Entropy at RHIC

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- **Calculation of Entropy density**
- **Final state Entropy from hadronic data**
- **Initial state Entropy from Lattice data**
- **Conclusions**

Entropy as an observable

- **Entropy Change**

$$\Delta S = \frac{Q}{T}$$

- **Lattice QCD predicts rapid increase in energy density at nearly constant $T \approx T_c$**

$\Rightarrow$ **QGP is a high entropy state**

- **Measurements confined to final state hadrons**
- **Entropy nearly conserved between initial thermalization and freeze-out**

$\Rightarrow$ S **a good candidate to probe initial state matter**

- **Viscous effects, shock waves, hadronization increase entropy**

$\Rightarrow$ S_{fin} **gives an upper bound for** S_{ini}

Entropy Rapidity Density

Entropy in Landau quasi-particle approximation

$$S = (2J+1)\int \frac{d^3r d^3p}{(2\pi)^3}\left[-f\ln f \pm (1\pm f)\ln(1\pm f)\right]$$

$$\approx (2J+1)\int \frac{d^3r d^3p}{(2\pi)^3}\left[-f\ln f + f \pm f^2/2\right] \quad \text{for } f \ll 1$$

Phase space density

$$f(\mathbf{r},\mathbf{p},t) = f_{\max}(\mathbf{p})\exp\left(-\frac{x^2}{2R_x^2} - \frac{y^2}{2R_y^2} - \frac{z^2}{2R_z^2}\right)$$

R_i's implicit function of p

No explicit chemical equilibration

Single-particle Spectra and Radii

$$\frac{dN}{d^3p} = (2J+1)\int \frac{d^3r}{(2\pi)^3} f(\mathbf{r},\mathbf{p})$$

$$f_{\max}(\mathbf{p}) = \frac{(2\pi)^{3/2}}{(2J+1)}\frac{dN}{d^3p}\frac{1}{R^3}$$

dS/dy from HBT radii and spectra

Two-particle correlation function (gaussian source)

$$C(\mathbf{K},\mathbf{q}) = \frac{P(\mathbf{p}_1,\mathbf{p}_2)}{P(\mathbf{p}_1)P(\mathbf{p}_2)}$$

$$= 1 + \exp\left(-q_x^2 R_x^2 - q_y^2 R_y^2 - q_z^2 R_z^2\right)$$

$\mathbf{K} = (\mathbf{p}_1 + \mathbf{p}_2)/2$ **and** $\mathbf{q} = \mathbf{p}_1 - \mathbf{p}_2$

In out-side-long frame: q_l: **along beam direction**; q_o: **parallel to total** K_T; q_s: **orthogonal to** q_l **and** q_o

$$C(\mathbf{K},\mathbf{q}) = 1 + \exp\left(-q_o^2 R_o^2 - q_s^2 R_s^2 - q_l^2 R_l^2\right)$$

