Schwinger's variational principle in a modified form

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Munich, 10'th March 2009

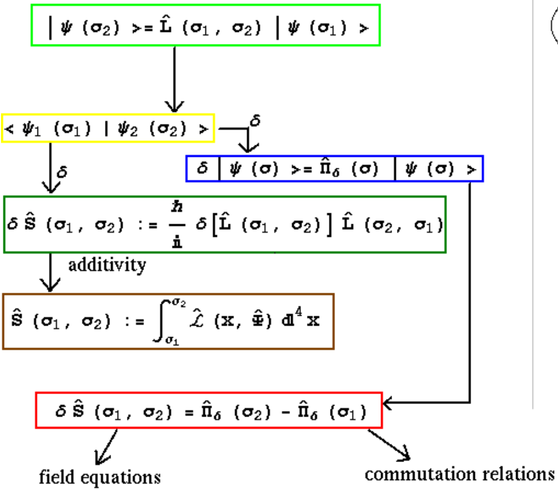
- Action principle in qunatum theory by Schinger (50’s and 60’s)
- Variation with respect to operators by Bozhidar (2002)
- Applications in quantum field theory by de Melo et al. (recently in the last years)

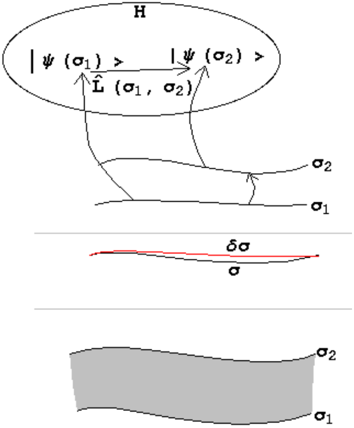
Advantages:

- generalization of Hamilton principle
- no real quantization, starting point lies in a quantum description
  - no ordering problem from the quantization
  - greater set of physical systems
- derivation of the quantum field equations and commutation relations

Disadvantages:

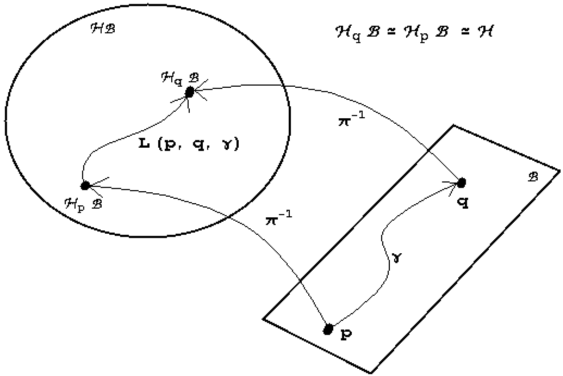
- no covariance in the changes of 3-dim hypersurfaces
- ill-defined variation with respect to operators (odering problem with the variation operators)


$|\psi(\sigma_2)\rangle = \hat{L}(\sigma_1,\sigma_2)\,|\psi(\sigma_1)\rangle$
$\langle\psi_1(\sigma_1)\,|\,\psi_2(\sigma_2)\rangle$
δ
$\delta\,|\psi(\sigma)\rangle = \hat{\Pi}_\delta(\sigma)\,|\psi(\sigma)\rangle$
δ
$\delta\hat{S}(\sigma_1,\sigma_2) := \frac{\hbar}{i}\,\delta\left[\hat{L}(\sigma_1,\sigma_2)\right]\hat{L}(\sigma_2,\sigma_1)$
additivity
$\hat{S}(\sigma_1,\sigma_2) := \int_{\sigma_1}^{\sigma_2}\hat{\mathcal{L}}(x,\hat{\Phi})\,d^4x$
$\delta\hat{S}(\sigma_1,\sigma_2) = \hat{\Pi}_\delta(\sigma_2) - \hat{\Pi}_\delta(\sigma_1)$
field equations
commutation relations

