{(1-\cos^2\theta)}{(1+\cos^2\theta)}$$



<u>Question:</u> Why is the light 100% transversely polarized at 90°?

<u>Ans.</u> : At 90° the current is up and down for the parallel case. Thus there is no component of the current transverse

Ans: continued ...

to the observation. Thus the cross section for this case vanishes