Hydrodynamics, freeze-out, and blast wave fits to flow spectra

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1. Hydrodynamics and Cooper-Frye prescription
2. Freeze-out systematics
3. Cooper-Frye spectra and "blast wave" models
4. Summary

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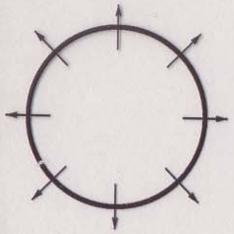
Flow – an unavoidable consequence of thermalization:

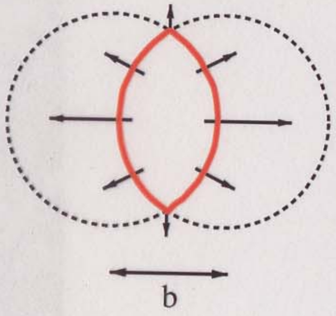
QGP $\Longrightarrow$ an (approximately) thermalized system of quarks and gluons
$\Longrightarrow$ thermal pressure gradients $\Longrightarrow$ **collective flow**

Radial flow:



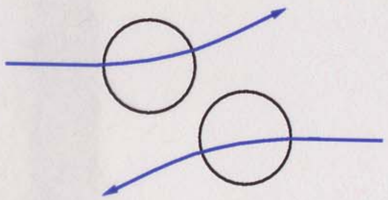
- $\rightarrow$ the only type of transverse flow in $b=0$ collisions between equal spherical nuclei
- $\rightarrow$ integrates pressure history over entire expansion stage
- $\rightarrow$ observable via effect of $\langle v_\perp \rangle$ on slope of $m_\perp$ spectra

Elliptic flow ($b \neq 0$ or collisions between deformed nuclei, e.g. U+U):



- $\rightarrow$ peaks at midrapidity
- $\rightarrow$ requires spatial deformation of reaction zone at thermalization
- $\rightarrow$ magnitude of signal probes degree and time of thermalization
- $\rightarrow$ shuts itself off as dynamics reduces deformation (H. Sorge)
- $\rightarrow$ sensitive to Equation of State during first $\sim 5\,\mathrm{fm}/c$

Directed flow ($b \neq 0$, $y \neq 0$):



- $\rightarrow$ generated very early while nuclei penetrate each other
- $\rightarrow$ dominated by early non-equilibrium processes
- $\rightarrow$ becomes weaker with increasing collision energy

Hydrodynamics – the natural tool to study flow:

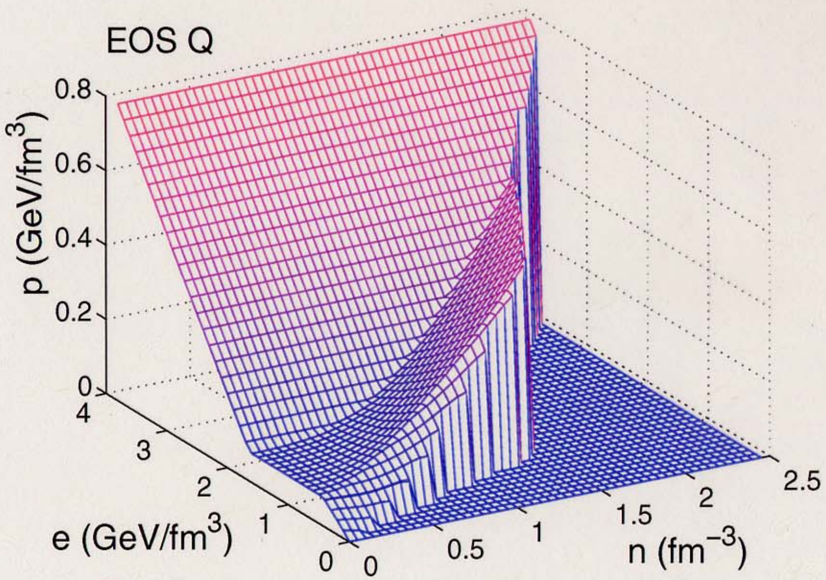
Relativistic Hydrodynamics:	Conservation of energy, momentum and baryon number	$\partial_\mu T^{\mu\nu} = 0$ $\partial_\mu j^\mu = 0$

with energy momentum tensor $T^{\mu\nu}(x) = (e(x)+p(x))\, u^\mu(x) u^\nu(x) - g^{\mu\nu} p(x)$
and baryon current $j^\mu(x) = n(x)\, u^\mu(x)$

Equation of state:

- EOS I: ultrarelativistic ideal gas, $p = \frac{1}{3} e$
- EOS H: hadron resonance gas, $p \sim 0.15\, e$
- EOS Q: Maxwell construction between EOS I and EOS H

critical temperature $T_{\text{crit}} = 0.164$ GeV
$\Rightarrow$ bag constant $B^{1/4} = 0.23$ GeV
latent heat $\Delta e = 1.15$ GeV/fm^3



Implement exact longitudinal boost invariance for simplicity ($Y \approx 0$ only)

Implement longitudinal scaling expansion

longitudinal expansion:

$\longrightarrow$ Bjorken scaling of flow: $v_z = z/t$ or $\eta_l = \eta$
($\eta_l = \frac{1}{2}\ln\frac{1+v_z}{1-v_z}$, $\eta = \frac{1}{2}\ln\frac{t+z}{t-z}$ and $\tau = \sqrt{t^2 - z^2}$)
implemented analytically ($\rightarrow$ Ollitrault 1992)

radial expansion:

$\longrightarrow$ transverse hydrodynamics, solved numerically:

$$
\begin{aligned}
\partial_\tau \tilde{T}^{\tau\tau} + \partial_x\left(\tilde{v}_x \tilde{T}^{\tau\tau}\right) + \partial_y\left(\tilde{v}_y \tilde{T}^{\tau\tau}\right) &= -p\,, \\
\partial_\tau \tilde{T}^{\tau x} + \partial_x\left(\bar{v}_x \tilde{T}^{\tau x}\right) + \partial_y\left(\bar{v}_y \tilde{T}^{\tau x}\right) &= -\partial_x \tilde{p}\,, \\
\partial_\tau \tilde{T}^{\tau y} + \partial_x\left(\bar{v}_x \tilde{T}^{\tau y}\right) + \partial_y\left(\bar{v}_y \tilde{T}^{\tau y}\right) &= -\partial_y \tilde{p}\,, \\
\partial_\tau \tilde{j}^{\tau} \;+\; \partial_x(\bar{v}_x \tilde{j}^{\tau}) \;+\; \partial_y(\bar{v}_y \tilde{j}^{\tau}) &= 0\,,
\end{aligned}
$$

with $\qquad \tilde{T}^{\mu\nu} = \tau T^{\mu\nu}\,, \quad \tilde{p} = \tau\, p\,, \quad \tilde{j} = \tau\, j\,,$

and energy flow velocities

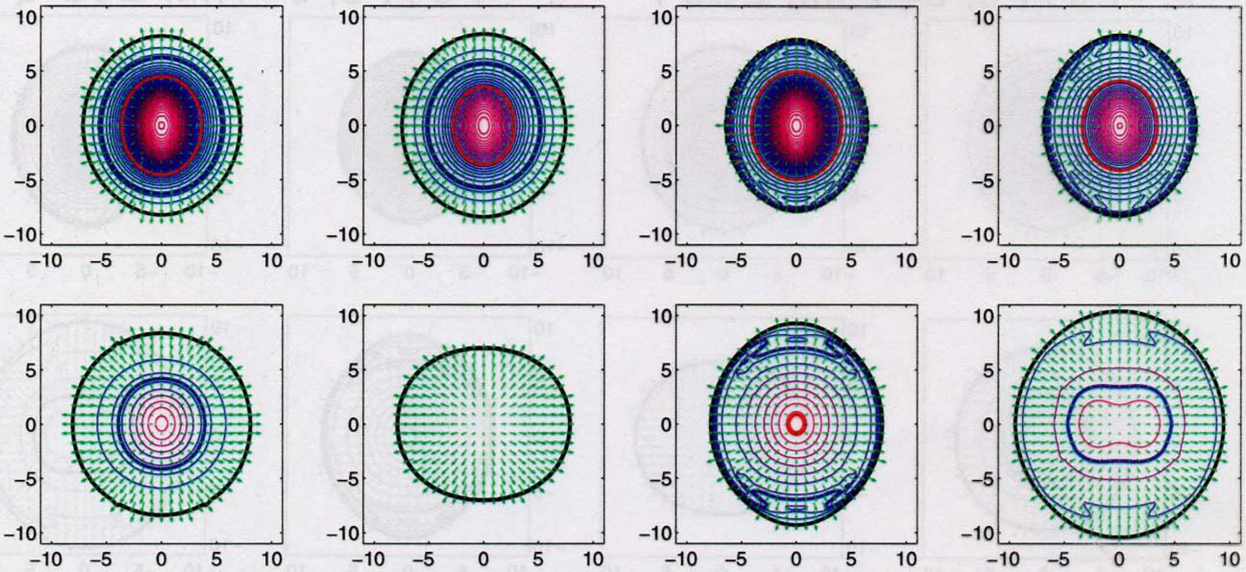
$$\bar{v}_i = v_i \cosh\eta\,, \quad \tilde{v}_i = \frac{T^{\tau i}}{T^{\tau\tau}}\,, \quad (i = x, y)$$

Study only $y = 0$ (midrapidity)

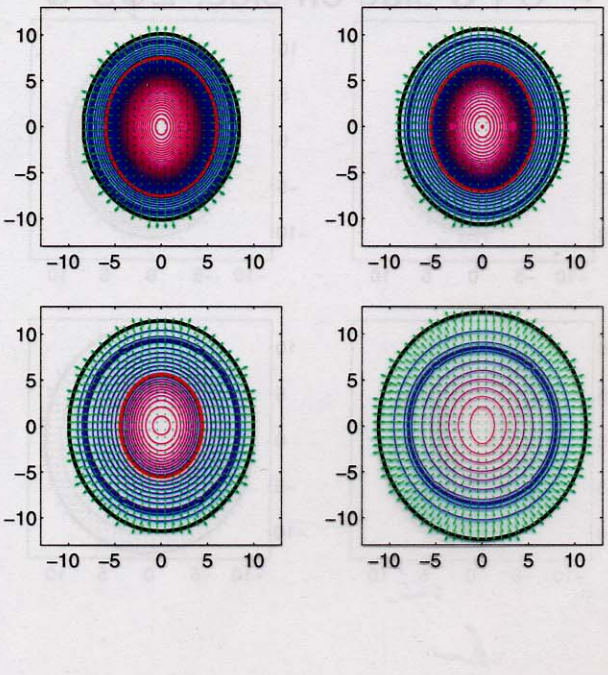
Evolution of energy density, $T_0 \approx 500 MeV$ @ $\tau_{equ} = 0.4$ fm/c

snapshots at τ=3.2, 4.0, 5.6 and 8.0 fm/c after initialization

Pb+Pb, b=7 fm, EOS I Pb+Pb, b=7fm, EOS Q



U+U side-on-side, EOS Q



Freeze-out:

- geometric freeze-out:

$$\lambda_i \approx R$$

$$\Rightarrow \quad \tau^{(i)}_{scatt} = \frac{\lambda_i}{\langle v_i \rangle} \approx \frac{1}{\sum_j \langle v_{ij}\, \sigma_{ij} \rangle \rho_j}$$

$$\approx \tau^{(i)}_{escape} = \frac{R}{\langle v_i \rangle} \approx \frac{R}{c}$$

- dynamical freeze-out:

time between collisions ≈ Hubble time

$$\Rightarrow \quad \tau^{(i)}_{scatt} \approx \tau_{expansion} = \frac{1}{\partial \cdot u(x)}$$

