Analyzing v_2 with Lee-Yang zeroes

Workshop on Collective flow and QGP properties
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nucl-th/0307018
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The final answer to an old problem

$$v_2 \equiv \langle \cos 2(\phi - \Phi_R) \rangle$$

where Φ_R, azimuth of the reaction plane, is

- unknown
- randomly distributed

How can one measure v_2?

$\rightarrow$ This is a well-posed problem

$\rightarrow$ There should be a simple solution without any additional assumption such as "all azimuthal correlations are due to flow"

Outline

- How is v_2 currently analyzed at RHIC ?
- A new simple recipe to analyze v_2
- Generating functions, cumulants, zeroes
- Relation with the Lee-Yang theory of phase transitions
- Summary and perspectives

How is v_2 analyzed at RHIC ? [1]

- Event-plane method

PHENIX nucl-ex/0305013
PHOBOS nucl-ex/0205021
STAR nucl-ex/0107003

- 2-particle correlations + mixed events

PHENIX nucl-ex/0204005

- Cumulants of 2, or 4, or 6 particle correlations

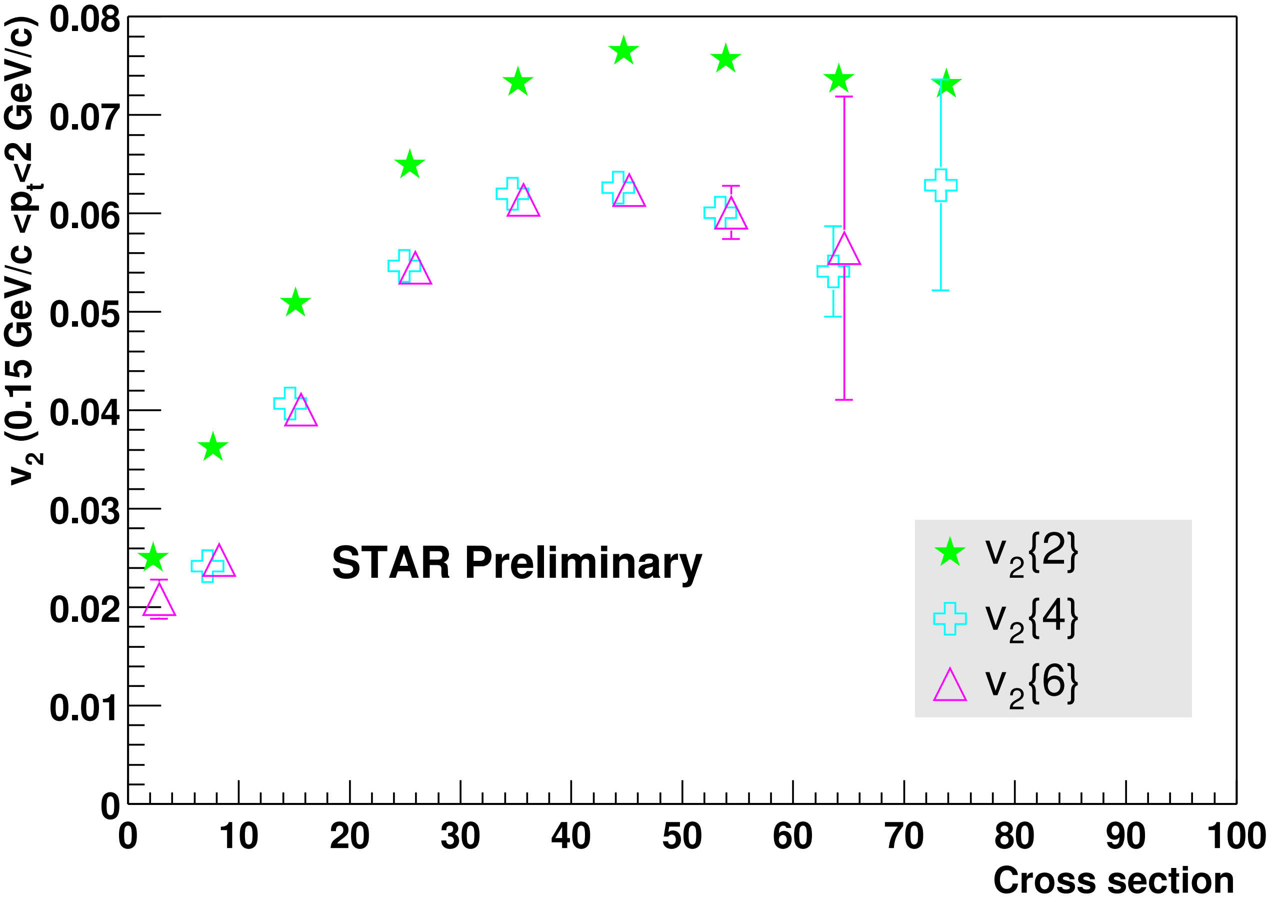
STAR nucl-ex/0308020

Event-plane, 2-particle correlations, 2-particle cumulants are essentially equivalent:

- Same statistical errors
- Same errors from nonflow effects
- Event-plane: only with azimuthally symmetric detector

Cumulants of 4, 6 particles eliminate most "non-flow" effects, but have larger statistical errors ($\times 2$ for midcentral@STAR)

How is v_2 analyzed at RHIC ? [2]



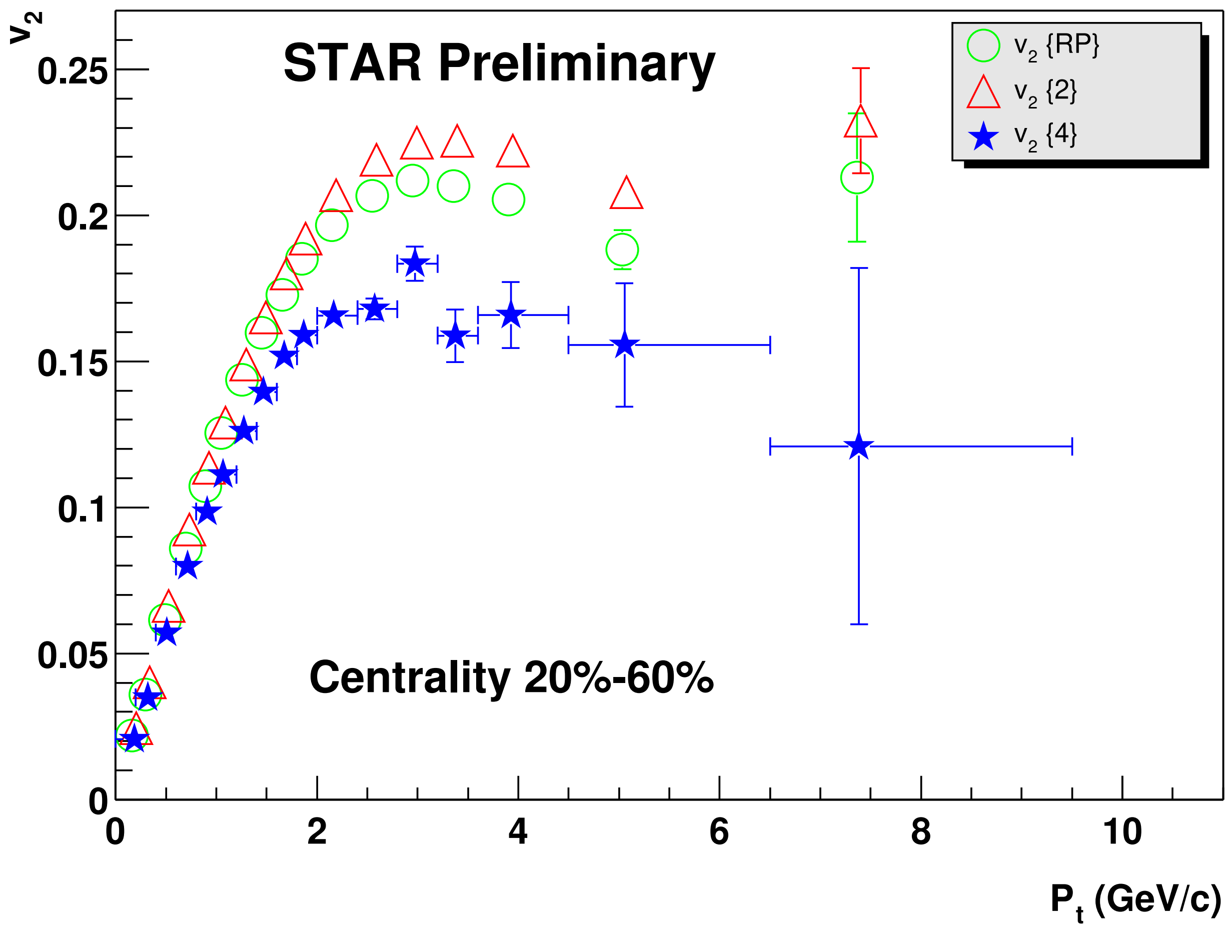
Cumulant analysis : A. Tang for STAR nucl-ex/0308020