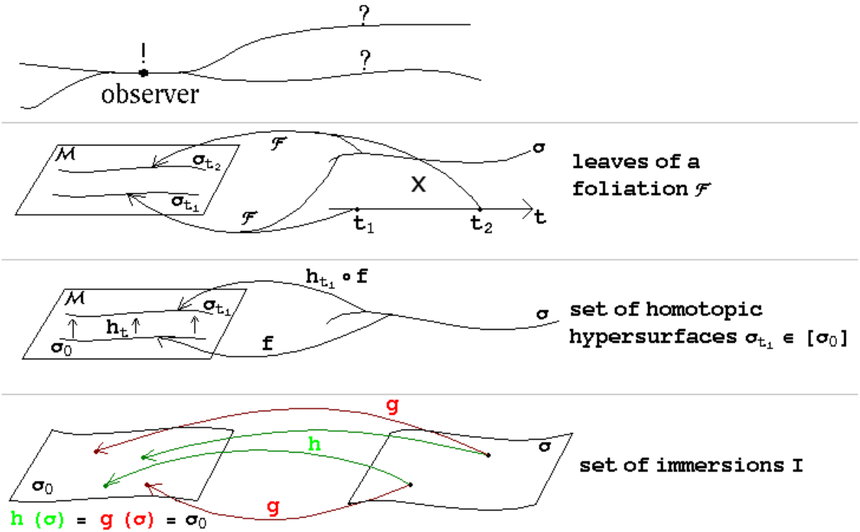

H
$|\psi(\sigma_1)\rangle$
$|\psi(\sigma_2)\rangle$
$\hat{L}(\sigma_1,\sigma_2)$
σ_2
σ_1
$\delta\sigma$
σ
σ_2
σ_1

We need:

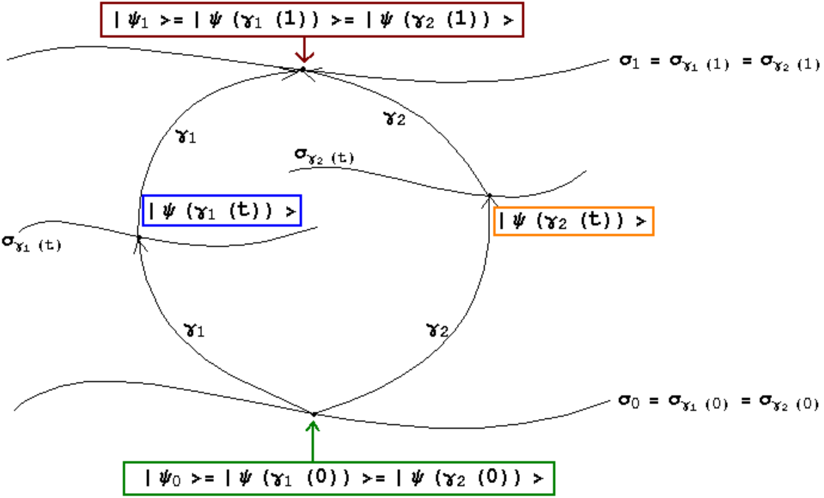
- Hilbert bundle
  - basis: set of immersions and set of field operator bases
  - standard fibre: Hilbert space
- differentiable linear transport
  $\Rightarrow$ kinematical covariant derivative



THE PROBLEM OF COVARIANCE IN THE CHANGE OF HYPERSURFACES

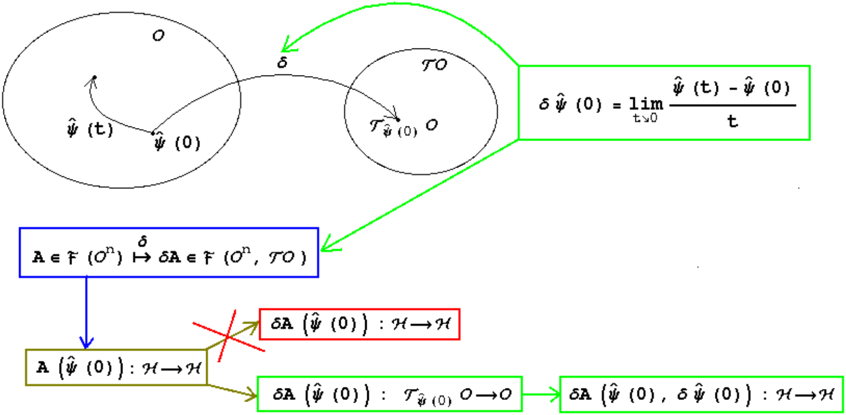


Path-independence of the linear transport



$$\Rightarrow L(\sigma_0, \sigma_1, \gamma_1) = L(\sigma_0, \sigma_1, \gamma_2)$$

$\mathbf{A} \in \mathcal{O} \;:\Leftrightarrow\; \mathbf{A} : \mathcal{H} \longrightarrow \mathcal{H}$ linear and other properties

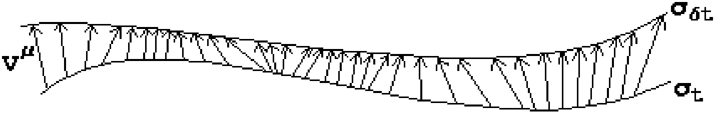


Momentum operators with respect to operator variations do not act on the Hilbert space but on the tangent operator space!