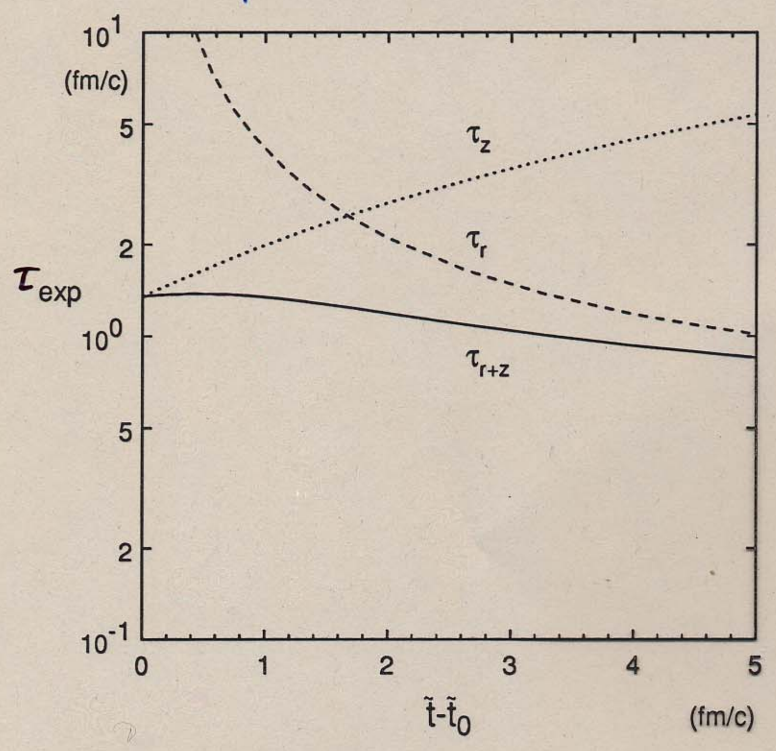
RHICs freeze out dynamically
(just as the early universe)

(Schnedermann + Heinz, PRC 50 (1994) 1675)

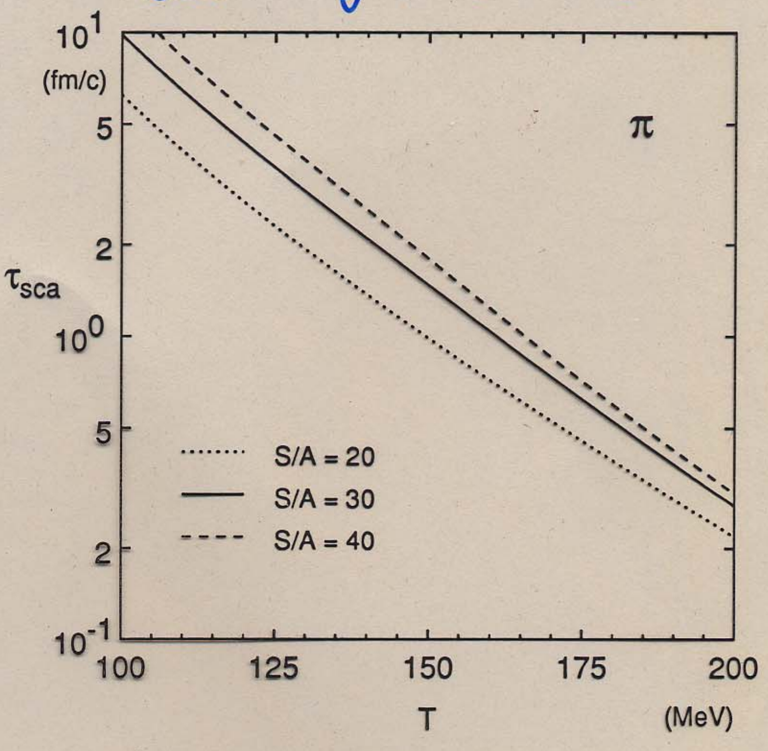
Freeze-out dominated by transverse expansion!

(Schnedermann + Heinz, PRC 50 (1994) 1675)

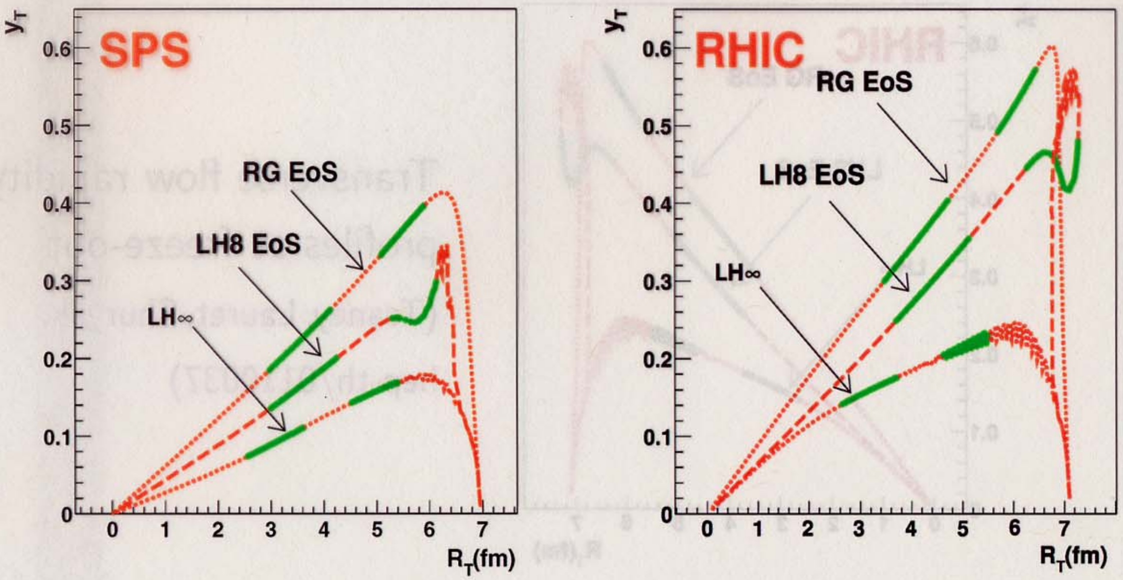
Expansion time scale



Scattering time scale



Expansion rates at the SPS and RHIC



Transverse flow rapidity profiles at freeze-out (Teaney, Lauret, Shuryak, hep-th/0110037)

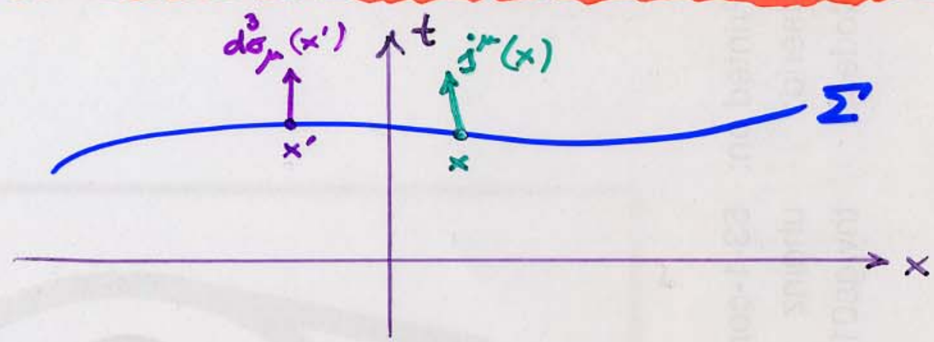
Transverse flow rapidity $y_\perp \approx \xi r$ with $\xi \approx 0.045\,\mathrm{fm}^{-1}$ at the SPS
and $\xi \approx 0.07\,\mathrm{fm}^{-1}$ at RHIC

Expansion rate $\partial \cdot u \approx \tau^{-1} + 2\xi$ (Kolb, nucl-th/0304036)

Expansion time scale $\frac{1}{\partial \cdot u}\Big|_{\mathrm{freeze-out}} = \tau_{\mathrm{exp}} \approx 6.3\,\mathrm{fm}/c$ in Pb+Pb at the SPS
and $\tau_{\mathrm{exp}} \approx 4.8\,\mathrm{fm}/c$ in Au+Au at RHIC

Momentum spectra from an expanding thermal source:

$$N = \int_\Sigma j^\mu(x)\, d^3\sigma_\mu(x)$$



$$= \int_\Sigma d^3\sigma_\mu(x) \frac{g}{(2\pi)^3} \int \frac{d^3p}{E}\, p^\mu\, f(x,p)$$

phase-space distr. function

Lorentz-invariant momentum int. measure

flux factor

$$\Rightarrow \quad E\frac{dN}{d^3p} = \frac{dN}{dy\, m_\perp dm_\perp\, d\phi_p} = \frac{g}{(2\pi)^3}\int_\Sigma p^\mu d\sigma_\mu(x)\, f(x,p)$$

$y = \frac{1}{2}\ln\frac{E+p_z}{E-p_z} = \frac{1}{2}\ln\frac{1+v_p}{1-v_p}$

rapidity

$m_\perp = \sqrt{m^2+p_\perp^2}$

transverse mass

$E = m_\perp \operatorname{ch} y$

$p_z = m_\perp \operatorname{sh} y$

$E^2 - p_\perp^2 - p_z^2 = m^2$

Cooper - Frye formula (1974)

- thermal equilibrium distribution:

$$f_{eq}(x,p) = \frac{1}{e^{[p\cdot u(x)-\mu(x)]/T(x)} \pm 1} = \sum_{n=1}^{\infty} (\mp)^{n+1}\, e^{-n[p\cdot u(x)-\mu(x)]/T(x)}$$

- collective (hydrodynamic) flow velocity:

η_L = longitudinal flow rapidity

$$u^\mu = \gamma_\perp (\mathrm{ch}\,\eta_L,\ v_x,\ v_y,\ \mathrm{sh}\,\eta_L)$$

$\gamma_\perp = 1/\sqrt{1-v_\perp^2}$, $v_\perp^2 = v_x^2 + v_y^2$

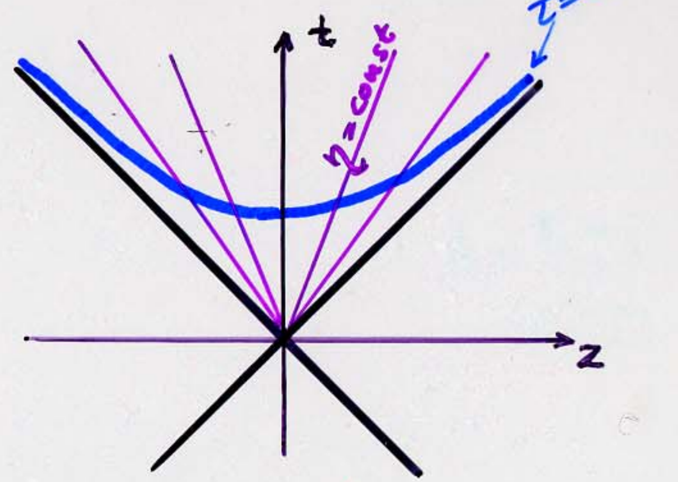
"Bjorken flow": $\eta_L(\vec{x},t) = \eta_L(\vec{x}_\perp,\eta,\tau) \overset{!}{=} \eta$

$\eta = \frac{1}{2}\ln\frac{t+z}{t-z}$ space-time rapidity

$\tau = \sqrt{t^2-z^2}$ long. proper time

$$\Rightarrow \frac{1}{2}\ln\frac{1+v_L}{1-v_L} \overset{!}{=} \frac{1}{2}\ln\frac{t+z}{t-z}$$

$$\Rightarrow \boxed{v_L = \frac{z}{t}}$$



- momentum $p^\mu = (m_\perp \mathrm{ch}\,y,\ p_x,\ p_y,\ m_\perp \mathrm{sh}\,y)$

- freeze-out surface $\Sigma^\mu = (\tau_f(\vec{x}_\perp)\,\mathrm{ch}\,\eta,\ \vec{x}_\perp,\ \tau_f(\vec{x}_\perp)\,\mathrm{sh}\,\eta)$

"longitudinal boost-invariance"

- normal vector $d^3\sigma_\mu = -\varepsilon_{\mu\nu\lambda\rho} \frac{\partial\Sigma^\nu}{\partial x} \frac{\partial\Sigma^\lambda}{\partial y} \frac{\partial\Sigma^\rho}{\partial \eta} \, dx\, dy\, d\eta$

$$= \left(\operatorname{ch}\eta, -\frac{\partial\tau_f}{\partial x}, -\frac{\partial\tau_f}{\partial y}, -\operatorname{sh}\eta\right) \tau_f(\vec{x}_\perp)\, d^2x_\perp\, d\eta$$

$$\Rightarrow \quad p\cdot d^3\sigma_\mu = \left(m_\perp \operatorname{ch}(y-\eta) - \vec{p}_\perp\cdot\vec{\nabla}_\perp\tau_f(\vec{x}_\perp)\right)\tau_f(\vec{x}_\perp)\, d^2x_\perp\, d\eta$$