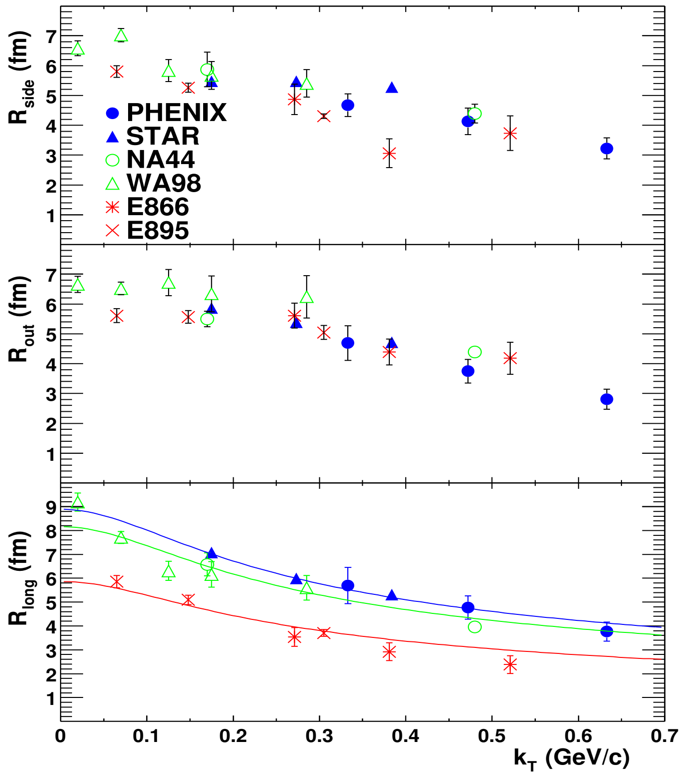
Final state entropy rapidity density

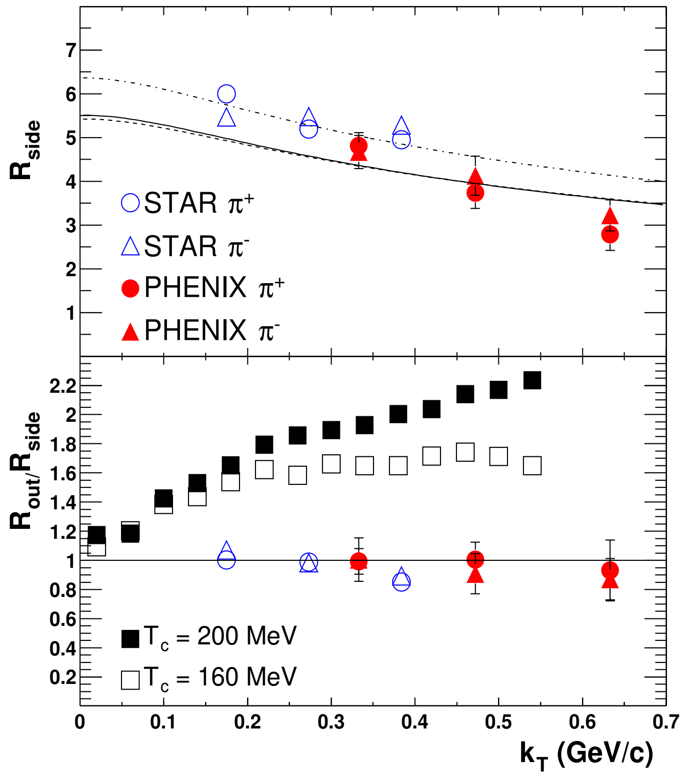
$$\frac{dS}{dy} = \int d^2p_T\, E\frac{dN}{d^3p}\left[\frac{5}{2} - \ln\mathcal{F}(\mathbf{p}) \pm \frac{1}{2^{5/2}}\mathcal{F}(\mathbf{p})\right]$$

$$\mathcal{F}(\mathbf{p}) = \frac{1}{m}\frac{(2\pi)^{3/2}}{(2J+1)}\, E\frac{dN}{d^3p}\frac{1}{R_{\mathrm{inv}}^3}, \quad R_{\mathrm{inv}}^3 = \frac{E_T}{m}R_o R_s R_l$$

dS/dy **in final state can be estimated from HBT radii** R_{inv} **and s.p. spectra** $E\, dN/d^3p$

Pion entropy rapidity density





$$R_l = \frac{3.32 \pm 0.04}{\sqrt{m_T}} \qquad R_s^2 = R_o^2 = \frac{R_{\mathrm{geom}}^2}{1 + \beta^2 \left(\frac{m_T}{T}\right)}$$

$$R_{\mathrm{geom}} = 6.7 \text{ fm}, \;\; \beta = 0.69, \;\; T = 125 \text{ MeV}$$

π^- s.p. spectra corrected for weak decay fitted to

$$E\, dN/d^3p = A'/[\exp(m_T/T_{\mathrm{eff}}) - 1]$$

$$\frac{dS}{dy}|_{\pi^-} = 680$$

Antiproton and Kaon entropy rapidity density

From coalescence arguments for reaction $a + b \leftrightarrow c$

$$R_{\rm inv}^3 = \pi^{3/2} \frac{(2J_c+1)}{(2J_a+1)(2J_b+1)} \left(\frac{E_a}{m_a}\frac{dN_a}{d^3p_a}\right)\left(\frac{E_b}{m_b}\frac{dN_b}{d^3p_b}\right) \Big/ \left(\frac{E_c}{m_c}\frac{dN_c}{d^3p_c}\right)$$

- **Antiproton dS/dy** $(\overline{p} + \overline{n} \leftrightarrow \overline{d})$

