Hydrodynamics of the the Quark-Gluon Plasma at RHIC

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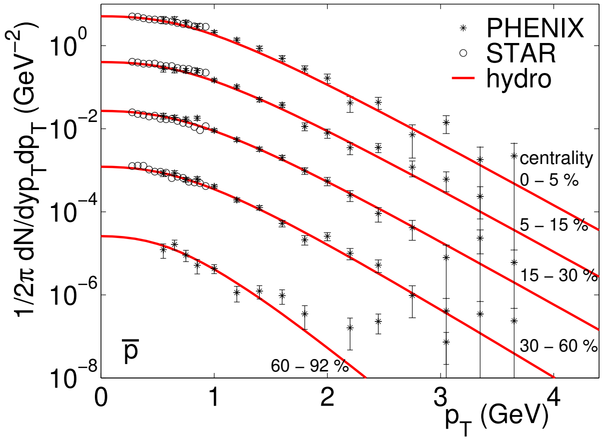
The hydrodynamics works very well, quantitatively describing for various secondaries: • The radial flow, • elliptic flow, v_2, v_4 ...

• Huge predictive power. If $l_{m.f.p.} << L$ it is the theory not the model

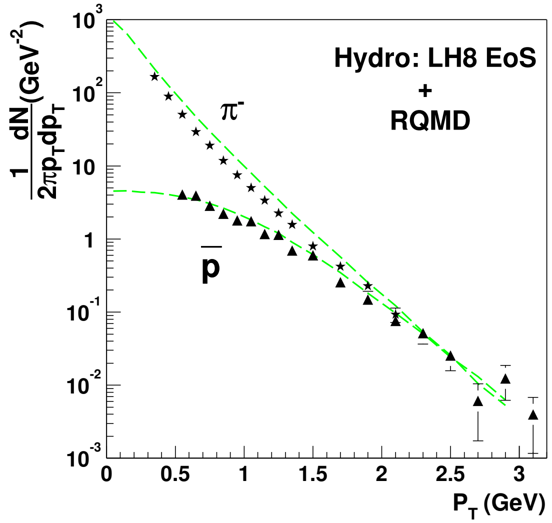
• The extracted EoS agrees well with the expected EoS from the theory/lattice

• However transport properties such as viscousity, mean free path are strongly overestimated by pQCD: they are surprisingly small: Why?

- Rethinking the QGP at $T = (1-3)T_c$: large cross sections at the zero-binding lines (ES+I.Zahed-03)
- New amasing connections to trapped ultracold (Li) atoms
- A strongly coupled plasma: the AdS/CFT duality => at the supercritical coupling $g^2 N_c >> 1$ it is the most perfect fluid known. New light bound states discovered in this problem as well (ES+I.Zahed-03b)

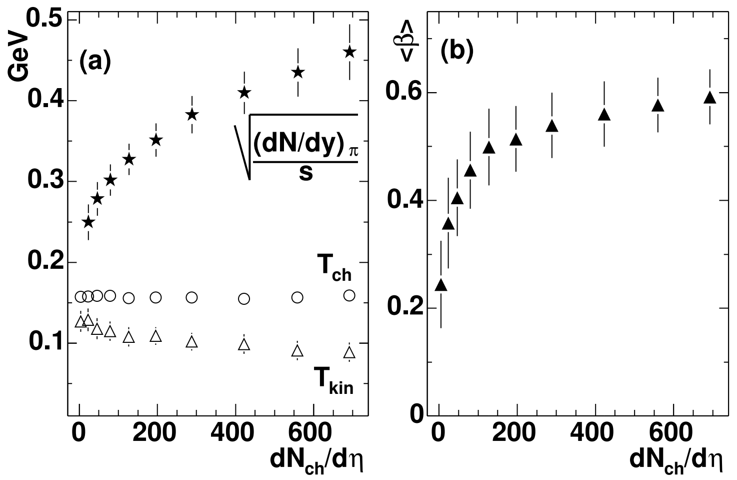


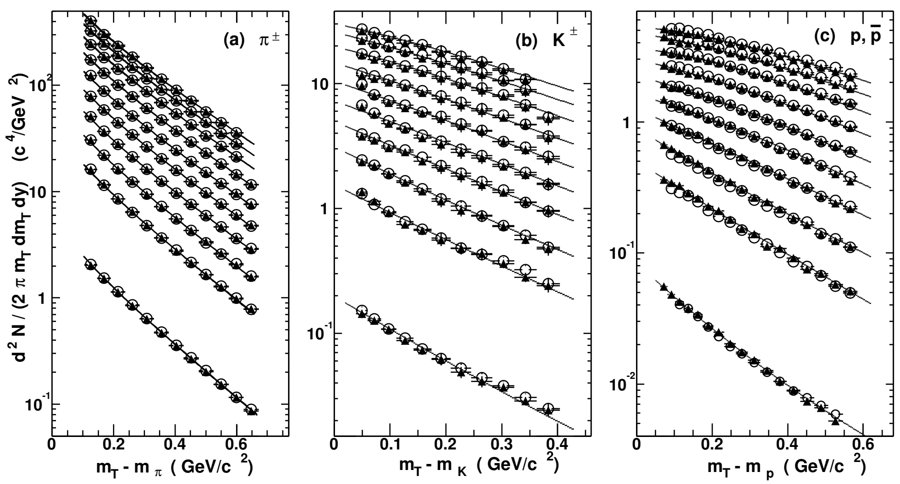
Teaney, Lauret, ES: at the hadronic stage the particle numbers (are chemically frozen). Picture (from recent work by P.Kolb and R.Rapp): excellent agreement between exps on $p, \bar{p}$, excellent description of the data.