Assumptions:

- covariance of the quantum description with respect to changes of immersions and field operator bases
- path independent, unitary linear transport L
- generator of hypersurface changes:

$$\begin{aligned}\frac{\hbar}{\imath}\partial_t|\psi(\sigma(t),\hat{\phi}^m)\rangle &= \hat{\Pi}_\alpha(\sigma(t),\hat{\phi}^m;v^\alpha)|\psi(\sigma(t),\hat{\phi}^m)\rangle \\ \hat{\Pi}_\alpha(\sigma(t),\hat{\phi}^m;v^\alpha) &= \int\limits_{\sigma(t)}\hat{\Lambda}(x,\sigma(t),\hat{\phi}^m)v^\alpha(x)d^3x_\alpha\end{aligned}$$



$$\delta\left[\langle\psi_2|\psi_1\rangle(\sigma(t),\hat{\phi}^m)\right]=0$$

$$\Downarrow$$

$$\int_{t_1}^{t_2}\langle\psi_2|\delta\left[\hat{\Pi}_\alpha(v^\alpha)\right]|\psi_1\rangle(\sigma(t),\hat{\phi}^m)dt=\left[\langle\psi_2|\hat{\Pi}_\delta|\psi_1\rangle(\sigma(t),\hat{\phi}^m)\right]_{t_1}^{t_2}$$

$$\Downarrow$$

$$0=\left(\frac{\partial\hat{\Lambda}}{\partial\hat{\phi}^a}-\hat{\pi}^\alpha_{a,\alpha}\right)(\sigma(t),\hat{\phi}^m,x;\delta\hat{\phi}^a)$$

$$\hat{\Pi}_{(\alpha,a)}(\sigma(t),\hat{\phi}^m;v^\alpha,\delta\hat{\phi}^a)=\int_{\sigma(t)}\left(\hat{\Lambda}v^\alpha+\hat{\pi}^\alpha_a(\delta\hat{\phi}^a)\right)(\sigma(t),\hat{\phi}^m,x)d^3x_\alpha$$

Momentum density operators: $\hat{\pi}^\mu_m=\frac{\partial\hat{\Lambda}}{\partial\hat{\phi}^m_{,\mu}}$

Path-independence of L	Momenta are not tensors but connexions
$\downarrow$	$\downarrow$
Zero-curvature of Hilbert bundle not space time	Momenta are the connexions of L
$\searrow$	$\swarrow$

$$0 = \left(\frac{\hbar}{\imath} \left(\tilde{\delta} \hat{\Pi}_{\delta} - \delta \hat{\Pi}_{\tilde{\delta}} \right) + \left[\hat{\Pi}_{\delta}, \hat{\Pi}_{\tilde{\delta}} \right]_{-} \right) (\sigma(t), \hat{\phi}^{m})$$

$\downarrow$

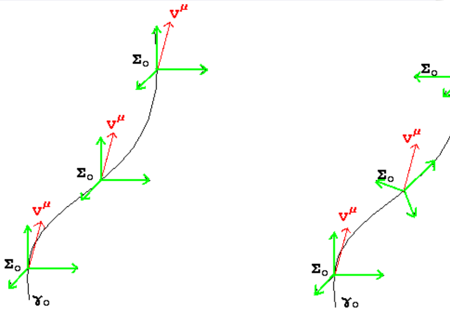
commutation relations

Fermi-Walker transport in quantum theory

Problem of time evolution: $\partial_t \langle \hat{A}(\sigma(t), \hat{\phi}^m) \rangle_{|\psi(\sigma(t),\hat{\phi}^m)\rangle} = 0$

Solution:

- Fermi-Walker transport (rigid coordinate system of the observer)
- Coordinate axes are observables = selfadjoint operators.



Time evolution with Fermi-Walker:

$$\imath\hbar\partial_t \langle \hat{A} \rangle_{|\psi(\sigma(t),\hat{\phi}^m)\rangle} = \langle \left[\hat{A}, \hat{P}_0(\sigma(t), \hat{\phi}^m) \right]_- \rangle_{|\psi(\sigma(t),\hat{\phi}^m)\rangle}$$

Thank you for your attention!