- Boltzmann exponent $p\cdot u = \gamma_\perp\left(m_\perp \operatorname{ch}(y-\eta) - \vec{v}_\perp\cdot\vec{p}_\perp\right)$

- $\Rightarrow$ Cooper-Frye spectrum (for long. boost invariant systems)

$$\boxed{\begin{aligned}\frac{dN}{dy\, m_\perp dm_\perp d\phi_p} &= \frac{g}{(2\pi)^3}\sum_{n=1}^{\infty}(\mp)^{n+1}\int d^2x_\perp\, \tau_f(\vec{x}_\perp)\, e^{n\mu(\vec{x}_\perp)/T(\vec{x}_\perp)}\, e^{n\gamma_\perp(\vec{x}_\perp)\vec{v}_\perp(\vec{x}_\perp)\cdot\vec{p}_\perp} \\ &\times \underbrace{\int_{-\infty}^{\infty} d\eta \left(m_\perp \operatorname{ch}(\eta-y) - \vec{p}_\perp\cdot\vec{\nabla}_\perp\tau_f(\vec{x}_\perp)\right) e^{-n\, m_\perp \operatorname{ch}(\eta-y)\,\gamma_\perp(x_\perp)/T(x_\perp)}}_{2\left(m_\perp K_1(n\beta_\perp) - \vec{p}_\perp\cdot\vec{\nabla}_\perp\tau_f(\vec{x}_\perp)\, K_0(n\beta_\perp)\right)}\end{aligned}}$$

$$\beta_\perp \equiv \frac{\gamma_\perp m_\perp}{T} = \beta_\perp(\vec{x}_\perp)$$

- $b=0 \to$ azimuthal symmetry $\to \phi$ int. can be done:

$$\boxed{\frac{dN}{dy\, m_\perp dm_\perp} = \frac{g\, m_\perp}{2\pi^2}\sum_{n=1}^{\infty}(\mp)^{n+1}\int_0^{\infty} r\,dr\, \tau_f(r)\, e^{n\mu(r)/T(r)}\left[K_1(n\beta_\perp) I_0(n\alpha_\perp) - \frac{p_\perp}{m_\perp}\frac{\partial\tau_f}{\partial r} K_0(n\beta_\perp) I_1(n\alpha_\perp)\right]}$$

$$\alpha_\perp = \gamma_\perp v_\perp p_\perp / T = \alpha_\perp(r)$$

Elliptic flow from Cooper-Frye formula:

$$v_2(p_\perp; b) = \langle \cos(2\phi_p) \rangle_b = \frac{\int d\phi_p \cos(2\phi_p) \frac{dN}{dy\, m_\perp dm_\perp d\phi}(b)}{\int d\phi_p \frac{dN}{dy\, m_\perp dm_\perp d\phi_p}}$$

$\vec{x}_\perp = (r, \phi_s)$

$$= \frac{\sum_{n=1}^{\infty} (\mp)^{n+1} \int_0^\infty r\,dr \int_{-\pi}^{\pi} d\phi_s\, \tau_f(r,\phi_s)\, e^{n\mu(r,\phi_s)/T(r,\phi_s)}\, \mathcal{N}_n(r,\phi_s; p_\perp)}{\sum_{n=1}^{\infty} (\mp)^{n+1} \int_0^\infty r\,dr \int_{-\pi}^{\pi} d\phi_s\, \tau_f(r,\phi_s)\, e^{n\mu(r,\phi_s)/T(r,\phi_s)}\, \mathcal{D}_n(r,\phi_s; p_\perp)}$$

with

$$\mathcal{N}_n(r,\phi_s;p_\perp) = \cos(2\phi_s)\left[m_\perp I_2(n\alpha_\perp) K_1(n\beta_\perp) - p_\perp \frac{\partial \tau_f}{\partial r} \frac{I_1(n\alpha_\perp) + I_3(n\alpha_\perp)}{2} K_0(n\beta_\perp)\right]$$
$$+ \sin(2\phi_s)\, p_\perp \frac{\partial \tau_f}{\partial \phi_s} \frac{I_1(n\alpha_\perp) - I_3(n\alpha_\perp)}{2} K_1(n\beta_\perp)$$

$$\mathcal{D}_n(r,\phi_s;p_\perp) = m_\perp I_0(n\alpha_\perp) K_1(n\beta_\perp) - p_\perp \frac{\partial \tau_f}{\partial r} I_1(n\alpha_\perp) K_0(n\beta_\perp)$$

U.H. + S.M.H. Wong, PRC66 (2002) 014907

- Boltzmann approximation: keep only $n=1$ term:
  ($\longrightarrow$ good for all hadrons except pions!)

$$\boxed{\frac{dN}{dy\, m_\perp dm_\perp} = \frac{g}{\pi^2}\int_0^\infty r\,dr\, \underbrace{n(r)}_{\substack{\uparrow \\ \text{radial density} \\ \text{profile}}} \left[m_\perp K_1\left(\frac{m_\perp \operatorname{ch}\rho}{T}\right) I_0\left(\frac{p_\perp \operatorname{sh}\rho}{T}\right) - p_\perp \frac{\partial \tau_f}{\partial r} K_0\left(\frac{m_\perp \operatorname{ch}\rho}{T}\right) I_1\left(\frac{p_\perp \operatorname{sh}\rho}{T}\right)\right]}$$

(Schnedermann, Sollfrank, Heinz, PRC 48 (93) 2462)

(independent of y $\leftrightarrow$ boost invariance)

- Looks $\approx$ exponential in $m_\perp$ with inverse slope T_{slope}:

$$\frac{dN}{m_\perp dm_\perp} \sim e^{-m_\perp / T_{slope}(m_\perp)}$$

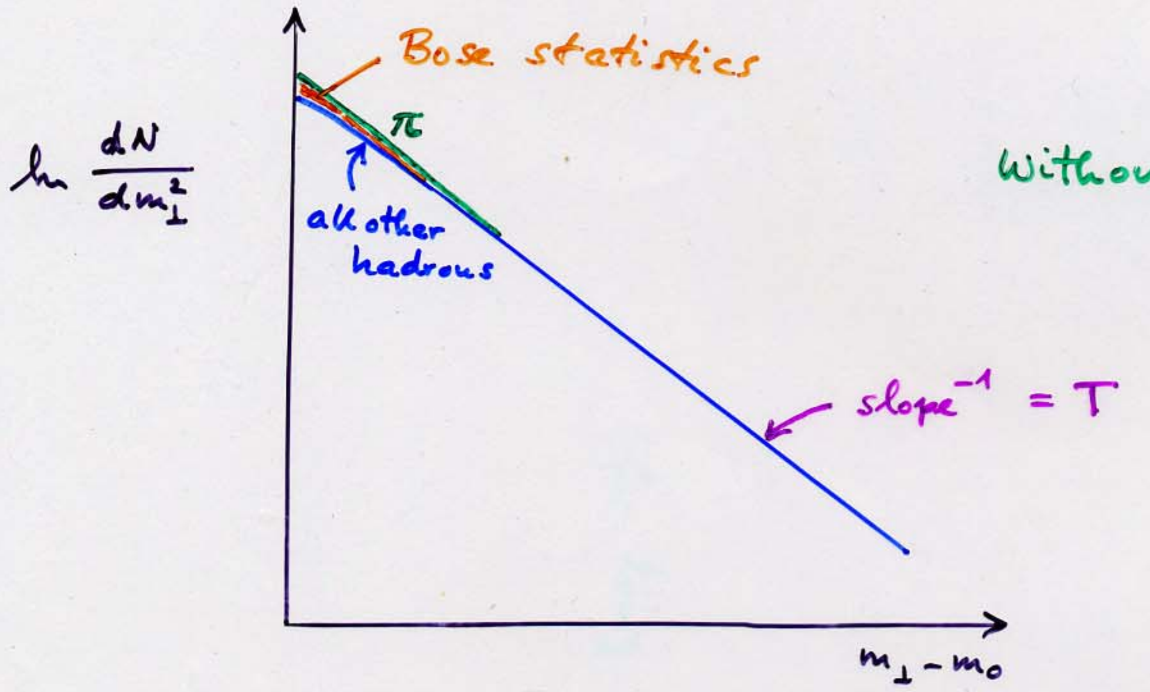
! not <u>really</u> exponential !

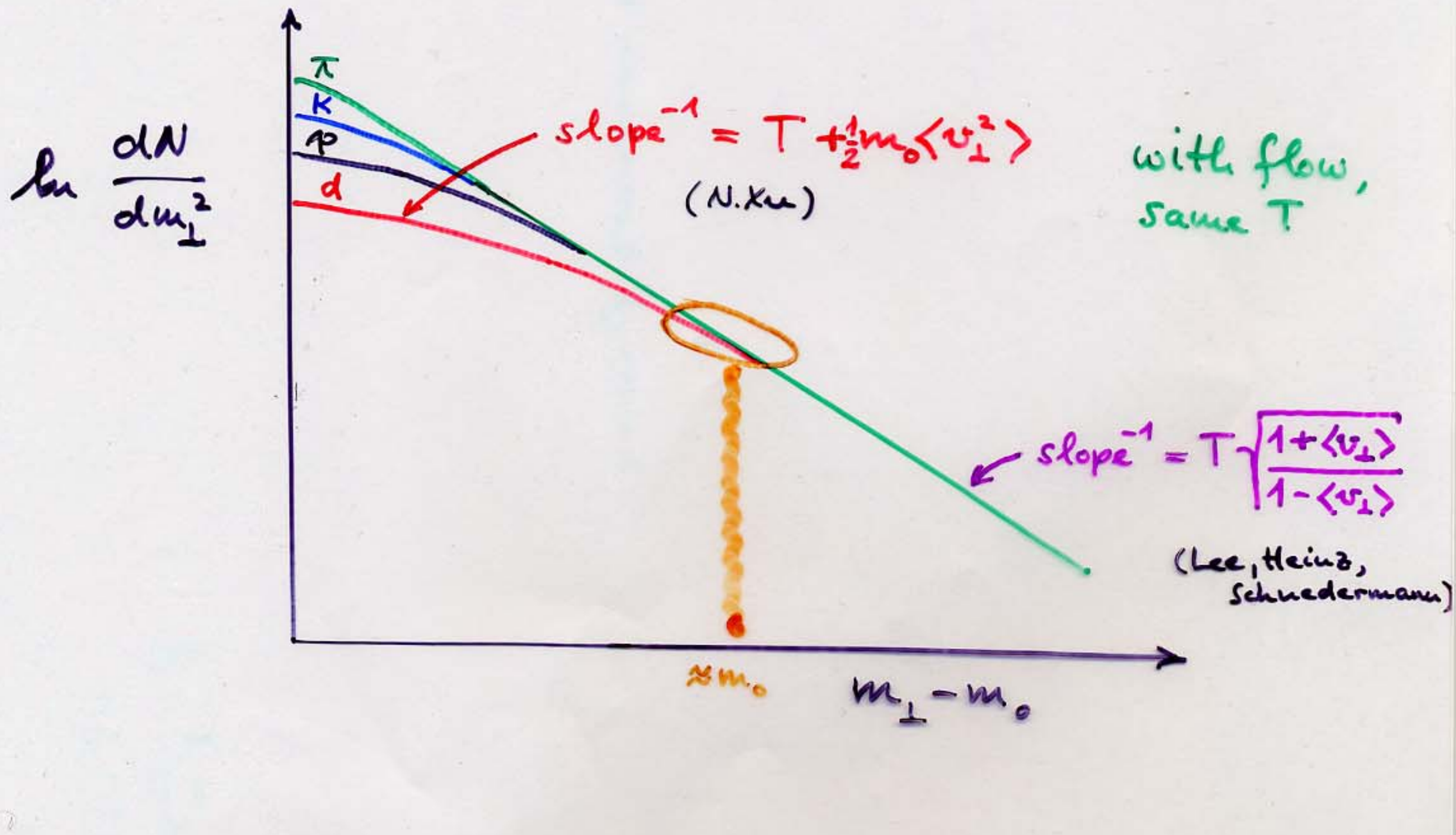
Two important limits:

- Nonrelativistic, $p_\perp \ll m_0$: $T_{slope} \approx T_f + \frac{1}{2} m \langle v_\perp \rangle^2$ (exact for Gaussian $n(r)$ + linear $v_\perp(r)$)
- Relativistic, $p_\perp > m_0$: $T_{slope} \approx T_f \sqrt{\frac{1+\langle v_\perp\rangle}{1-\langle v_\perp\rangle}}$ "blueshift"

Main effect of radial flow on single-particle spectra:

flattening of $m_\perp$-spectra ("blueshift")

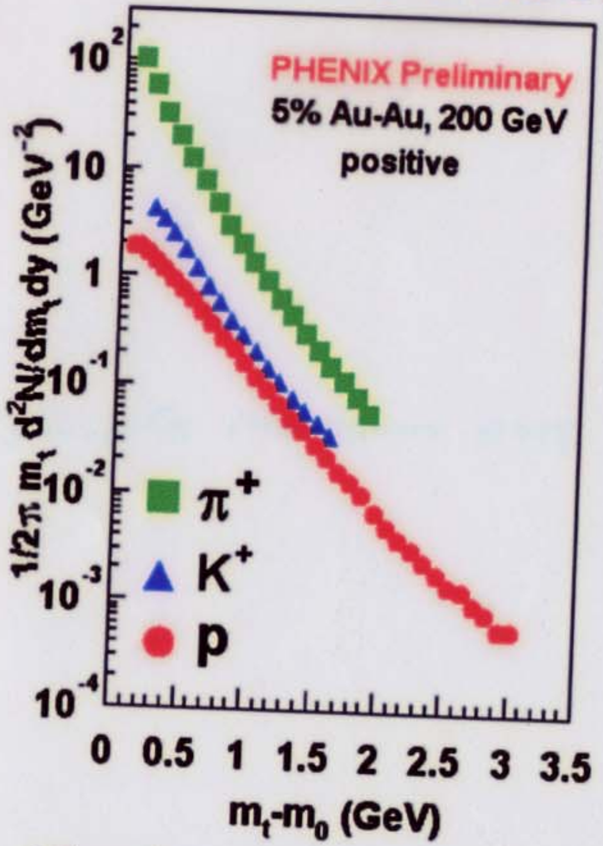


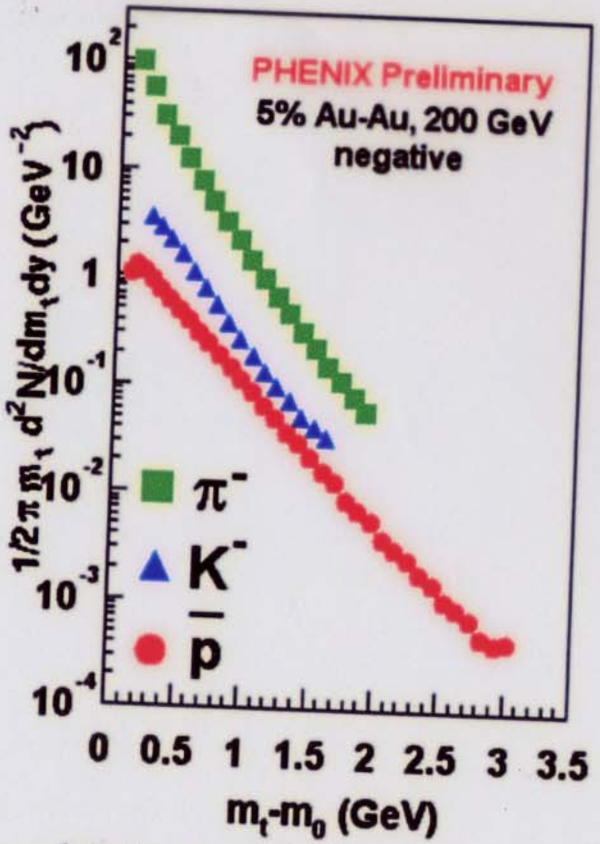


Rhic:

Transverse Kinetic Energy Spectra

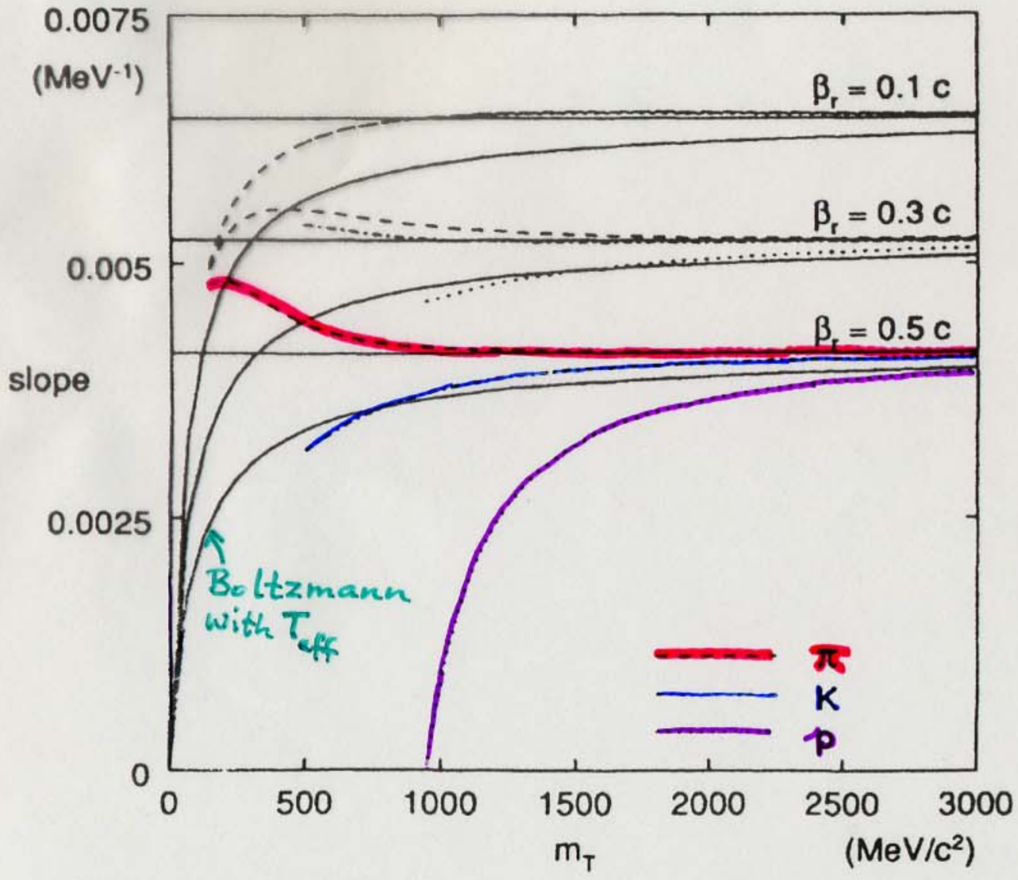
5% Au-Au at $\sqrt{s_{NN}} = 200$ GeV





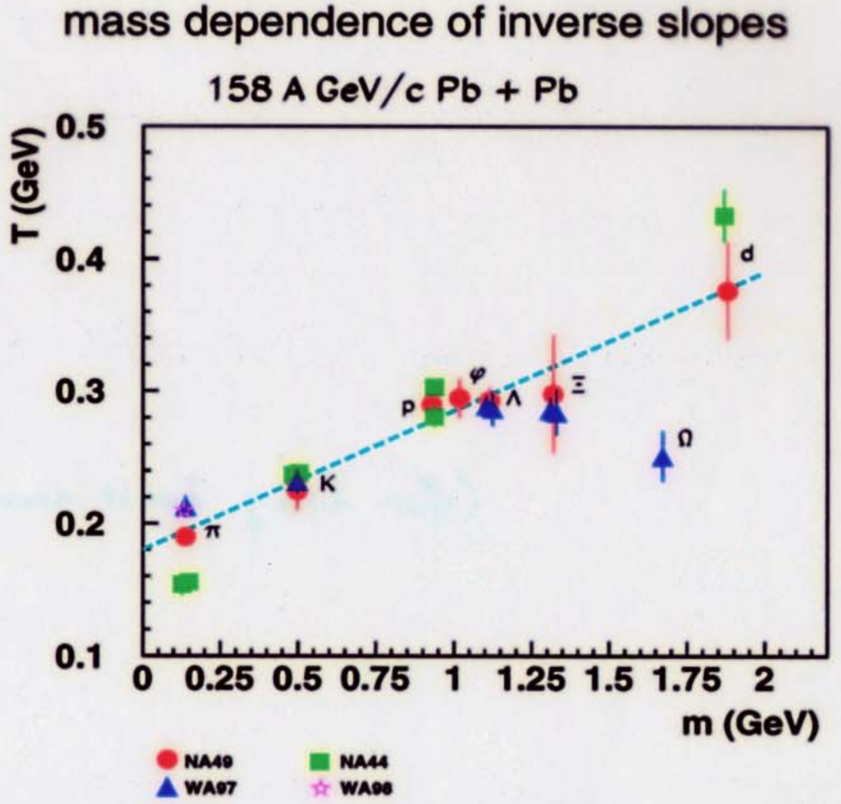
- Kinetic energy spectra broaden with increasing mass, from π to K to p (they are not parallel).

local $m_\perp$ slopes:



(Schnedermann et al, PRC 48 (1993) 2462)

Strong Collective Expansion – "The Little Bang"



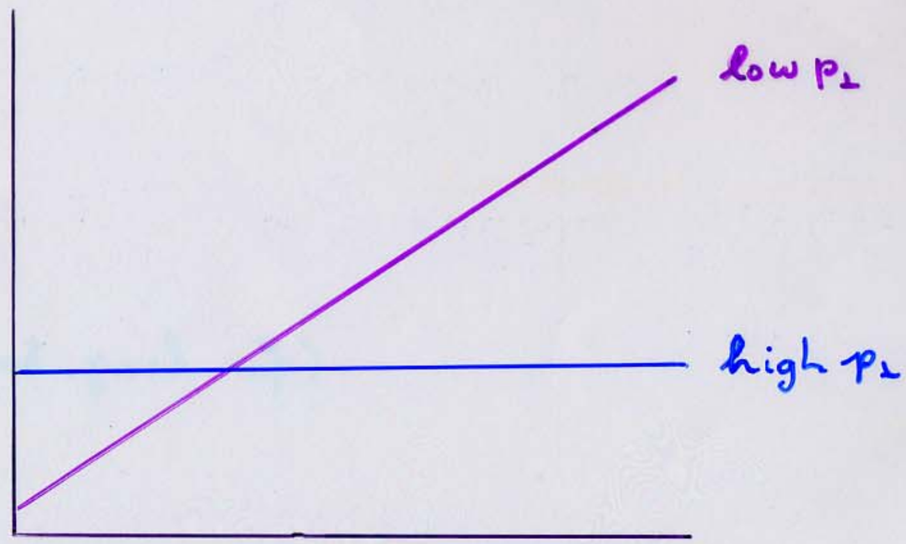
Simultaneous analysis of slope of this plot and two-particle correlations yields expansion velocity $\langle v_\perp \rangle \approx 0.55\,c$ at hadronic decoupling.