At RHIC the $\bar{p}$ cross with π^- at $p_t \approx 2GeV$, exactly as hydro boosts predicts!

How far in p_t hydro should work?

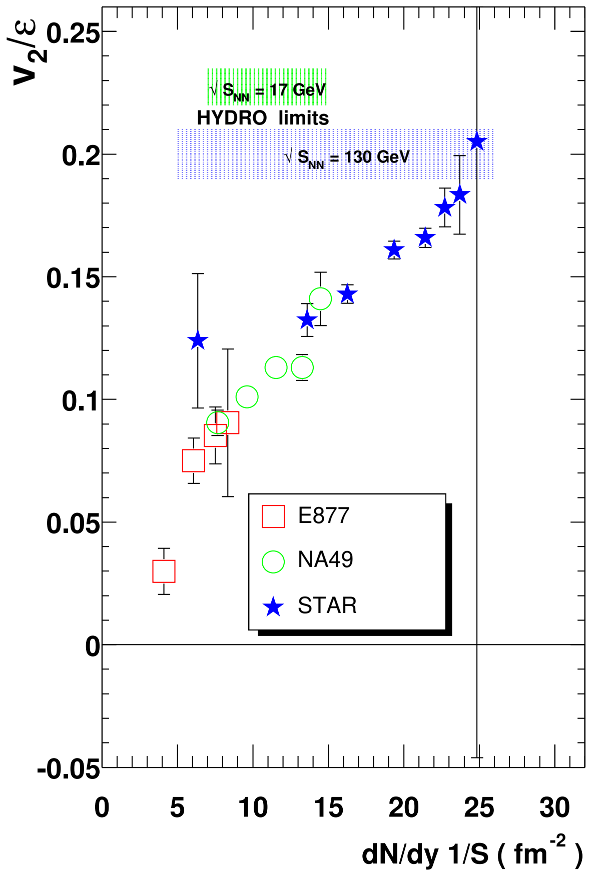


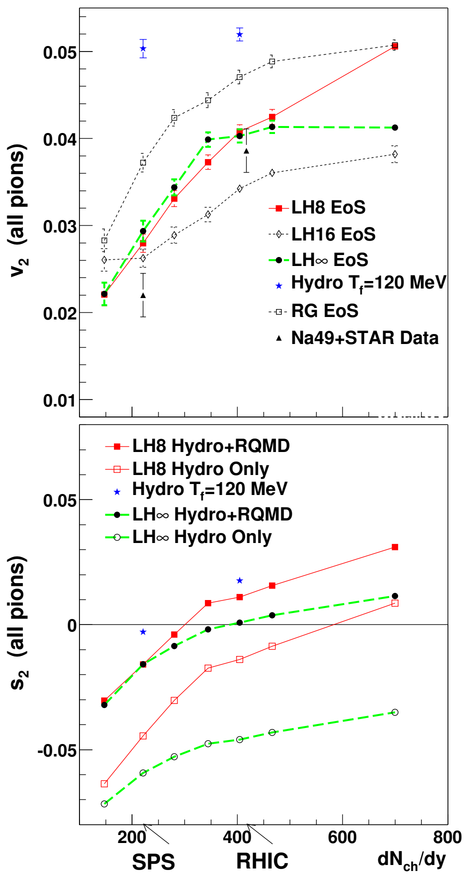


- Blust wave fits to RHIC spectra (STAR): all species can be well fitted with the same velocity and T_{kin}.
- The chemical freezeout T_{ch} is indepedent on centrality, while T_f decreases to about 90 MeV.
- Hung+ES,96: larger systems cool further.

At RHIC we are making progress in the proverbial "rocket science" getting as strong as possible conversion of the internal energy into flow

The energy (multiplicity) dependence of the elliptic flow, data vs hydro.



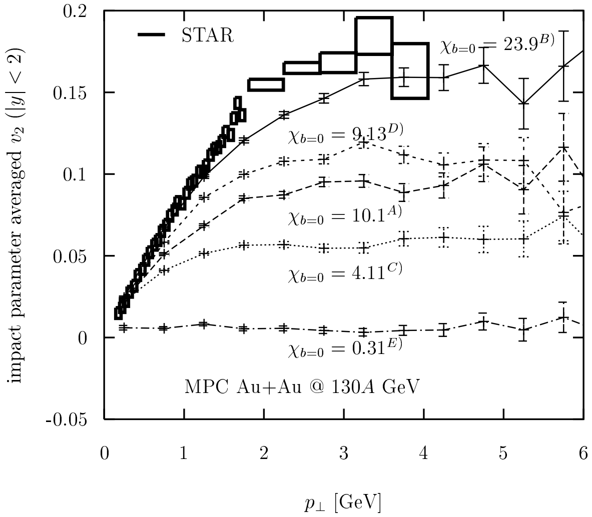


From Teaney, Lauret, ES (shown already at QM99, so it is well documented prediction). The rize is due to large QGP pressure. The red curve has the latent heat (LH) $=800\ MeV/fm^3$, it is is the closest to the lattice EoS is also the best fit to all flow data at SPS and RHIC

Comment: "hydro limit" on the STAR fig is for ideal hydro with wrong $T_{freezeout} = const(b)$, while the theory has correct freezeout (via RQMD).