$v_2\{2\} \neq v_2\{4\}$: There are nonflow effects (or fluctuations, cf talk by Poskanzer)

$v_2\{4\} = v_2\{6\}$: No nonflow effects beyond 4-particle cumulants.

This shows that going beyond 4-particle cumulants may be somewhat academic at RHIC. However, the "improved" method will turn out to be simpler to implement, and formally more elegant than cumulants.

How is v_2 analyzed at RHIC ? [3]



Aihong Tang for STAR nucl-ex/0308020

Nonflow effects could be larger at high p_T.
$\rightarrow$ Cumulants are relevant to high p_T physics.

- v_2 at high p_T may be due to jet quenching

  X.N.Wang nucl-th/0009019

- v_2 is a background to jet correlations

  STAR nucl-ex/0206006

A new simple recipe to analyze v_2

Define sum over particles seen in the event

$$Q \equiv \sum_j \cos 2\phi_j$$

azimuthal angles measured in the laboratory

The relevant information is contained in the probability distribution of Q, $p(Q)$. But where ?

A histogram mixes relevant+irrelevant information.

See talk by S. Voloshin, and E877 hep-ex/9405003

Rather compute the Fourier transform of the distribution (it's even easier than a histogram!)

$$G(ir) \equiv (1/N_{\rm evts}) \sum_{\rm evts} e^{irQ}$$

The first minimum of $|G(ir)|$, at $r = r_0$ is

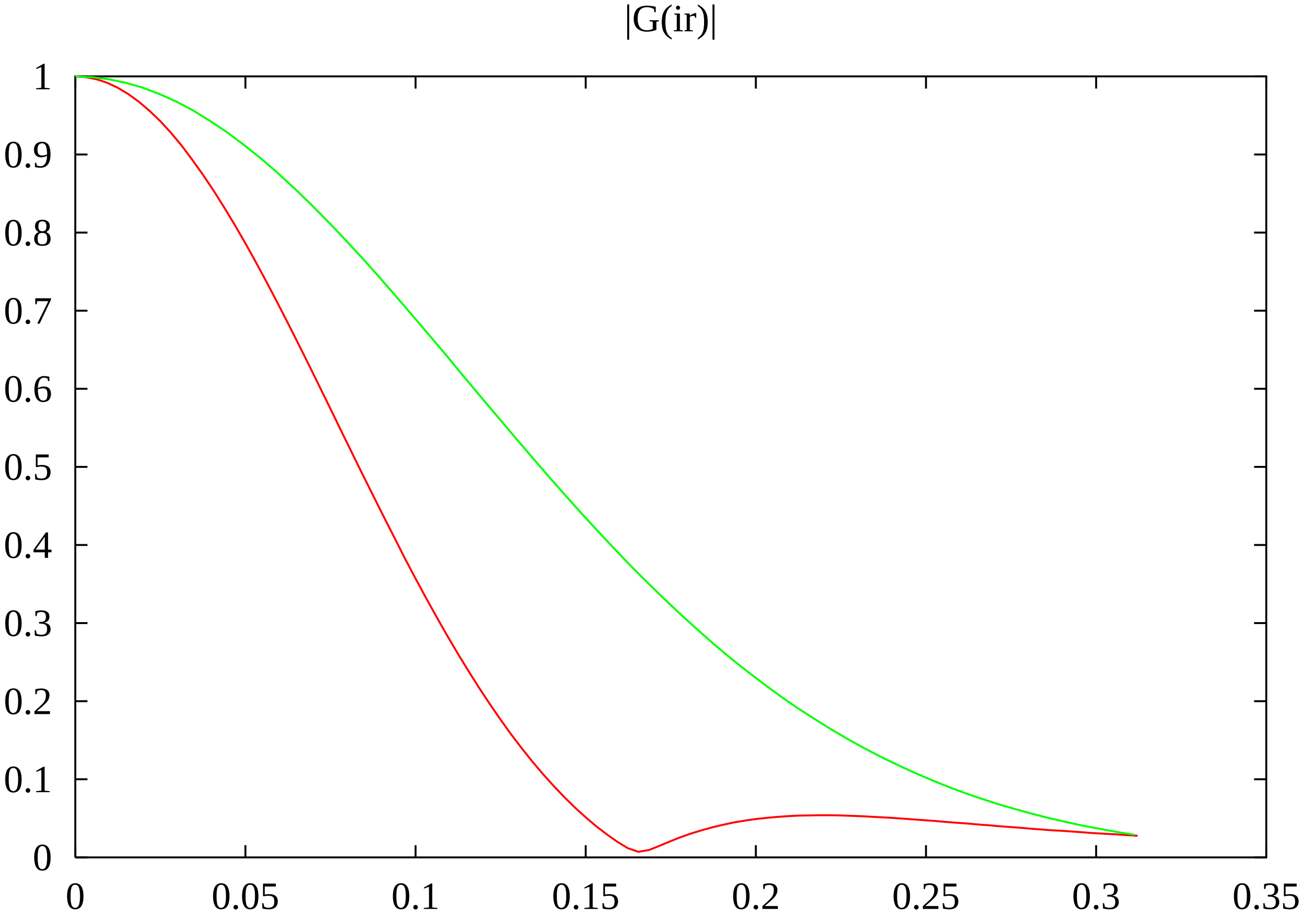
- insensitive to nonflow correlations
- directly related to $v_2 = 2.405/M r_0$
  (M = mean event multiplicity)