Assume $f_{\overline{d}}(\mathbf{p}_{\overline{d}} = 2\mathbf{p}_{\overline{p}}, \mathbf{r}) = f_{\overline{p}}(\mathbf{p}_{\overline{p}}, \mathbf{r}) f_{\overline{n}}(\mathbf{p}_{\overline{n}}, \mathbf{r}) = f_{\overline{p}}^2(\mathbf{p}_{\overline{p}}, \mathbf{r})$

$\Rightarrow$ $R_{\rm inv}$ **as a function of** p_T **can be extracted**

$\overline{p}$ **s.p. spectra corrected for weak decay fitted to**

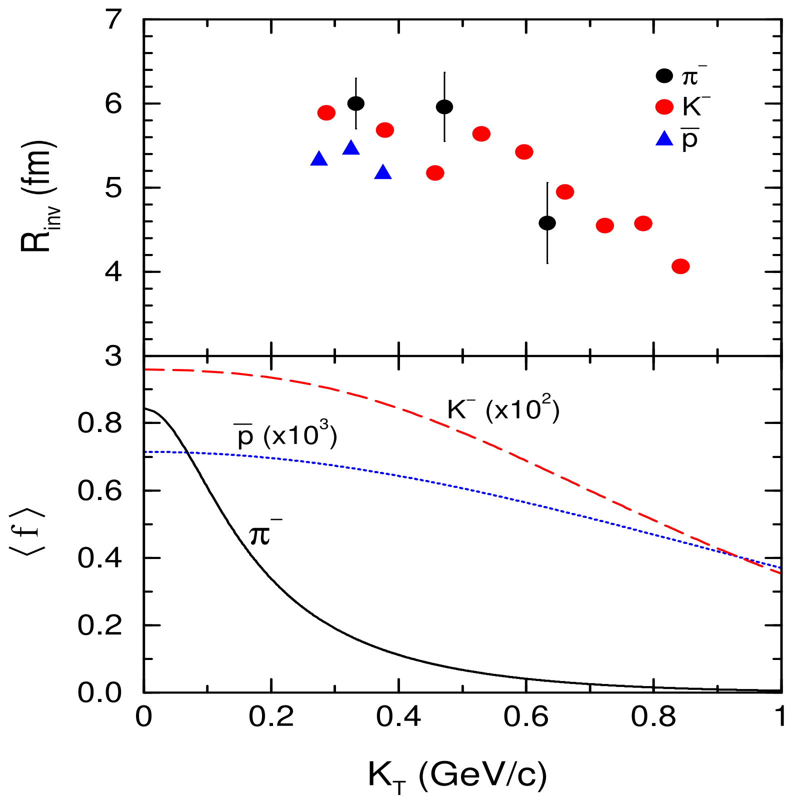
$$E\, dN/d^3p \sim dN/dy \exp(-p_T^2/2\sigma^2)$$

$$\frac{dS}{dy}|_{\overline{p}} = 118$$

- **Kaon dS/dy** $(K^+ + K^- \leftrightarrow \phi)$

$$\frac{dS}{dy}|_{K^-} = 256, \qquad \frac{dS}{dy}|_{K^+} = 286$$

HBT Radii and Phase Space Density



- π multiplicity at RHIC 70% larger than at SPS
- π radii identical at RHIC and SPS
- High π final phase space density at RHIC
- HBT radii $R_{\mathrm{inv}} = A/\sqrt{m_T}$ for $\pi, K, \bar{p}$ not consistent with one scaling coefficient