It is only possible to get elliptic flow if the quasiparticle rescattering is increased by big factor ~ 50 relative to the pQCD expectations.

The Figure (from Gyulassy and Molnar) shows how the measured effect (boxes) can be reached if matter opacity of matter grows. The smallest value, roughly corresponding to gg scattering in pQCD, shows no collective effects whatsoever, and the rescattering cross section should be boosted by a big factor before the effect appears.



(Gluon elliptic flow for Au+Au at $\sqrt{s} = 130A$ GeV with transport opacities $\chi_{b=0} = 0.31,\ 4.11,\ 9.13,\ 10.1$ and 23.9.)

Viscous $\sim l/L$ corrections to hydro

• Ideal hydrodynamics assumes that all the dynamics is local, it ignores the finiteness of the mean free path l (or other nonzero correlation lengths).

• Including the first $O(l/L)$ dissipative terms, e.g. into boost-inv. (Bjorken) and axially symmetric hydro eqn:

$$\frac{1}{\epsilon + p}\frac{d\epsilon}{d\tau} = \frac{1}{s}\frac{ds}{d\tau} = -\frac{1}{\tau}\left(1 - \frac{\Gamma_s}{\tau}\right)$$

one finds in the r.h.s. exactly the combination which also appears in the sound attenuation length $\Gamma_s = \frac{4}{3}\eta/(\epsilon + p) = \frac{4}{3}(\eta/Ts)$

• according to Derek Teaney, hep-ph/0301099 $\Gamma_s \sim .1 - .2 fm$ or $\eta/s \sim .1$, which is much lower than for all known liquids.

The theory of viscosity

- Developed for long time in the weak coupling perturbative framework: large, $\eta/T^3 \sim const/g^4 log(1/g) >>> 1, const \sim 100$ at small $g <<$ 1
If so, $l \sim few\, fm$ and no hydro at RHIC ! (Recall the Gyulassy-Molnar plot here)
- However, in the strong coupling ($\mathcal{N}$ = 4 supersymmetric Yang-Mills or CFT) Polycastro, Son, Starinets, Phys. Rev. Lett. **87** (2001) 081601 (not in QCD!) their value for the sound attenuation length $T\Gamma_s = \frac{4}{3}\frac{\eta}{s} = \frac{1}{3\pi}$. If used for RHIC QGP $\Gamma_s \sim .1\, fm$ If so, excellent hydro, but no parton cascades...
Son and Starinets (03) found several gravity backgrounds in which similar calculation is possible and found the same $\frac{\eta}{s} = \frac{1}{4\pi}$ which is conjectured to be the universal lower bound for the most perfect liquid. The best known liquid, He^4 at high pressure, has $\frac{\eta}{s \sim 1}$, water at notmal conditions about 40.

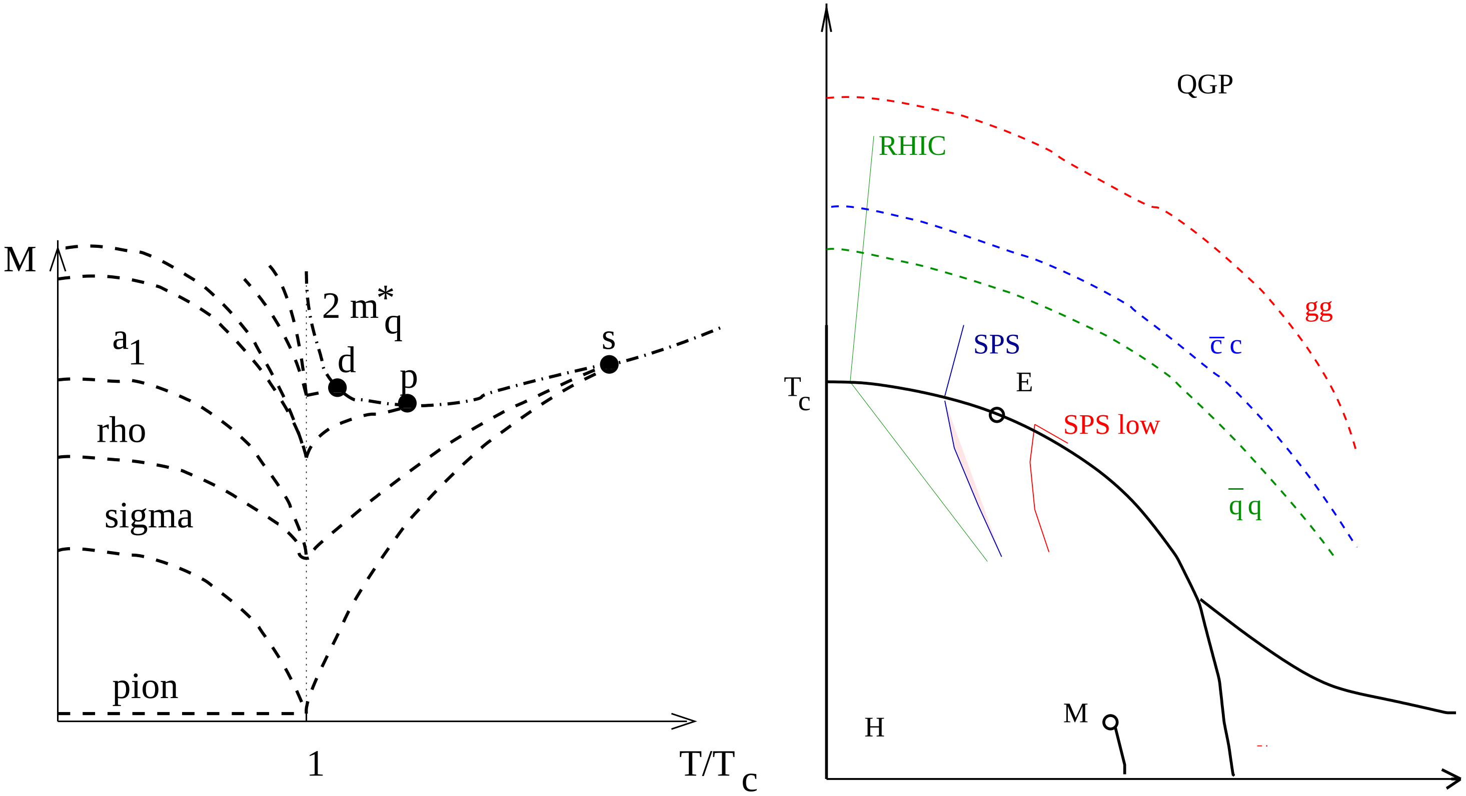
Are there hadrons above T_c, in Quark-Gluon Plasma?