And that's it! No "flattening" of azimuthal distributions, no "subevents", no hidden details !

Monte-Carlo simulation

- 10000 events
- Event multiplicity $125 < M < 175$
- —— $v_2 = 0$

  —— $v_2 = 10\%$



Generating functions [1]: definitions

Define

$$G(z) \equiv \left\langle \exp\left(z \sum_{j=1}^{M} \cos 2\phi_j \right) \right\rangle$$

$\langle \ldots \rangle$ = average over events,
$\sum_j$ = sum over particles in an event.

Or, equivalently

$$\tilde{G}(z) \equiv \left\langle \prod_{j=1}^{M} (1 + z \cos 2\phi_j) \right\rangle$$

(everywhere in this talk, $\tilde{G}$ can be used instead of G)

Cumulants c_k are defined by

$$\ln G(z) = \sum_{k=1}^{+\infty} c_k \frac{z^k}{k!}$$

Naive order of magnitude: $c_k \sim M^k$
(M = event multiplicity)

Generating functions [2]: no flow

No collective behaviour: the system is made of (a large number of) independent subsystems
(typically, particles are correlated within small clusters)

$$
\begin{aligned}
G(z) &= \prod_{\text{subsystems}} G_{\text{subsystem}}(z) \\
\ln G(z) &= \sum_{\text{subsystems}} \ln G_{\text{subsystem}}(z) \\
c_k &= \sum_{\text{subsystems}} (c_k)_{\text{subsystem}}
\end{aligned}
$$

The cumulant c_k scales like the system size M, not like M^k.

(this is why cumulants were created)

Generating functions [3]: flow

Elliptic flow is defined as a correlation with the reaction plane Φ_R:

$$v_2 \equiv \langle \cos 2(\phi_j - \Phi_R) \rangle \quad (\langle \sin 2(\phi_j - \Phi_R) \rangle = 0)$$

We decompose $\cos 2\phi_j$ as follows:

$\cos 2\phi_j = \cos 2(\phi_j - \Phi_R)\cos 2\Phi_R - \sin 2(\phi_j - \Phi_R)\sin \Phi_R$

First average over many particles and events with the same Φ_R:

$$\begin{aligned} \left\langle \cos 2\phi_j \right\rangle &= v_2 \cos 2\Phi_R \\ \ln G(z) &= zMv_2 \cos 2\Phi_R + z^2 \cdots \\ G(z) &= \exp\left(zMv_2 \cos 2\Phi_R\right) \times \cdots \end{aligned}$$

Finally average over Φ_R:

$$G(z) = I_0(zMv_2) \times \cdots$$

z goes with a factor M, so z^k goes with M^k

The cumulant c_k scales like M^k due to elliptic flow.

