Gauge topology and confinement: an update

Edward Shuryak
Stony Brook
(CPOD 2014, Bielefeld)

Stony Brook University

Our group's logo emphasizing our interest in topology

outline

- nonzero holonomy => instanton-dyons, interactions
- classical dyon-antidyon interaction **(new R.Larsen+ES)**
- back reaction to holonomy potential => confinement
  ES and T.Sulejmanpasic,arXiv:1305.0796, inspired by Poppitz, Schafer and Unsal,arXiv:1212.1238
- Numerical simulations of the dyon ensemble
  (new R.Larsen+ES)
- fermionic zero modes of the dyons, dyon-antidyon pairs
  ES and T.Sulejmanpasic,arXiv:1201.5624, R.Larsen and ES, in progress
- dyon ensemble+fermions => chiral symmetry breaking P.Faccioli+ES,archive 1301.2523**Phys. Rev. D 87, 074009 (2013)**

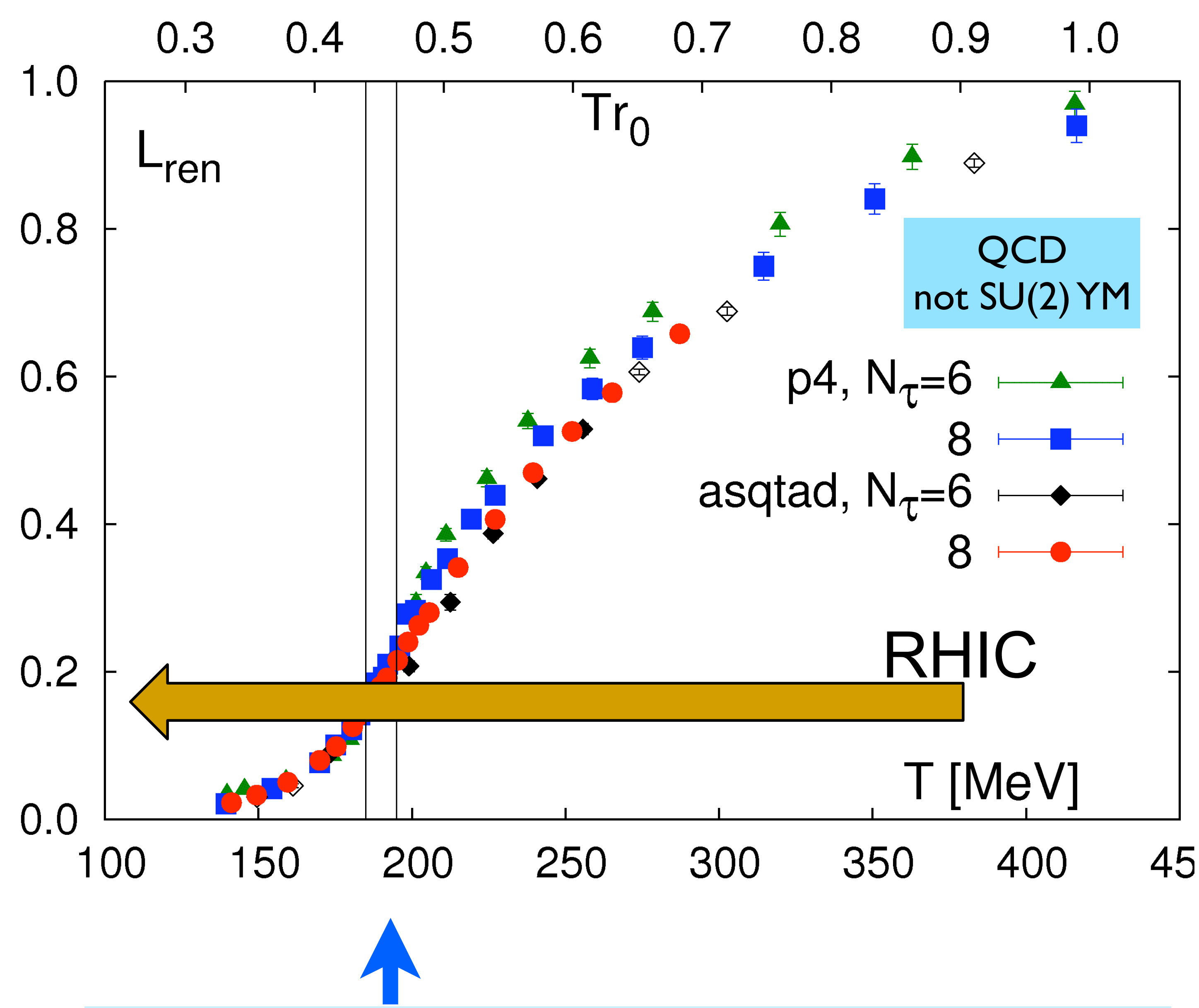
holonomy: P=>0 is the onset of confinement

$$L =< P >=< \frac{1}{N_c} Tr P exp(i \int d\tau A_0) >$$
$$= e^{-F(quark)/T}$$

The Polyakov loop

L=1 => A0=0 high T full QGP
L=1/2 "semi-QGP" (Pisarski)
L=>0 no quarks or onset of confinement