- Old point of view: most hadrons (including $J/\psi = \bar{c}c$ but not $\Upsilon = \bar{b}b$) melt there.
- Exceptions: the pion (+sigma etc chiral multiplet) was believed to survive as resonance e.g.NJL-based papers or T.Schafer+ES, PLB 356:147,1995

- New lattice results from Bielefeld group and Asakawa-Hatsuda (using the maximal entropy method) found bound η_c at $1.5T_c$, very recently Asakawa-Hatsuda (hep-lat/0309001) have argued that the de-binding point is $1.6 < T_\psi < 1.9T_c$.

- One important point: near T_c the q,g quasiparticles are not light, then their mass decreases and at high T grows again as predicted by pQCD $M \sim gT$

ES and I.Zahed,hep-ph/0307267



A general view of the endlines for $\bar{q}q$ and also colored composites like qg, gg. Another famous colored dimer is the qq, the Cooper pair of color superconductivity

Why some hadrons may survive above T_c?

ES and I.Zahed,hep-ph/0307267

- **The loophole in the old argument: α_s kept frozen in the QGP as in-vacuum, at $\alpha_s \sim 1/3$.)**
- **New idea: at $T > T_c$ the charge continues to run to larger values, stopped by the Debye screening only. $\alpha_s \sim 1$ is reached**

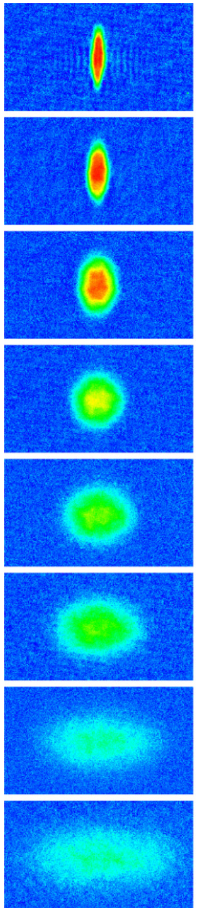
G.Brown,C.H.Lee, M.Rho and ES, in progress $\bar{q}q$ bound states for $T_c < T < T_{zerobinding}$: relativistic effects $(1 - \vec{v}_1\vec{v}_2)+$ spin-spin, plus the nonperturbative forces due to instanton-antiinstanton molecules provide massless σ, π at $T => T_c$

- **The main idea: large (unitarity limited) cross sections at the endlines $=>$ "sticky molasses"**

Elliptic flow with trapped Li^6 atoms:

K.M.O'Hara et al, Science 298,2179, 2002

T.Bourdel et al, PRL 91 020402 , July 11 2003



Magnetic field $B \sim 800G$ shifts (via the Feschbach resonance $|f = 1/2, m_f = 1/2 ><=> |f = 1/2, m_f = -1/2 >$) and makes the 38-th vibrational Li_2 state to exactly zero energy => infinite scattering length a, very large size and lifetime ~ 1 sec.

Normally gas is transparent, $l \ll L$, and expands without collisions isotropically

But in the strong coupling regime $l \ll L$ it explodes hydrodynamically !, see the figure

Cross section can be changed by many orders of magnitude, but the EoS changes by $\sim 20\%$ only ! (like in QGP and CFT... why?)