Generating functions [4]: zeroes

Nonflow $c_k \propto M$
Flow $c_k \propto M^k$

Identify the flow: construct c_k for large k

$$\ln G(z) = \sum_{k=1}^{+\infty} c_k \frac{z^k}{k!}$$

How do I compute the asymptotic behaviour of the coefficients of a power-series expansion ?

Hint :

$$\ln\left(1 - \frac{z}{z_0}\right) = \sum_{k=1}^{+\infty} \frac{z^k}{k {z_0}^k}$$

The asymptotic behaviour of c_k is completely determined by z_0, i.e. the zero of the function in the log.

This is a general result : the first zero of $G(z)$ in the complex plane completely determines large-order cumulants.

Very generally, studying collective motion amounts to finding the zero of the generating function $G(z)$ which is closest to the origin in the complex plane of z.

Generating functions [5]: zeroes

Where is the first zero ?

$$G(z) = I_0(zMv_2) \times \cdots$$

Zeroes are on the imaginary axis. The first is at

$$z_0 = \frac{2.405\, i}{Mv_2}.$$

Determine z_0 experimentally, thus obtain v_2.

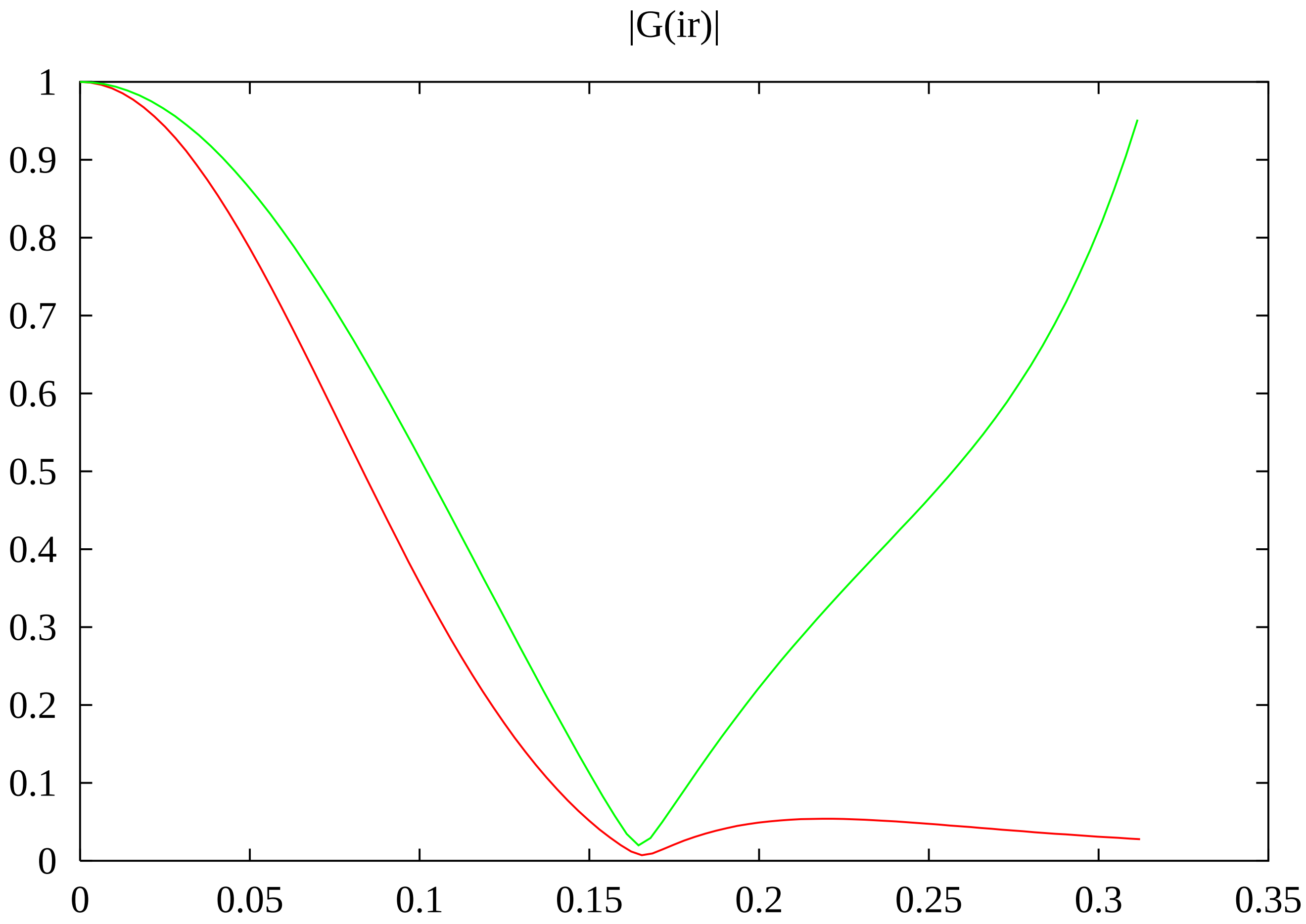
In practice $G(z)$ is obtained from an average over a finite number of collisions, and the detector is not perfectly isotropic:
$\rightarrow$ The zeroes of $G(z)$ don't lie exactly on the imaginary axis.
$\rightarrow$ Rather than the first zero, we determine the first minimum of $|G(z)|$ on the imaginary axis.