$T_f = 120$ MeV

$\langle v_\perp \rangle = 0.55c$

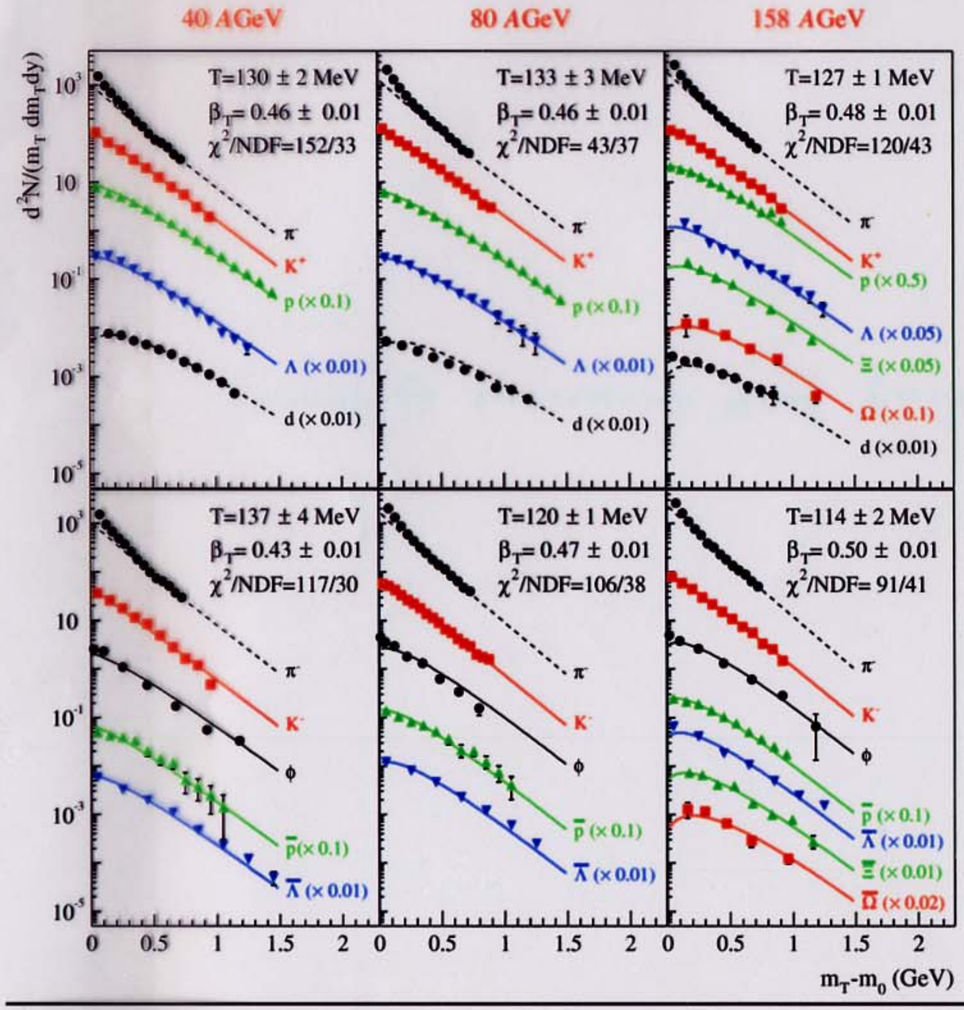
$T_{eff} = T_f \sqrt{\frac{1+\langle v_\perp \rangle}{1-\langle v_\perp \rangle}}$

$T_{eff} = T_f + \frac{1}{2} m \langle v_\perp \rangle^2$



Midrapidity $m_\perp$-spectra at the SPS:

NA49 Coll., M. van Leeuwen, Quark Matter 2002



- Two-parameter flow-fit with (Schnedermann, Sollfrank, U.H., PRC 48 (1993) 2462)

$$\frac{dN}{dy\, m_\perp dm_\perp} \sim m_\perp K_1\left(\frac{m_\perp \mathrm{ch}\rho}{T}\right) I_0\left(\frac{p_\perp \mathrm{sh}\rho}{T}\right)$$

$(\beta_T = \mathrm{th}\rho)$

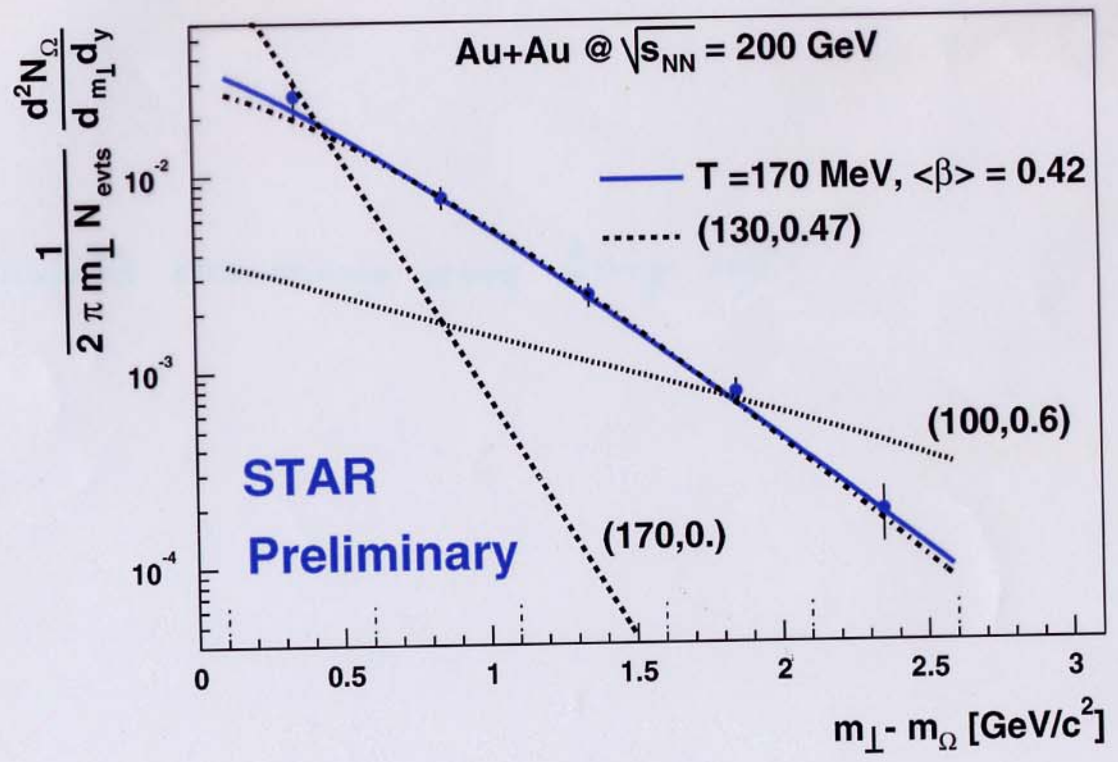
- At all three beam energies:

$$T \approx 120 - 130\,\mathrm{MeV}$$
$$\beta_T \approx 0.4 - 0.5$$

- Fit also describes Ξ, Ω and deuteron spectra

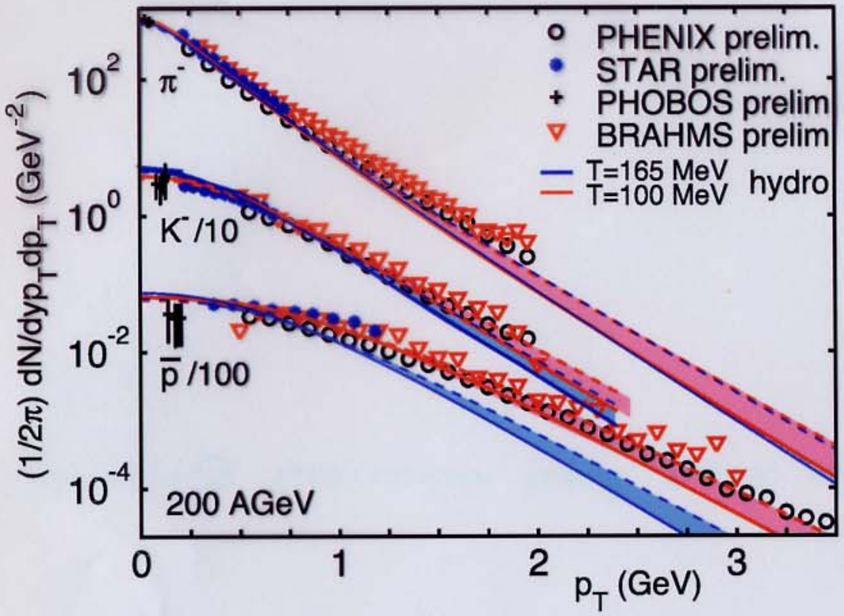
Blast wave fits to Omega spectra:

Au+Au @ $\sqrt{s} = 200\,A$ GeV [C. Suire (STAR), NPA 715 (2003) 470c]:

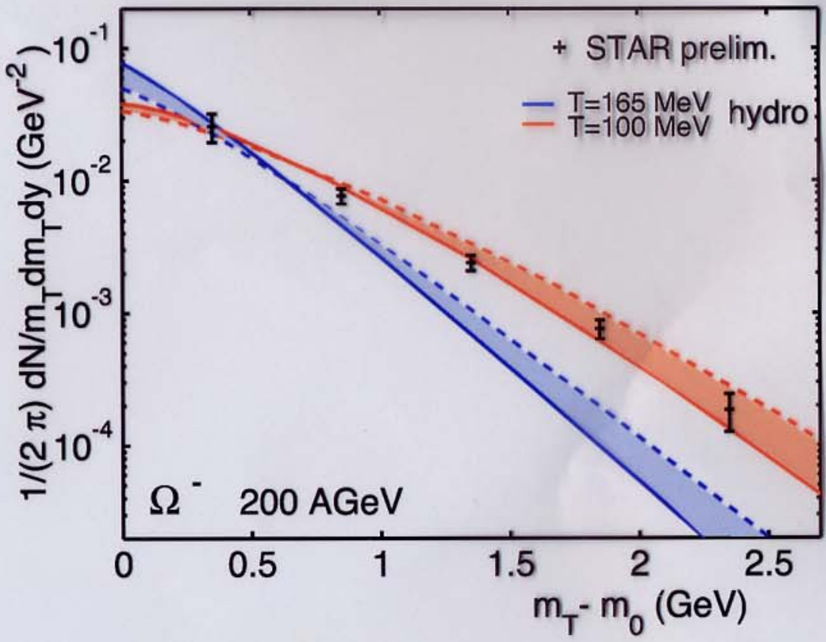


200 A GeV Au+Au spectra and hydrodynamics (I)

hydro: Kolb & Rapp, PRC 67 (2003) 044903



C. Suire (STAR), NPA 715 (2003) 470c



Hydro parameters: $\tau_{\rm eq} = 0.6\,{\rm fm}/c$, $s_0 \equiv s_{\rm max}(b{=}0) = 110\,{\rm fm}^{-3}$, $s_0/n_0 = 250$

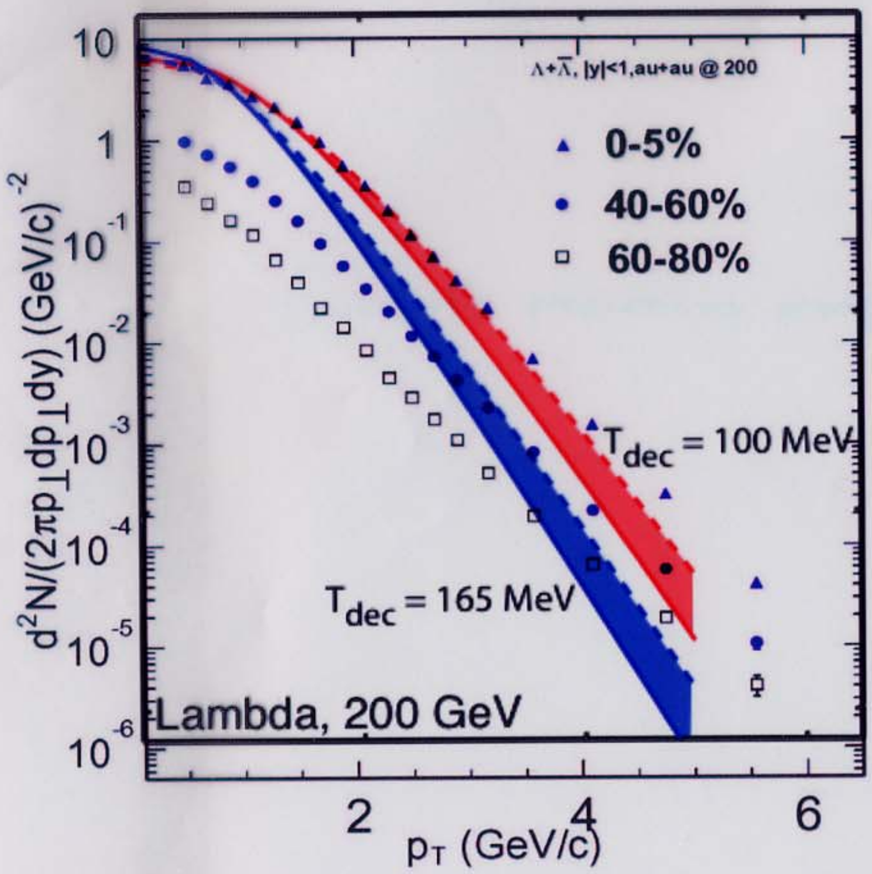
$T_{\rm chem} = T_{\rm crit} = 165\,{\rm MeV}$, $T_{\rm dec} = 100\,{\rm MeV}$

Note:
- Hydro does not create enough radial flow already at T_c to describe baryon spectra
- Multistrange baryons seem to fully participate in continued radial flow build-up during late hadronic phase!