The phenomenon of the Adiabatic capture

- Very recent important discovery with trapped Li atoms J.Cubizolles et al, cond-mat/0308018, K.Strecker et al,cond-mat/0308318 all in PRL
- If one changes the magnetic field so that the molecular level moves from unbound into bound domain, nearly all atoms (~ 85 percents) are turned into Li_2 molecules, all of course in the same relative state near zero.
- Only a bit more cooling is needed to get BEC of molecules
- The phenomenon is reversible which proves that no entropy is produced: going back one finds molecules dissolved
- Going further into the bound region one finds that binding energy goes into heating the gas

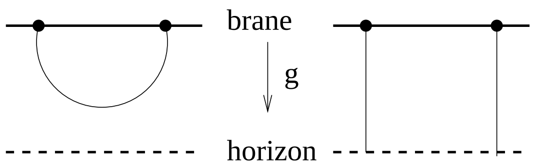
Hadroization by the Adiabatic capture?

- The adiabatic path in heavy ion collision also crosses the no-biding line in this direction.
- the reheating was predicted as a zig-zag path on the phase diagram
- Can the “hadronization” happen at this line, not at $T = T_c$?
- At least that would be enough to explain why we do not see large fluctuations related to quasi-first order transition: no “clumps”, the matter remains homogeneous at all times

A plasma phase at a supercritical coupling

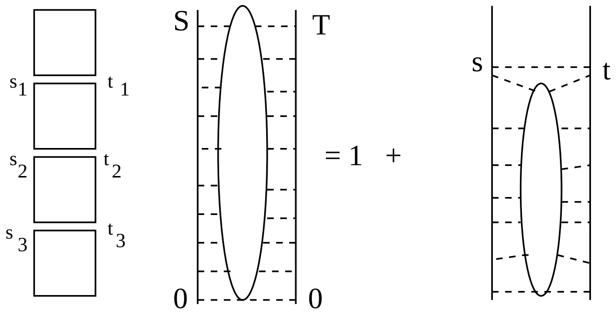
- The $\mathcal{N}=4$ SUSY Yang Mills gauge theory is conformal (CFT) (the coupling does not run). At finite T it is in a plasma phase at ANY coupling, including very strong $\lambda = g^2 N_c \gg 1$.
- Unexpected help from the string theory AdS/CFT correspondence by Maldacena turned the strongly coupled gauge theories to a classical problem in gravity albeit in 10 dimensions
- Example: a modified Coulomb's law (by Maldacena)

$$V(L) = -\frac{4\pi^2}{\Gamma(1/4)^4}\frac{\sqrt{\lambda}}{L}$$



- Free energy at large λ is $F = (3/4 + O(1/\lambda^{3/2}))F_{free}(T)$

G. Semenoff and K.Zarembo,hep-th/0202156. have shown that a modified Coulomb law can be understood by the resummed gluonic ladder



(a)
(b)

$\Gamma(S,T) = 1 + \frac{\lambda}{4\pi^2}\int_0^S ds \int_0^T dt \frac{1}{(s-t)^2+L^2}\Gamma(s,t)$

$\frac{\partial^2\Gamma}{\partial S\,\partial T} = \frac{\lambda/4\pi^2}{(S-T)^2+L^2}\Gamma(S,T)$

Change variables to $x = (S-T)/L$ **and** $y = (S+T)/L$

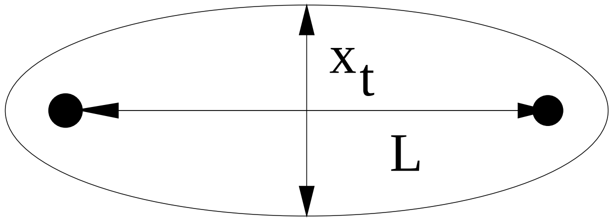
Expansion of the kernel in the first power of $(S-T)^2$ **leads to an oscillator potential and the problem is easily solved**

$\Gamma(x,y) =\approx \mathrm{C}_0\, e^{-\sqrt{\lambda}\,x^2/4\pi}\, e^{\sqrt{\lambda}\,y/2\pi}$

$V_{\mathrm{lad}}(L) = -\lim_{T\to+\infty}\frac{1}{T}\Gamma\,(T,T) = -\frac{\sqrt{\lambda}/\pi}{L}$