Until now, we have only used the $\cos 2\phi_j$. In order to use also the information on $\sin 2\phi_j$,

- We repeat the whole procedure with $\cos 2(\phi_j - \theta)$ where θ is a fixed angle $\rightarrow$ we obtain an estimate of v_2 for each value of θ;
- We average the result over the values of θ.

Monte-Carlo simulation

- 10000 events
- Event multiplicity $125 < M < 175$
- Input $v_2 = 10\%$
- — $|\tilde{G}(ir)|$
  — $|G(ir)|$



Although the two generating functions differ, the minima are the same.

Analogy with Lee and Yang

A classic paper: theory of phase transitions from the zeroes of the grand partition function
(see textbooks, e.g. *Statistical Mechanics*, by K. Huang)

Lee Yang Phys.Rev.87 (1952) 404

Grand partition function for fixed $\beta = 1/T, V$:

$$\mathcal{G}(\mu) = \sum_{N=0}^{+\infty} Z_N e^{\beta\mu N}$$

Choose a reference value for the chemical potentiel μ_c, and define $\beta\mu = \beta\mu_c + z$. Define

$$G(z) = \frac{\mathcal{G}(\mu)}{\mathcal{G}(\mu_c)} = \sum_{N=0}^{+\infty} P_N e^{zN},$$

where P_N=probability to have N particles in the system at $\mu = \mu_c$. Finally,

$$G(z) = \left\langle e^{zN} \right\rangle .$$

Phase transition at $\mu = \mu_c$
$\leftrightarrow$ zeroes get closer to $z = 0$ as V increases.
(furthermore, they are often on the imaginary axis).

Analogy $V \leftrightarrow M \quad N \leftrightarrow \sum_{j=1}^{M} \cos 2\phi_j$

Differential elliptic flow

- So far, I have shown how to measure v_2, integrated over all particles ("integrated flow", $M\langle v_2\rangle$).
- We can now use this as a reference to analyze v_2 for identified particles, as a function of p_t, y,... As an exemple, to analyze v_2 for kaons,

$$v_2(K) = \underbrace{\mathrm{Re}}_{\text{real part}}\left[\underbrace{\frac{M\langle v_2\rangle}{\langle Qe^{ir_0Q}\rangle}}_{\text{normalization constant}}\underbrace{\left\langle \cos 2\phi(K)e^{ir_0Q}\right\rangle}_{\text{average over } K \text{ particles}}\right]$$

In this equation,

- $Q = \sum_j \cos 2\phi_j$
- r_0 is the previously determined minimum

"Autocorrelations" need not be removed !

Summary and perspectives

- First application of Lee-Yang theory to experimental data
- Isolate directly genuine collective behavior in heavy ion collisions.
- Simple to implement
- Can be used with azimuthally asymmetric detector (not shown here)
- For STAR, statistical errors will not be larger than with 4-particle cumulants
- Can also be applied to the analysis of directed flow at SIS
- It will the best method to analyze v_2 at LHC
- Could be extended to other observables in order to look for critical fluctuations