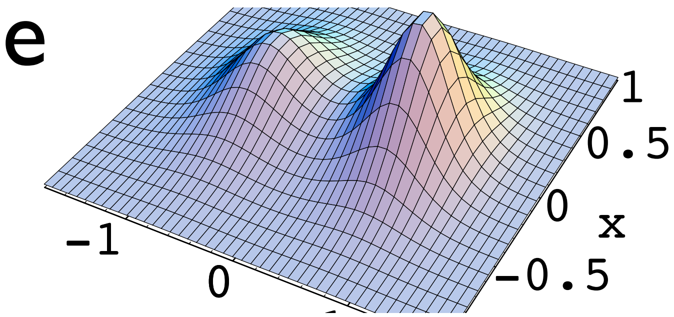
popular models like PNJL and PSM , make semi-QGP quantitative

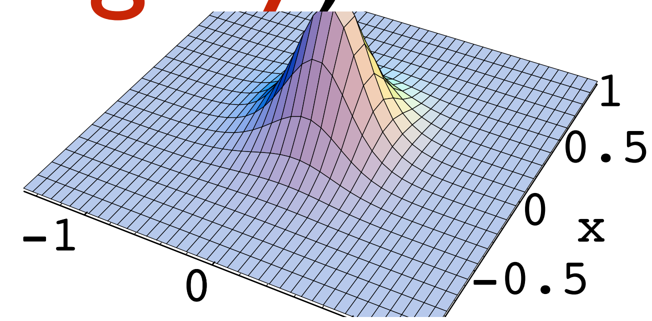


The approximate width of the phase transition in thermodynamical quantities, energy and entropy is small, but P changes between Tc and 2Tc

Instantons => Nc selfdual dyons (KvBLL, Pierre van Baal legacy)

<P> nonzero Polyakov line
=> <A_4> nonzero
=> new solutions





Instanton liquid
4d+short range

Dyonic plasma
3+1d long range

instanton-dyons in SU(2)

name	E	M	mass
M	+	+	v
$\bar{M}$	+	-	v
L	-	-	$2\pi T - v$
$\bar{L}$	-	+	$2\pi T - v$

calorons=M+L are E and M neutral

TABLE I: The charges and the mass (in units of $8\pi^2/e^2T$) for 4 SU(2) dyons.

terminology

- **particle-monopoles,** 3d particle-like objects with nonzero magnetic charge. Its Bose condensate makes ``dual superconductor'' and confinement. Not a solution in pure gauge, not to be discussed in this talk, though
- **instanton-***

*=dyon (Diakonov et al, ES et al)
*= monopole (Unsal et al)
*=quark (Zhitnitsky et al)

the same object

(anti)selfdual 3d YM solution at nonzero holonomy with electric and magnetic charges, a constituent of the instanton. Not a particle=> no paths or condensates, Z is an integral over locations only

terminology

- **particle-monopoles,** 3d particle-like objects with nonzero magnetic charge. Its Bose condensate makes ``dual superconductor'' and confinement. Not a solution in pure gauge, not to be discussed in this talk, though
- **instanton-***

*=dyon (Diakonov et al, ES et al)
*= monopole (Unsal et al)
*=quark (Zhitnitsky et al)

the same object

(anti)selfdual 3d YM solution at nonzero holonomy with electric and magnetic charges, a constituent of the instanton. Not a particle=> no paths or condensates, Z is an integral over locations only

! in N=2 SYM (Seiberg-Witten theory) when both are under control, and can prove that stat.sum Z over particle-monopoles and instanton dyons **are equal !**
(being low and high-T approaches to the same physics)

terminology

- **particle-monopoles,** 3d particle-like objects with nonzero magnetic charge. Its Bose condensate makes ``dual superconductor'' and confinement. Not a solution in pure gauge, not to be discussed in this talk, though
- **instanton-***

