RELATIVITY (PHY408), HOMEWORK 2, DUE SEPT.17

1. (10) For hyperbolic rotation in 2 dimensions (a) prove that their product in any order is the same and $a(\theta_1)a(\theta_2) = a(\theta_2)a(\theta_1) = a(\theta_1 + \theta_2)$ (b) since velocity of the frame is related to the hyperbolic angle by $v = tanh(\theta)$ and the angles are additive, derive Einstein's velocity addition formula from $tanh(\theta_1 + \theta_2)$.

2.(10) Two brothers separated by distance D start simultaneously (in frame (t, x)) walking toward each other, with velocity $v = c/2$ each. The starting events are called A and B, and the meeting event is called C. Draw the space-time world lines AC,BC in this frame, and then translate them (events A,B,C) into the frame $(\bar{t}, \bar{x})$ co-moving with one of the brothers.

3.(20) A lightweight pole 20 m long is on the ground next to a barn 15 m long. An olympic athlete picks up the pole and runs with it toward the barn with velocity $v = 0.8c$. The friend at the door notice the **front end** of the pole passing him (the event A) and when **the end** of the pole passes him as well (the event B), he shuts the door. The front end of the pole hit **the back wall of the barn** (the event C). Draw space-time diagram in two frames, (i) that of the friend (rest) and (ii) co-moving with the runner. What happened earlier, B or C, in original frame? What is earlier in the comoving frame? What is the interval between B and C, is it timeline, space like or null? Was at some time the 20 m pole been inside the 15 m barn? What the friend and the runner would answer to this question?

4.(30) Futuristic police chase a criminal in a car. The police car can reach the speed $v_p = c/4$ but the criminal's one can reach $v_c = (3/4)c$. (a) Policeman shoots his gun which (relative to him) provides the speed for a bullet $v_v = 1/2$. Will the bullet catch with the criminal?

(b) when the distance between them is $L = 10^8 km$, the policeman shoots his laser gun (light from it has speed c). Draw the world lines of the police car, the criminal's car, the bullet and laser light in all three frames (rest, and both cars) and decide if the light reaches the criminal.

5.(10) A hypothetical rocket has n boosters. When the first of them finished its work and separates, the rest gets to the velocity $0.6c$, after the second to the velocity $0.6c$ relative to the previous one, and so on. What is the rapidity η_n, the velocity v_n and the Lorentz factor $\gamma_n = cosh(\eta_n)$ after $n = 10$ such steps?