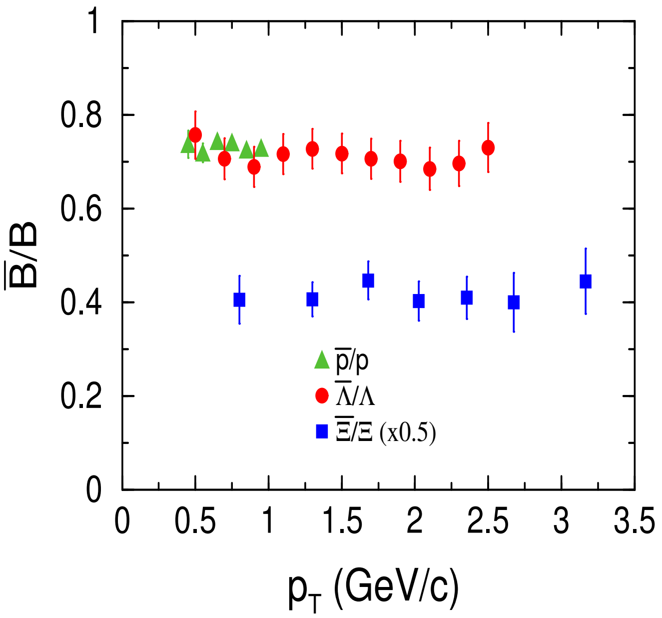
Final State Entropy Rapidity Density

dS/dy **of baryons:**

$\overline{p}/p = 0.72$, $\overline{\Lambda}/\Lambda = 0.71$

$\overline{\Xi}^+/\Xi^- = 0.83$, $\overline{\Omega}^+/\Omega^- = 0.95$

Slope T and HBT radii of B and $\overline{B}$ assumed same



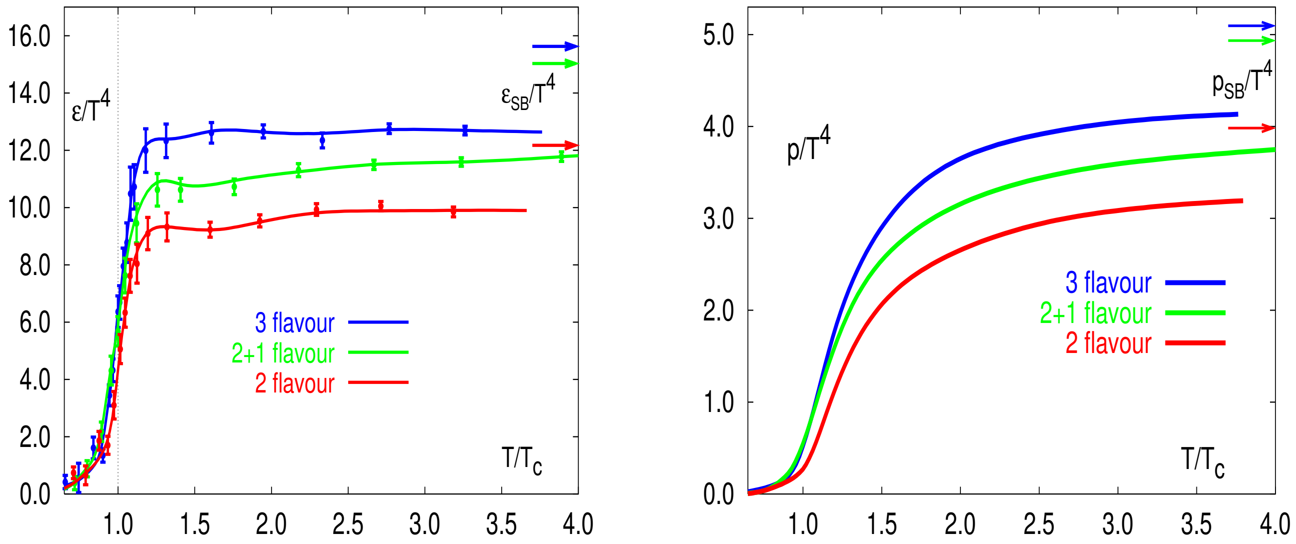
Final State Entropy rapidity density of hadrons

π^-	η (η')	K^- (K^+)	$\overline{p}$ (p)	$\overline{\Lambda}+\overline{\Sigma}^0$ ($\Lambda+\Sigma^0$)	$\overline{\Sigma}^\pm$ ($\Sigma^\pm$)	$\overline{\Xi}^\pm$ ($\Xi^\pm$)	$\overline{\Omega}^+$ (Ω^-)
680	256 (69)	256 (286)	118 (159)	81 (111)	31 (42)	23 (27)	5 (5)

Final State Total $dS/dy = 4451$ (model indep)