*=dyon (Diakonov et al, ES et al)
*= monopole (Unsal et al)
*=quark (Zhitnitsky et al)

the same object

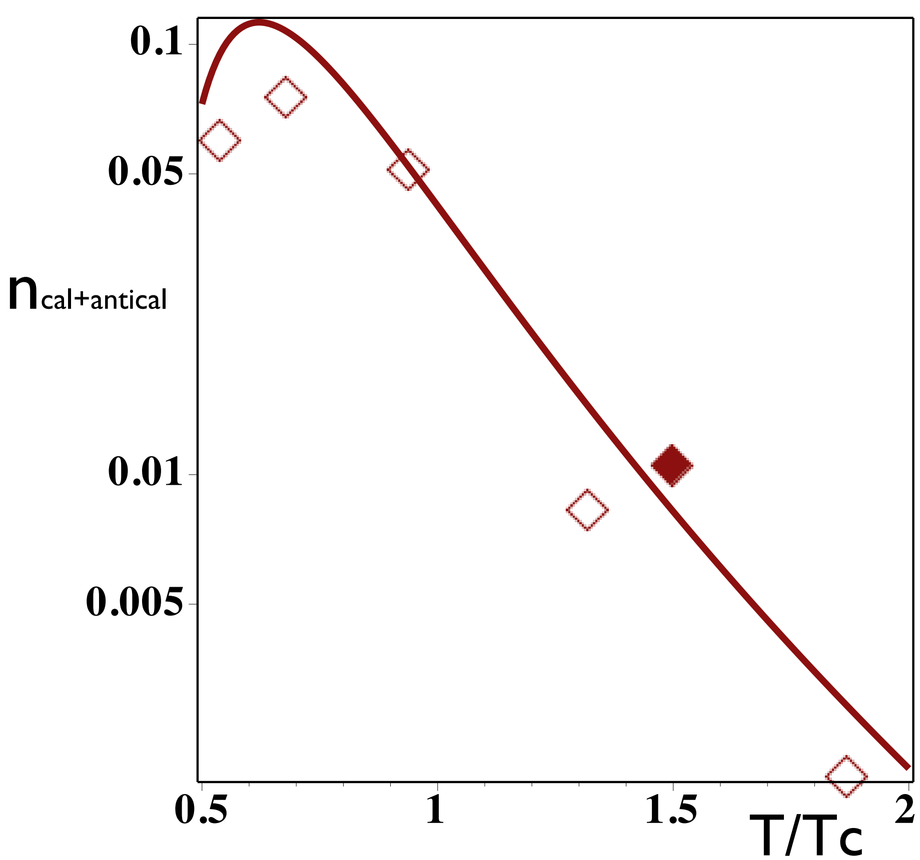
(anti)selfdual 3d YM solution at nonzero holonomy with electric and magnetic charges, a constituent of the instanton. Not a particle=> no paths or condensates, Z is an integral over locations only

! monopoles were used before to understand confinement
instantons were used to understand chiral breaking,
and instanton-dyons seem to be able to do **both!**

calorons (finite-T) were located on the lattice

Illgenfritz et al, Gattringer...

are instanton-dyons semiclassical?



$$n_{cal+\bar{cal}} = K S_{cal}^4 e^{-S_{cal}}, \ S_{cal} = \frac{22}{3} \ln\left(\frac{T}{\Lambda}\right) \qquad (1)$$

with parameters[1] $K = 0.024, \Lambda/T_c = .36$. The caloron action at T_c is 7.50, so per dyon it makes $S_d = S_{cal}/2 = 3.75$, which gives an idea how semiclassical the discussed objects are. (SU(3) instantons have actions $S_{cal} \approx 12$ or $S_d = S_{cal}/3 \approx 4$, quite close in magnitude.) After those parameters are fixed, one knows semiclassical densities of the dyons and their pairs, as we explain in detail below.

dimensionless
density n/T^4
vs lattice data

so Sd=4>>1 or exp(-4)
is my semiclassical parameter

at T>Tc M are lighter than L,
both were identified
on the lattice

Statmech of the dyons

The screening by the plasma (ES,Pisarski-Yaffe, Diakonov)

$$Z = \int \{dX_i\} e^{-S_c} \det G \, \det F_{zm} \frac{\det' F_{nzm}}{\sqrt{\det' B}}$$

The moduli space metric (Atiyah,Hitchin,Diakonov)

in a dilute case provides electric and magnetic Coulomb with natural charges

If dense produces regularization and repulsive core

Fermionic determimant in zero mode approximation (ES.Sulejmanpasic), only for L dyons if fermions are anti-periodic

Statmech of the dyons

The screening by the plasma (ES,Pisarski-Yaffe, Diakonov)

$$Z = \int \{dX_i\} e^{-S_c} \det G \, \det F_{zm} \frac{\det' F_{nzm}}{\sqrt{\det' B}}$$

The moduli space metric (Atiyah,Hitchin,Diakonov)

in a dilute case provides electric and magnetic Coulomb with natural charges

If dense produces regularization and repulsive core

Fermionic determimant in zero mode approximation (ES.Sulejmanpasic), only for L dyons if fermions are anti-periodic

classical dyon-antidyon interaction was still unexplored!

confinement

(where the holonomy potential comes from?)