Single Particle Spectra: Strangeness

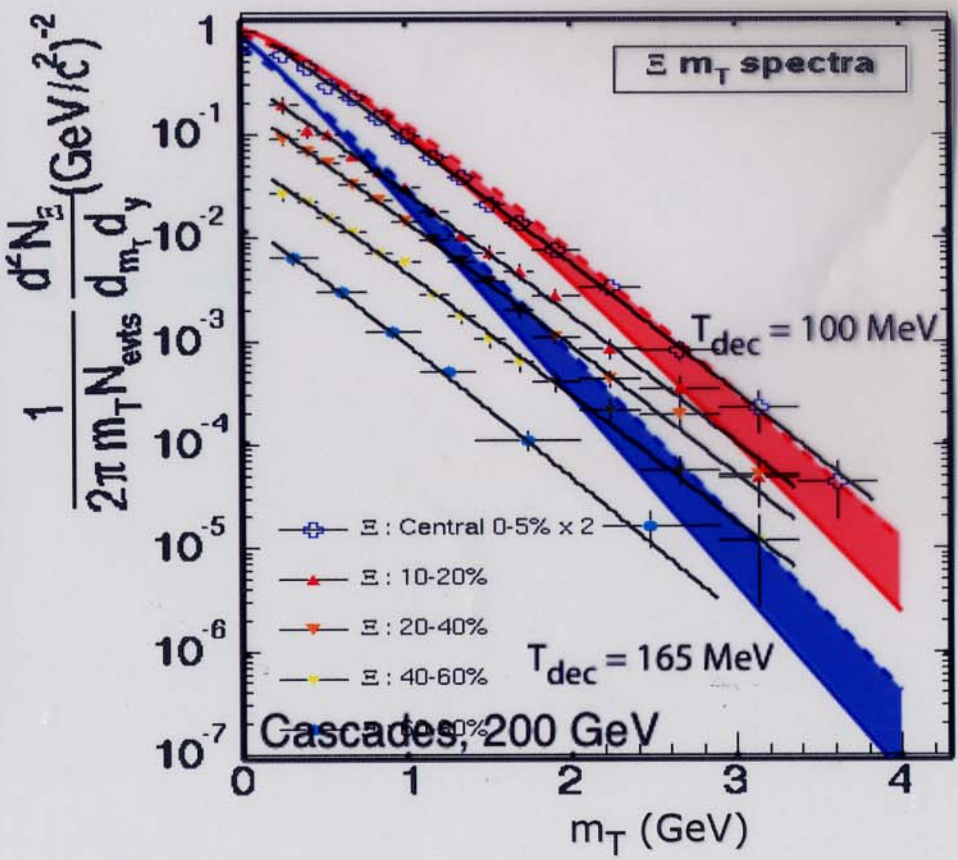
Lambda

H. Caines, STAR Collab., talk at SQM 2003

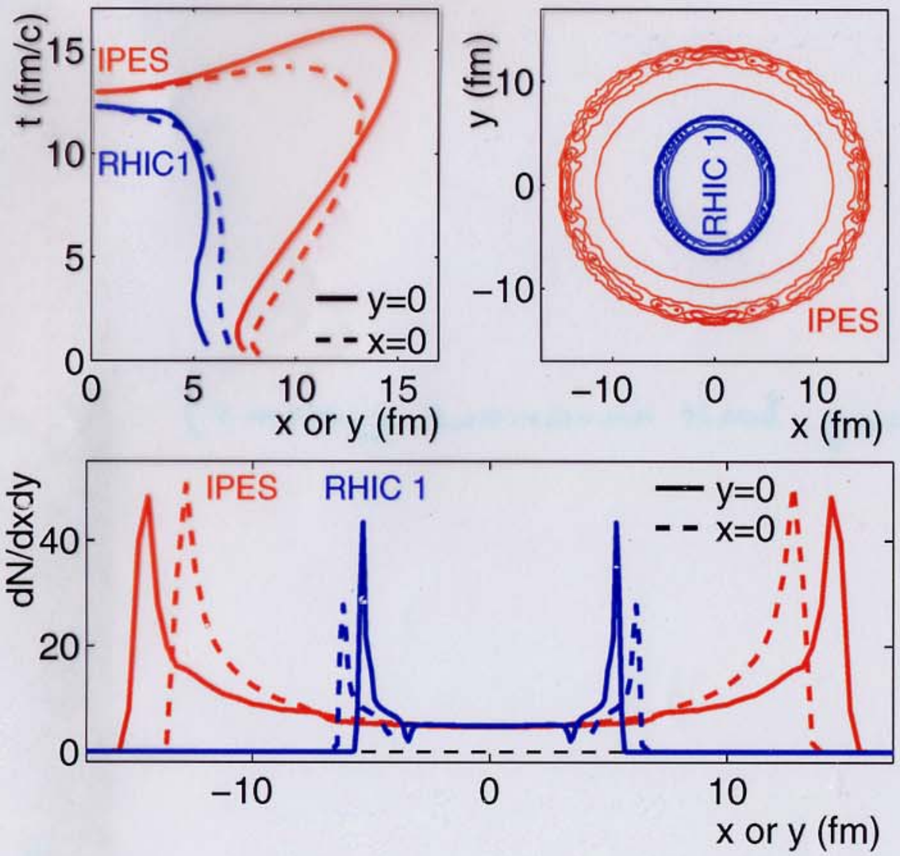


Cascades

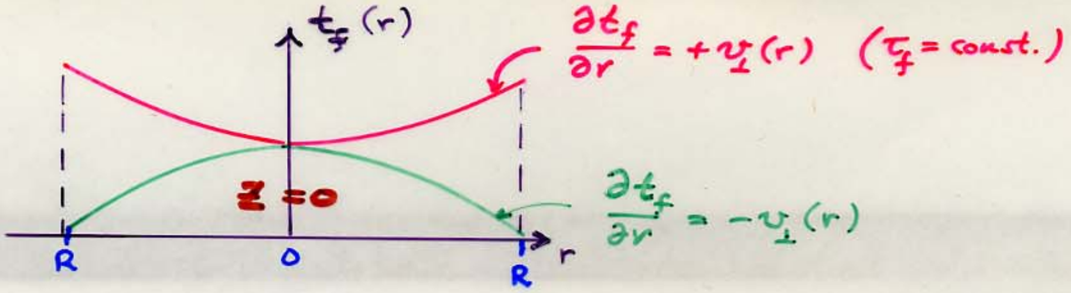
J. Castillo, STAR Collab., talk at HIC 2003



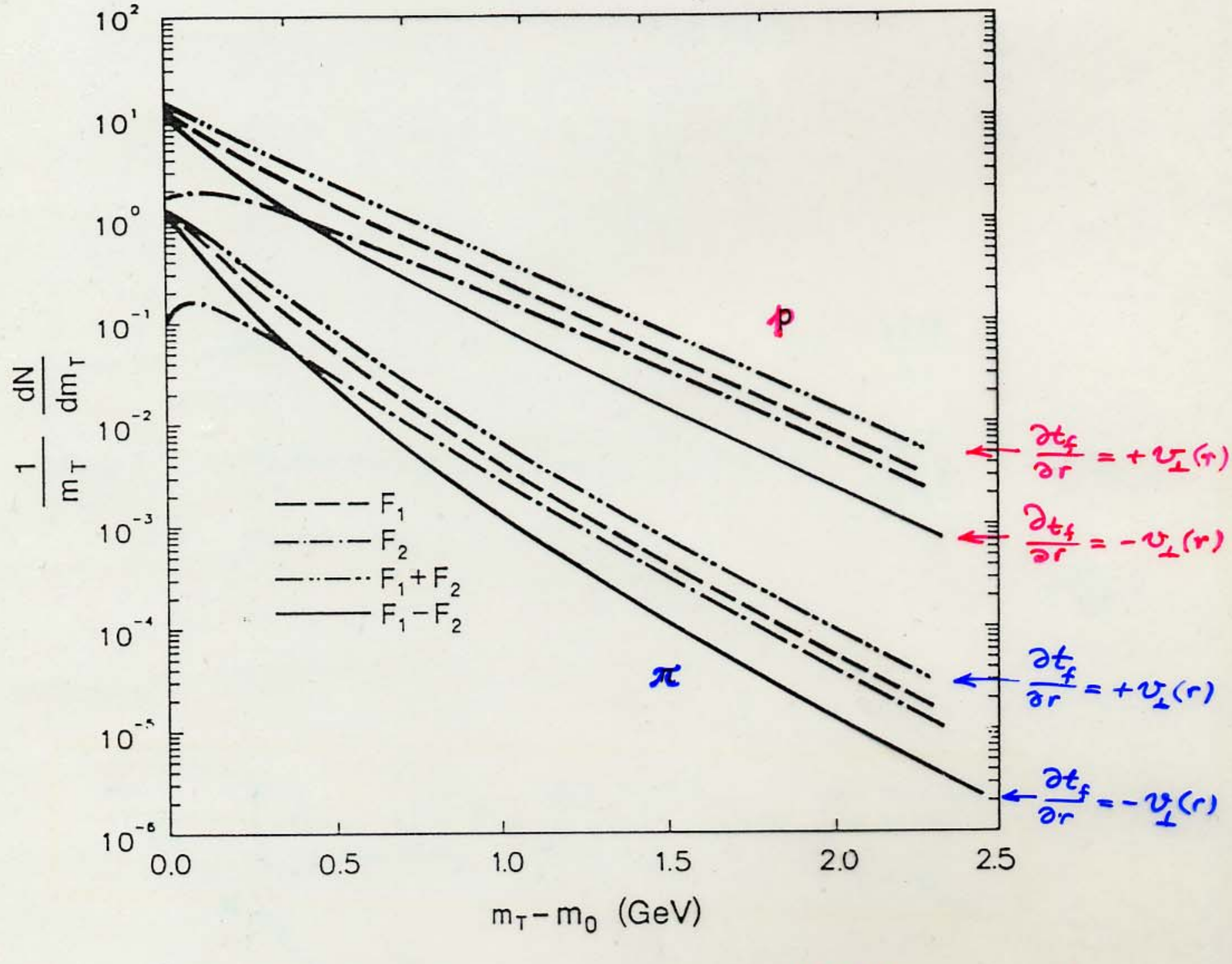
Probing the freeze-out distribution with HBT:



- RHIC1: $T_{0,\max} = 340\,\mathrm{MeV}$
  IPES: $T_{0,\max} = 2\,\mathrm{GeV}$
- RHIC1 source still out-of-plane elongated, IPES much larger and in-plane elongated
- onset of transverse "inside-out cascade" pattern at LHC energies
- emission strongly surface dominated; "opacity" strongest at RHIC energies



$\Rightarrow$ same data require larger $\langle v_\perp \rangle$, lower T
for $\frac{\partial t_f}{\partial r} < 0$ than for $\frac{\partial t_f}{\partial r} > 0$!