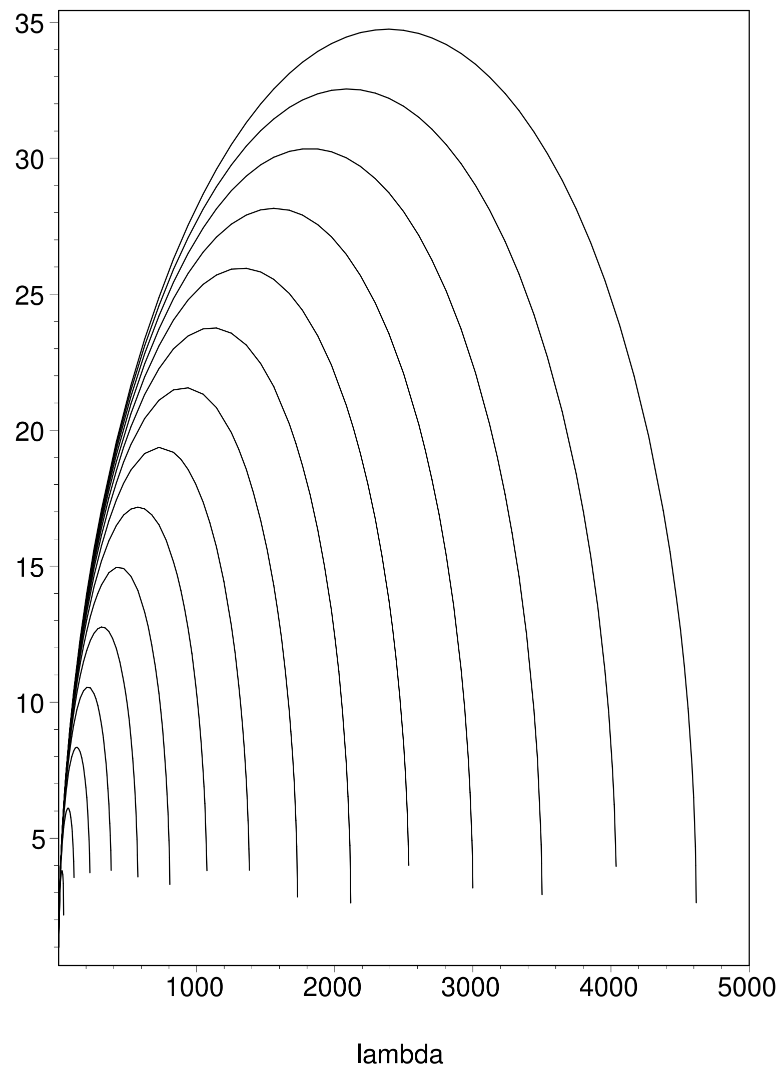
the same parametric form but different coefficient: $1/\pi = 0.318$ **while the exact Maldacena value is 0.228.**

- We observed that the exchanged gluons move with **the superluminal speed** $v \sim \delta t/L \sim \lambda^{1/4} \gg 1$
- Some higher order diagrams have extra $\lambda d^4 x \sim O(1)$: multi-body Bethe-Salpeter
- vertices distributed in a **quasi-string** regime



- **The main idea: the modified Coulomb law can be used even for relativistic bound states, with $v \sim 1$.**

- **Using a Klein-Gordon eqn $(E - V)^2 - m^2 = p^2, V = -C/r$ with a Coulomb potential, by WKB or exactly, one can find the spectrum. (Known from about 1930).**



$V = -\frac{C}{r}$

$E_{nl} =$

$m\left[1 + \left(\frac{C}{n+1/2+\sqrt{(l+1/2)^2-C^2}}\right)^2\right]^{-1/2}$

Small C - nonrelat. atoms, Balmer series... New regime at large $C >> 1$: families of relativistic deeply bound states, with large orbital momentum balancing the supercritical Coulomb

- These states in fact dominate the matter: their masses are $\sim T$ much smaller than those of original quasiparticles $\sim \lambda^{1/2}T$.

At strong coupling only those compound states are thermally excited,

- Furthermore, the density of states can be shown not to depend on λ.

The composites carry large angular momentum $l \approx \sqrt{\lambda} + 1/\sqrt{\lambda}$ and $\int_{l_{min}}^{l_{max}} dl^2 = l_{max}^2 - l_{min}^2 \approx \lambda^0$ thus the free energy $\mathrm{F} = \mathrm{C}N_c^2 T^4$ and without dependence on λ

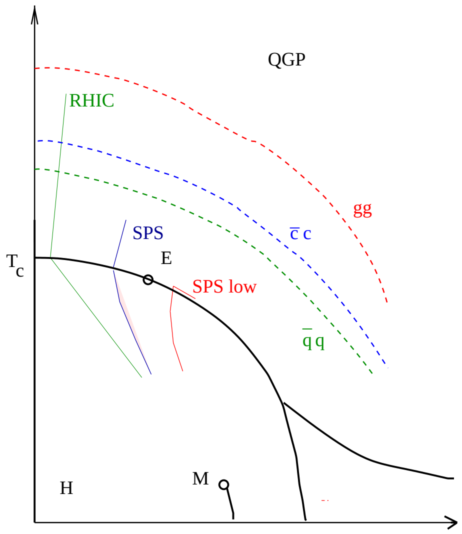
- (The oscillator-like spectrum $M_n \sim nT$ have been inferred by Starinets (and also Teaney, upublished) before our work, by a direct calculation of the stress tensor correlator -- via graviton propagator in AdS metrics.)

Summary

• Viscosity corrections in QGP (for large b and at early time relevant for v_2) seem to be small $\Gamma_s/\tau < .1$

• This disagrees with pQCD or weak coupling but agrees better with the strong coupling regime

• All s-wave mesons and glueballs do not melt at T_c, but at around 1.5-3 T_c, which generates strong (unitarity limited) rescattering e.g. $\bar{q}q <=>$ meson



• Amazing connection to trapped Li^6 atoms, the coolest matter known behaves as the hottest one, QGP

• Connection to string theory via Ads/CFT correspondence by Maldacena. The puzzles of the CFT plasma at the supercritical coupling are explained by the multiple deeply bound highly rotational states with masses $M \sim T$

additional slides

$$\frac{d^2\chi}{dx^2} + \left(\kappa^2 + \frac{4m_c}{3M_D}\frac{\alpha_s(x)}{x}e^{-x}\right)\chi = 0$$

The appearance of a level, at zero binding $E = 0, \kappa = 0$, is at $\frac{4m_c}{3M_D}\alpha_s > 1.68$

For example, using $4/3\alpha_s = 0.471$ from the Cornell potential and $m_c = 1.32\,\mathrm{GeV}$, as Satz et al did long ago, one finds that J/ψ melts at T_c.