- $dS/dy|_\pi < 1/2\ dS/dy|_{\text{tot}}$ **though mult.** $N_\pi \approx 2/3\ N_{\text{tot}}$
- **Baryons compensate the lack of π entropy**

dS/dy at RHIC from Lattice data



Lattice QCD prediction

Quark flavor $n_f = 3$, QGP transition at $T_c = 154$ MeV

Bjorken estimate for energy density in early stage

$$\overline{\epsilon_{\text{Bj}}(\tau)} = \frac{dE_T}{dy}\frac{1}{\tau\pi R^2} \approx 4.5 \text{ GeV/fm}^3, \quad \tau = 1 \text{ fm/c}$$

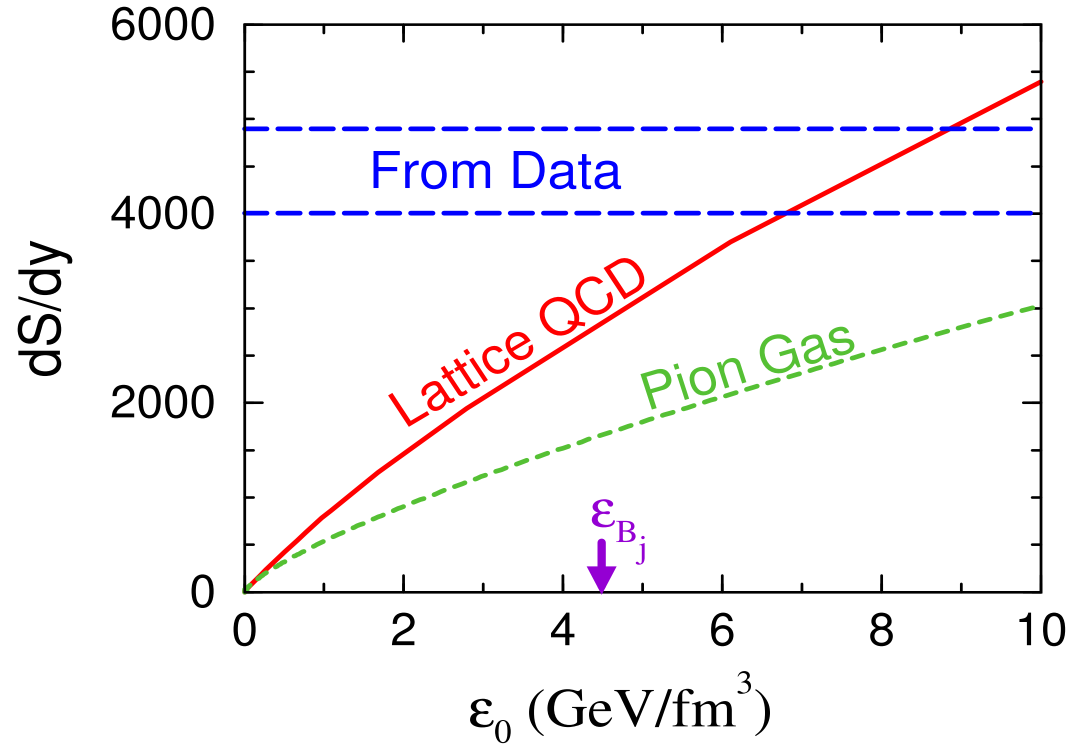
$T \approx 230$ MeV, $p \approx 3T^4 = 1.1 \text{ GeV/fm}^3$

Entropy density $s = (\epsilon + p)/T = 24.3 \text{ fm}^{-3}$

Average entropy rapidity density at $\tau = 1$ fm/c

$$\frac{dS}{dy}|_{\text{QGP};n_f=3} = \tau \int d^2x_T\, s \ \simeq 2922$$

Entropy rapidity density from Lattice QCD



Adding local longitudinal motion, viscosity, transverse flow, longitudinal work result in $\overline{\epsilon(\tau)} \simeq 7 \pm 2$ **GeV/fm**^{3}

$\star$ **Final state dS/dy consistent with a thermalized QGP at RHIC**

- **For a massless gas of quarks and gluons:**

$$\varepsilon = \left(N_B + \frac{7}{8} N_F\right) \frac{\pi^2}{30} T^4$$

Conclusions

- **Pions have large phase space density and carry less than half the total entropy**

- **Hadrons heavier than pions contribute more than 4 units of entropy**

- **Finite state total $dS/dy = 4451$ is consistent with lattice gauge results for a thermalized QGP at RHIC**