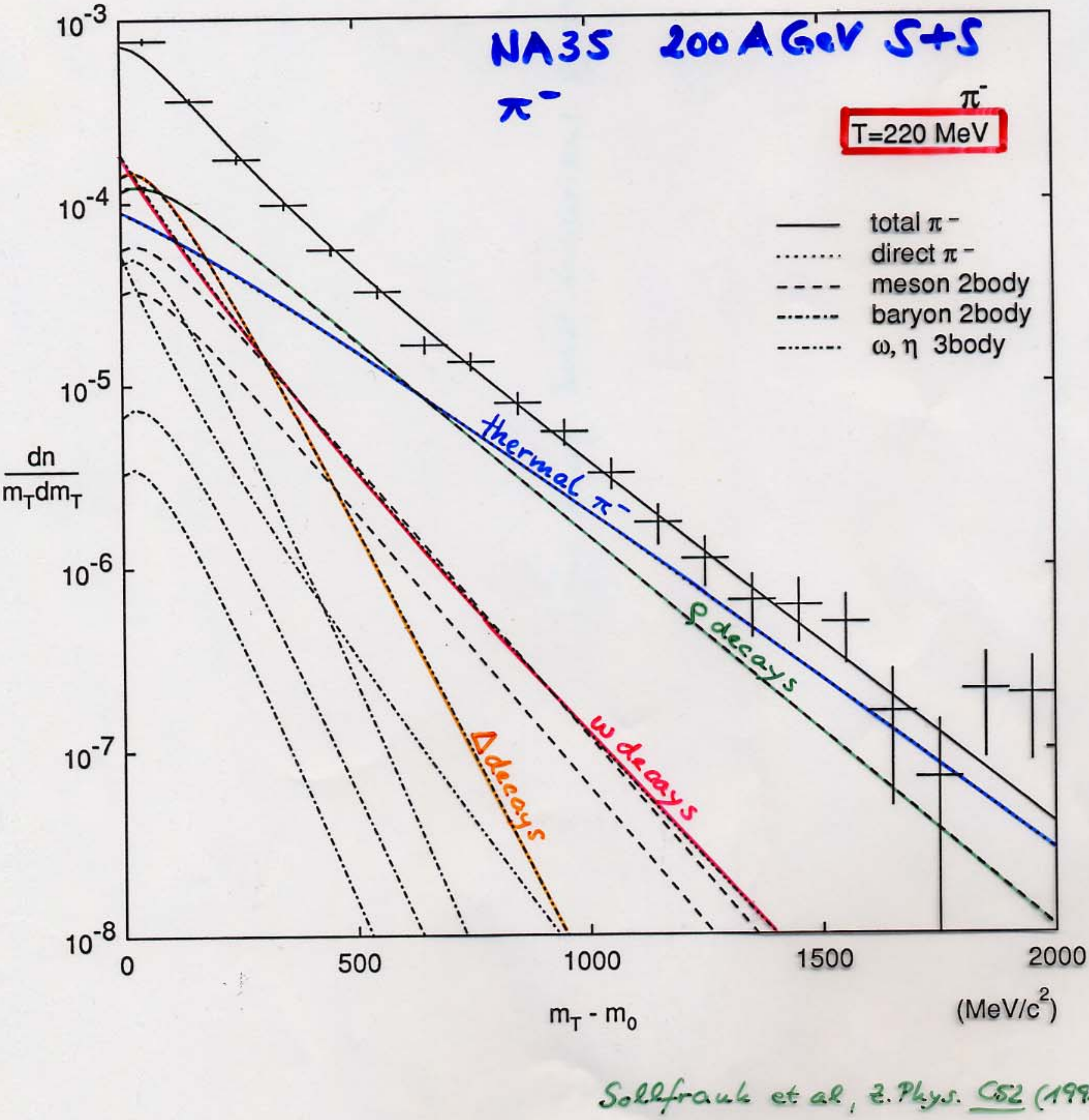


$T_f = 103\,\text{MeV}$, $v_\perp(r) = \beta_s \left(\frac{r}{R}\right)^2$, $\beta_s = 0.79$

Figure 1

Lee, Heinz, Schnedermann,
Z. Phys. C48 (1990) 525

Thermal fit + resonance decays
No flow

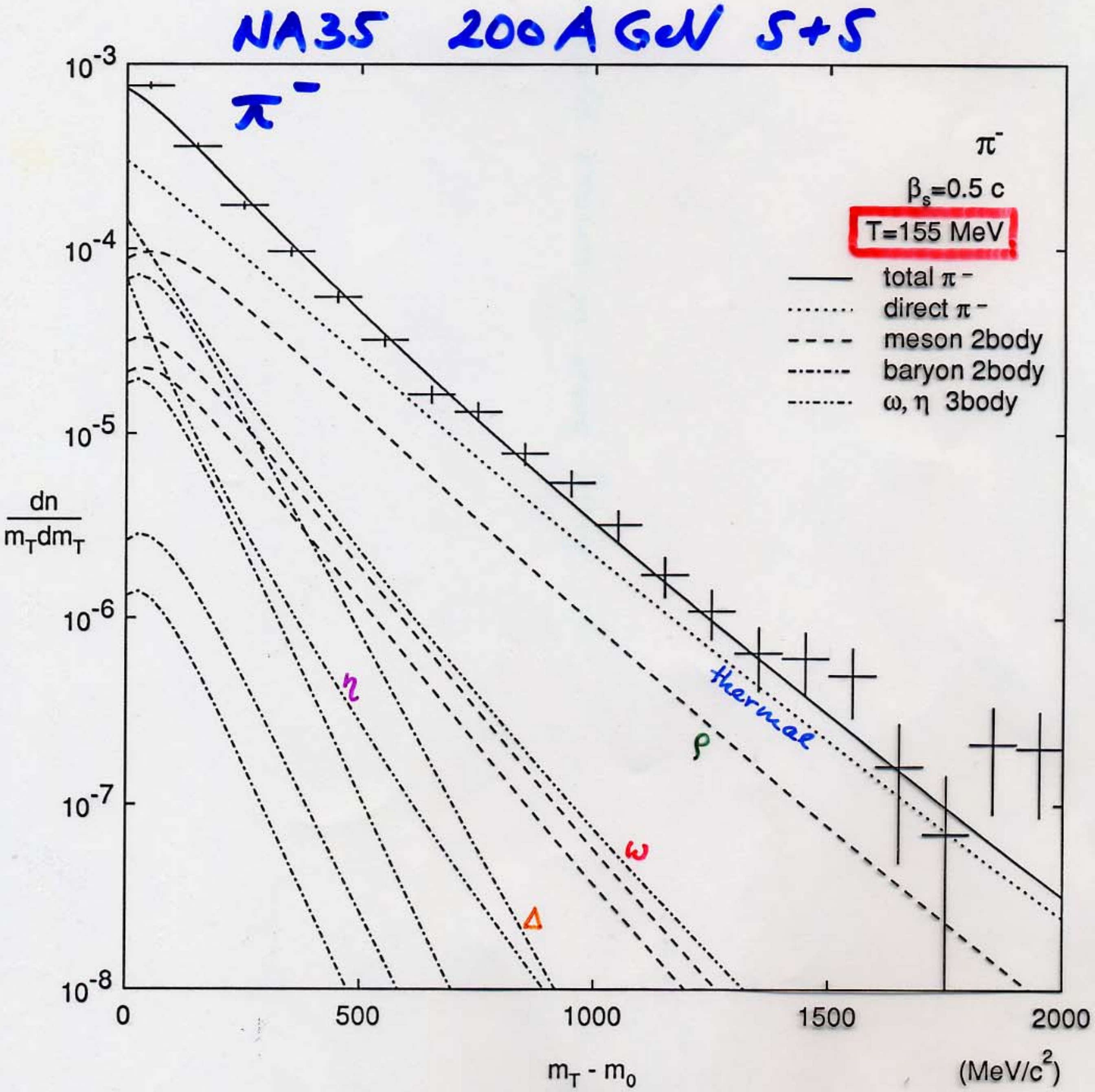


Sollfrank et al, Z. Phys. C52 (1991) 593

Thermal + transverse flow

Intermediate case: $\beta_s = 0.5c$

$(\langle\beta\rangle = 0.25c)$



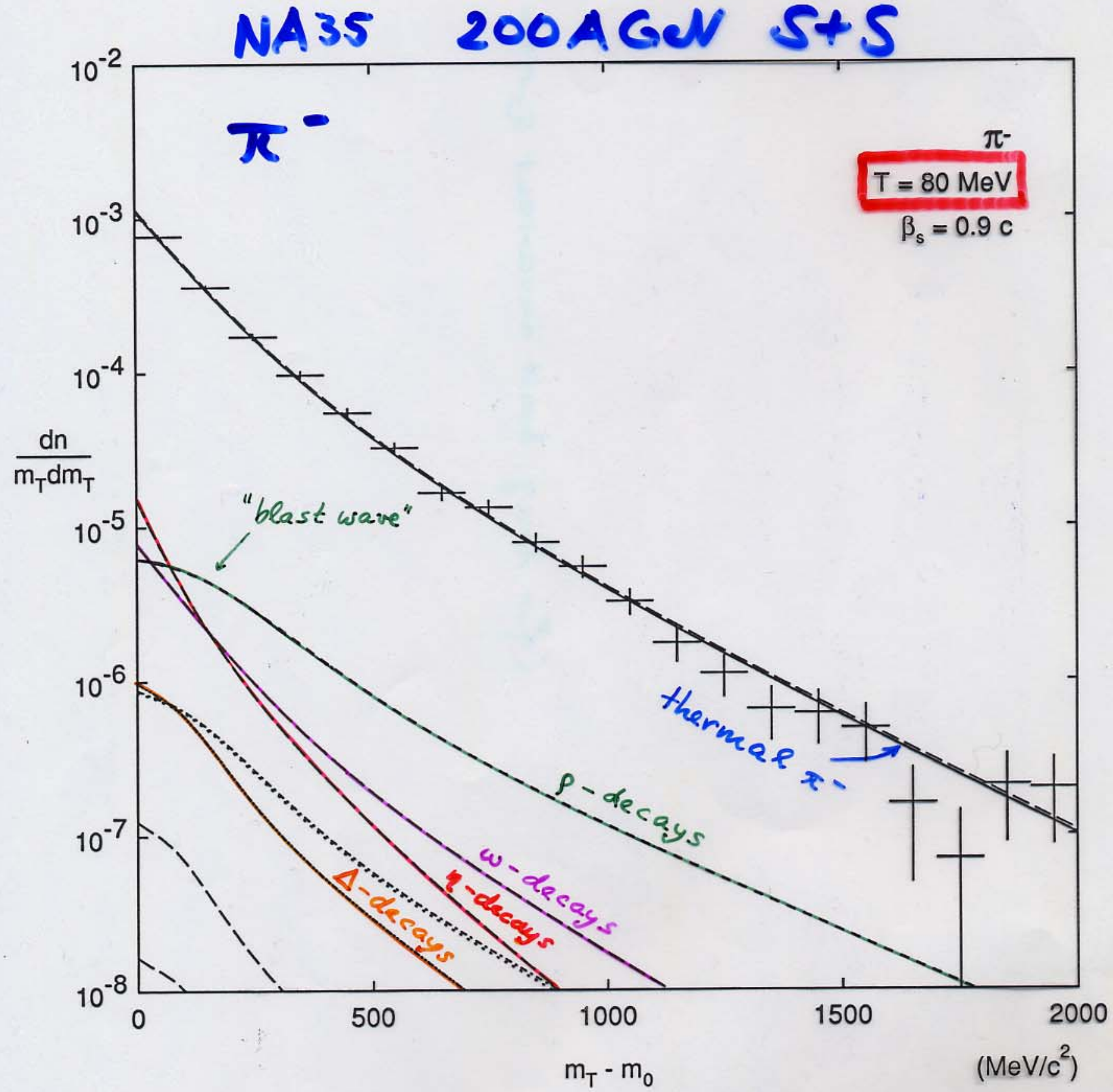
$$T_{eff} = T\sqrt{\frac{1+\beta_\perp}{1-\beta_\perp}}$$

"blueshifted temperature"

(E. Schnedermann + J. Sollfrank)