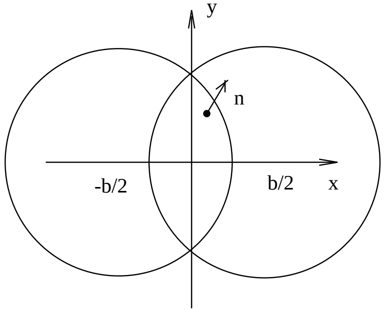
- **But, if the running of the QCD coupling continues till the screening length, we found that charmonium gets unbound only at $T = 1.62T_c$. Recent lattice results (Karsch etal, also Asakawa-Hatsuda hep-lat/0309001: it is between 1.6 and 1.9 T_c.**

Introductory remarks

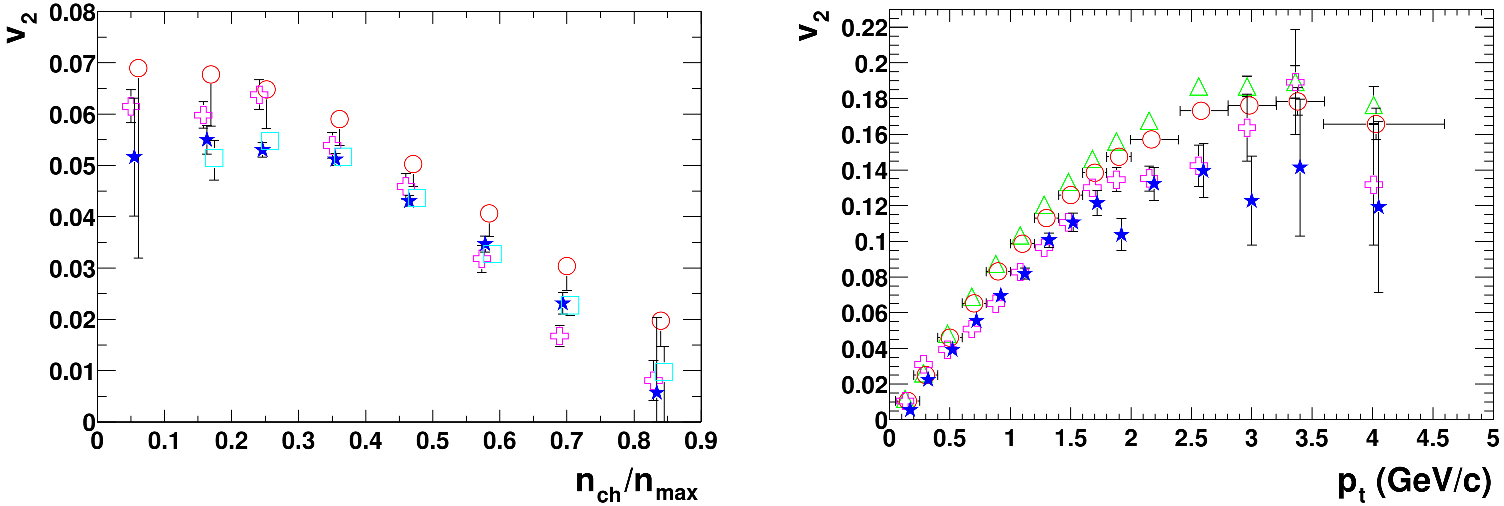
E.Shuryak

- History: no collective flow in e^+e^-,pp
- Hydrodynamics: the theory, not a model => Indeed $l \ll L$
- EoS predicted by the theory (lattice QCD) is confirmed quantiatively => $T_c \approx 170\, MeV$, latent heat $\approx .7 - .8 GeV/fm^3$.
- QGP $\neq$ weakly interacting quasiparticle gas
- transport properties (such as viscosity η) should still be quantified, but it is clear already that QGP is the most perfect liquid known $\eta/s \sim .1$ is 2 orders of magnitude less than for water!
- In fact QGP at RHIC ($T = (1-2)T_c$) is a strongly interacting matter, a relative to other (recently discovered) examples such as
Trapped ultracold (Li) atoms at $a \to \infty$
$\mathcal{N} = 4$ SUSY YM at the supercritical coupling $g^2 N_c >> 1$ (AdS/CFT duality <=> string theory)

Elliptic flow starts with the "initial almond" (for non-central collisions) and is described by Fourier harmonics



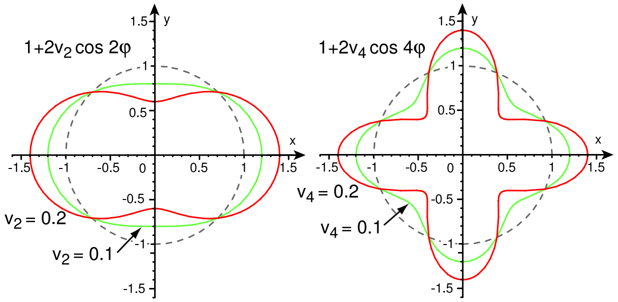
$$\frac{dN}{d\phi} = \frac{v_2}{2\pi} + \frac{v_2}{\pi}\cos(2\phi) + \frac{v_4}{\pi}\cos(4\phi) + \cdots$$