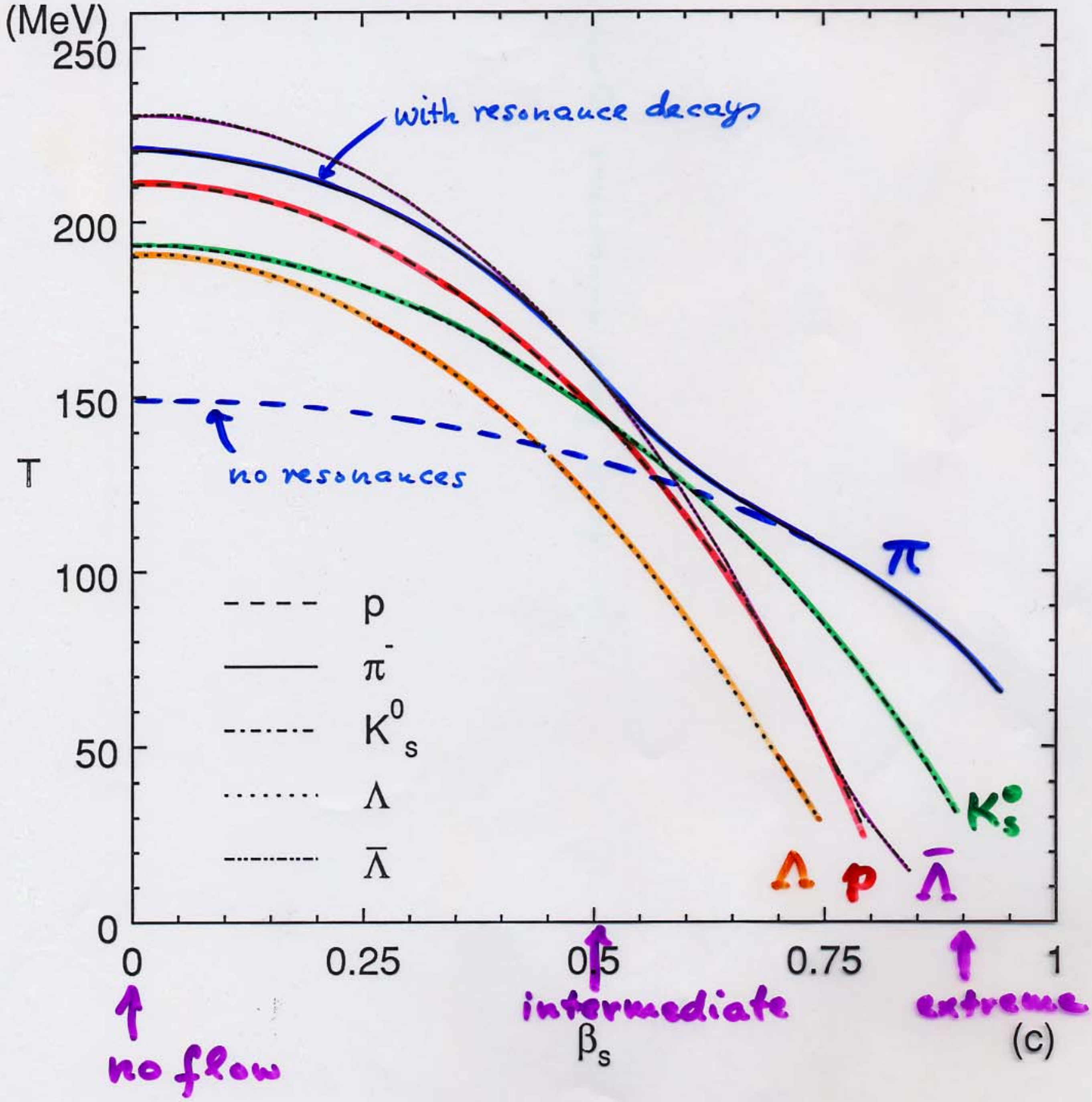
Thermal fit + transverse flow

Extreme example: $\beta_s = 0.9c$



Summary of best fits (T, β_s) to $\frac{dn}{dm_\perp^2}$

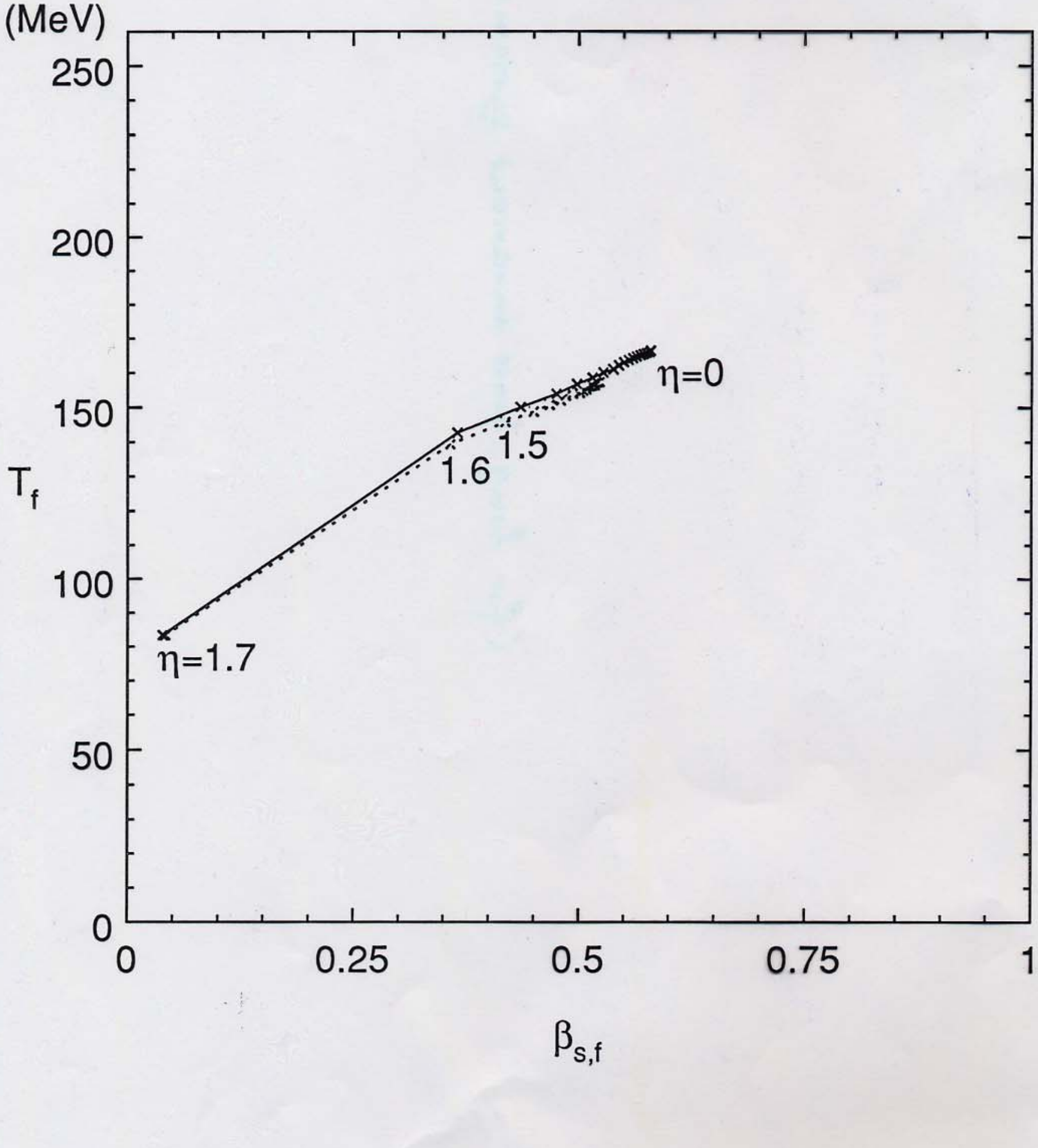
NA35 S+S @ 200 A GeV



Tradeoff flow ↔ T

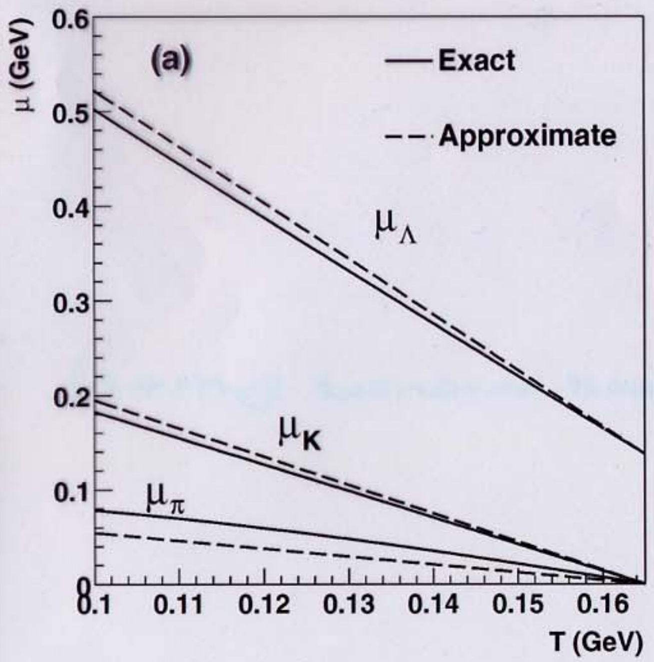
(Schnedermann, Sollfrank, Heinz, PRC 48 (1993) 2462)

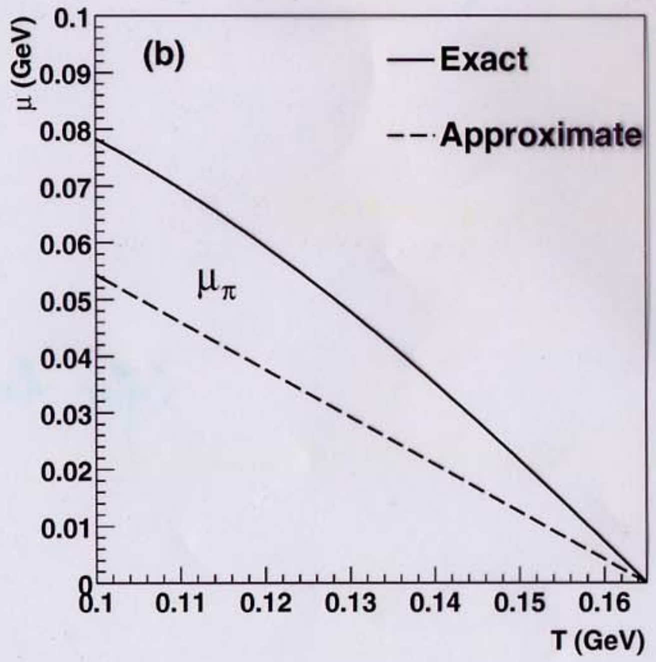
(T_f, β_s) pairs consistent with hydro + freeze-out :



Non-Equilibrium chemical potentials at $T < T_{\mathrm{chem}}$:

For Pb+Pb at the SPS (D. Teaney, nucl-th/0204023):





For RHIC see T. Hirano, PRC 66 (2002) 054905

These chemical potentials must be included in resonance feeddown!

Conclusions:

- Blast wave fits have not only statistical, but also systematic errors, due to assumptions on the details of the blast wave model. Good data quality implies that these days the systematic uncertainties dominate!
- Blast wave fits must include resonance feeddown contributions in a selfconsistent way, taking into account non-equilibrium chemical potentials at $T < T_{\rm chem}$.
  Neglecting decay pions biases the fits towards lower temperatures.
- Blast wave fits must take into account the shape of the freeze-out surface.
  Assuming freeze-out at constant proper time $\tau = \sqrt{t^2 - z^2 - r_\perp^2}$ biases the fits towards larger temperatures and less flow.
- Blast wave fits should explore the sensitivity to the flow-velocity distribution $dN/dv_\perp$ and include this in the systematic error.
- Blast wave fits are not the end, but the beginning of a dynamical understanding. They are only meaningful if the fit parameters are dynamically consistent. If they aren't, the fits should not be overinterpreted, but taken as an indication that more thought is needed.