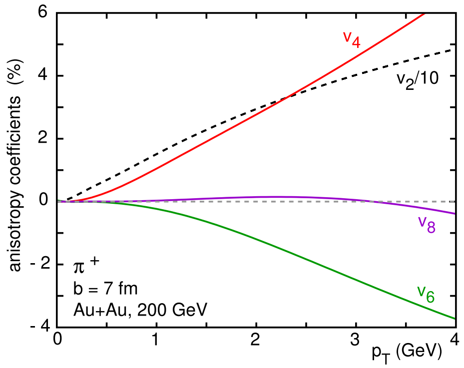


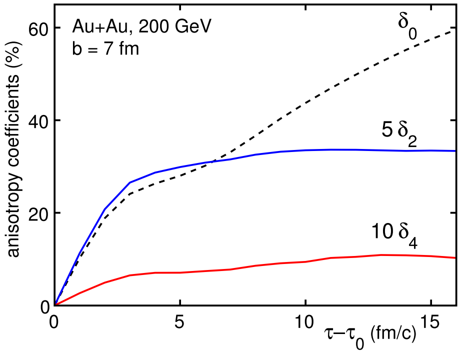
Elliptic flow versus (a) centrality and (b) p_t for Au + Au at $\sqrt{s_{NN}} = 130$ GeV from STAR at RHIC. Note large effects at semi-peripheral and large p_t

The next moment v_4

- D.Teaney+ES, PRL 1999, The unusual “nutcracker” solution at $b > 8\,fm$ which is more square than elliptic
- Recently P.Kolb 2003 calculated $v_n(b, p_t)$ in a systematic way





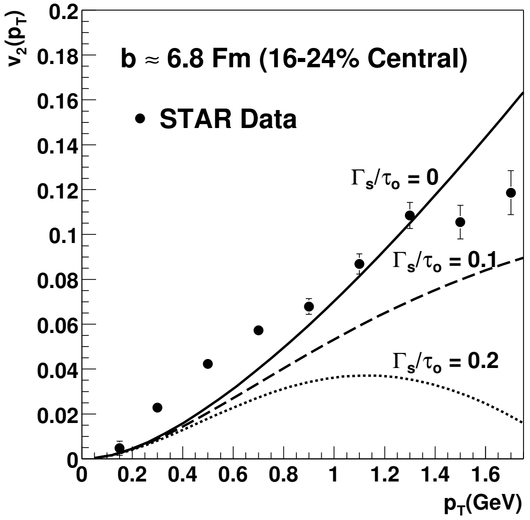


Rumors are this is seen by STAR, in good agreement with hydro once again...

• Following Derek Teaney, hep-ph/0301099, one may try to extract it from the data, by looking at deviations from ideal hydro predictions.

• the viscous correction to particle distribution is completely determined $f = f_o(1 + \frac{\eta}{s} p^\alpha p^\beta \left\langle \nabla_\alpha u_\beta \right\rangle)$. Consequence: Corrections are $\sim \frac{\Gamma_s}{4\tau} \left(\frac{p_T}{T}\right)^2$

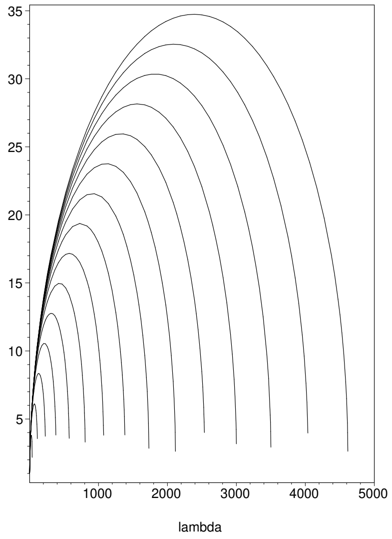
• We need QGP viscosity at earlier times.: thus we look at elliptic flow. the fireball is spatially anisotropic only for $\tau < 3 fm/c$, when matter is in the QGP phase.



Elliptic flow v_2 as a function of p_T for different values of Γ_s/τ_o. The data points are four particle cummulant data from the STAR collaboration [**?**]. Only statistical errors are shown.

The results for different Γ_s/τ is shown The experimental data deviate from ideal hydro curve at $p_\perp \approx 1.6\,GeV$ which indicates $\Gamma_s/\tau \sim 0.05$ or so. Substituting here the relevant time $\tau \sim 3fm/c$ we get surprisingly small $\Gamma_s \sim .15fm$.

News (from ES+ I.Zahed, hep-th/0308073): The CFT plasma at the supercritical coupling is made mostly of the binary composites



• From Klein-Gordon eqn in modified Coulomb potential the families of relativistic deeply bound states follow, in which large orbital momentum balances the relativistic Coulomb attraction

$$E_{nl} = \pm m \left[1 + \left(\frac{C}{n+1/2+\sqrt{\tilde{l}^2 - C^2}} \right)^2 \right]^{-1/2}$$

• These states in fact dominate the matter: their masses are $\sim T$ much smaller than those of original quasiparticles $\sim \lambda^{1/2} T$. At strong coupling only those compound states are thermally excited