Holonomy potential and confinement from a simple model of the gauge topology

E. Shuryak[1,*] and T. Sulejmanpasic[2,†]
[1] *Department of Physics and Astronomy, Stony Brook University, Stony Brook 11794,USA*
[2] *Institut für Theoretische Physik, Universität Regensburg, D-93040 Regensburg, Germany*

Close $\bar{I}I$ pairs correspond to weak fields, which cannot be treated semiclassically and should be subtracted from the semiclassical configurations. This physical idea has been implemented in the Instanton Liquid Model via an "excluded volume", which generates a repulsive core and stabilizes the density.

In a few important cases, in which the partition function is independently known, such subtraction can be performed exactly, *without* any parameters. The $\bar{I}I$ pair contribution to the partition function in QM instanton problem has been done via the analytic continuation in the coupling constant $g^2 \to -g^2$ by Bogomolny [14] and Zinn-Justin [15] (BZJ), who verified it via known semi-classical series. Another analytic continuation has been used by Balitsky and Yung [13] for supersymmetric quantum mechanics.

Recently Poppitz, Schäfer and Ünsal (PSU) [16, 17] used BZJ approach in the $N = 1$ Super-Yang-Mills theory on $R^3 \times S^1$, observing that the result obtained matches exactly the result derived via supersymmetry [18]. PSU papers are the most relevant for this work, as they focus on the instanton-dyons (referred to as

v=<A0> is Higgs VEV
shifted and rescaled,
v=0 trivial limit (high T)
b=0 confining (T<Tc)

$$\frac{1}{2}\mathrm{Tr}\, P(x) = \cos\left(\frac{v(x)}{2T}\right) \ , \quad b = \frac{4\pi^2}{g^2}\left(\frac{v}{\pi T} - 1\right)$$

Similarly to electric holonomy
Polyakov introduced magnetic
one <C0>=sigma

b magnetic holonomy
sigma - magnetic one

$$V_{eff} = 2n_d \left(-2\cos\sigma \cosh b + n_d \mathcal{A} \cosh(2b)\right) \ .$$

$$\mathcal{M} \sim e^{-b+i\sigma-S_d} \ , \qquad \bar{\mathcal{M}} \sim e^{-b-i\sigma-S_d}$$

$$\mathcal{L} \sim e^{b-i\sigma-S_d} \ , \qquad \bar{\mathcal{L}} \sim e^{b+i\sigma-S_d} \ .$$

4 dyon amplitudes

Poppitz+Schafer+Unsal idea:

If the quadratic term is repulsive, (+ sign) it can lead to confinement at sufficiently high density

b magnetic holonomy
sigma - magnetic one

$$V_{eff} = 2n_d\left(-2\cos\sigma\cosh b + n_d\mathcal{A}\cosh(2b)\right) \ .$$

$$\mathcal{M} \sim e^{-b+i\sigma-S_d}\ , \qquad \bar{\mathcal{M}} \sim e^{-b-i\sigma-S_d}$$
$$\mathcal{L} \sim e^{b-i\sigma-S_d}\ , \qquad \bar{\mathcal{L}} \sim e^{b+i\sigma-S_d}\ .$$

4 dyon amplitudes

Poppitz+Schafer+Unsal idea:
If the quadratic term is repulsive, (+ sign)
it can lead to confinement
at sufficiently high density

b magnetic holonomy
sigma - magnetic one

$$V_{eff} = 2n_d\left(-2\cos\sigma\cosh b + n_d \mathcal{A}\cosh(2b)\right) \ .$$

$$\mathcal{M} \sim e^{-b+i\sigma-S_d}\ , \qquad \bar{\mathcal{M}} \sim e^{-b-i\sigma-S_d}$$
$$\mathcal{L} \sim e^{b-i\sigma-S_d}\ , \qquad \bar{\mathcal{L}} \sim e^{b+i\sigma-S_d}\ .$$

$$V_{pert} = \frac{\pi^2}{12}\left(1-\frac{b^2}{S_d^2}\right)^2$$

4 dyon amplitudes

Poppitz+Schafer+Unsal idea:

If the quadratic term is repulsive, (+ sign) it can lead to confinement at sufficiently high density

b magnetic holonomy
sigma - magnetic one

$$V_{eff} = 2n_d \left(-2\cos\sigma \cosh b + n_d \mathcal{A} \cosh(2b)\right) \; .$$

$$V_{pert} = \frac{\pi^2}{12}\left(1 - \frac{b^2}{S_d^2}\right)^2$$

$$\mathcal{M} \sim e^{-b+i\sigma-S_d} \, , \qquad \bar{\mathcal{M}} \sim e^{-b-i\sigma-S_d}$$

$$\mathcal{L} \sim e^{b-i\sigma-S_d} \, , \qquad \bar{\mathcal{L}} \sim e^{b+i\sigma-S_d} \, .$$

4 dyon amplitudes

unlike in the SUSY setting of PSU, non-SUSY theories have non-zero perturbative holonomy potential to be overcomed!

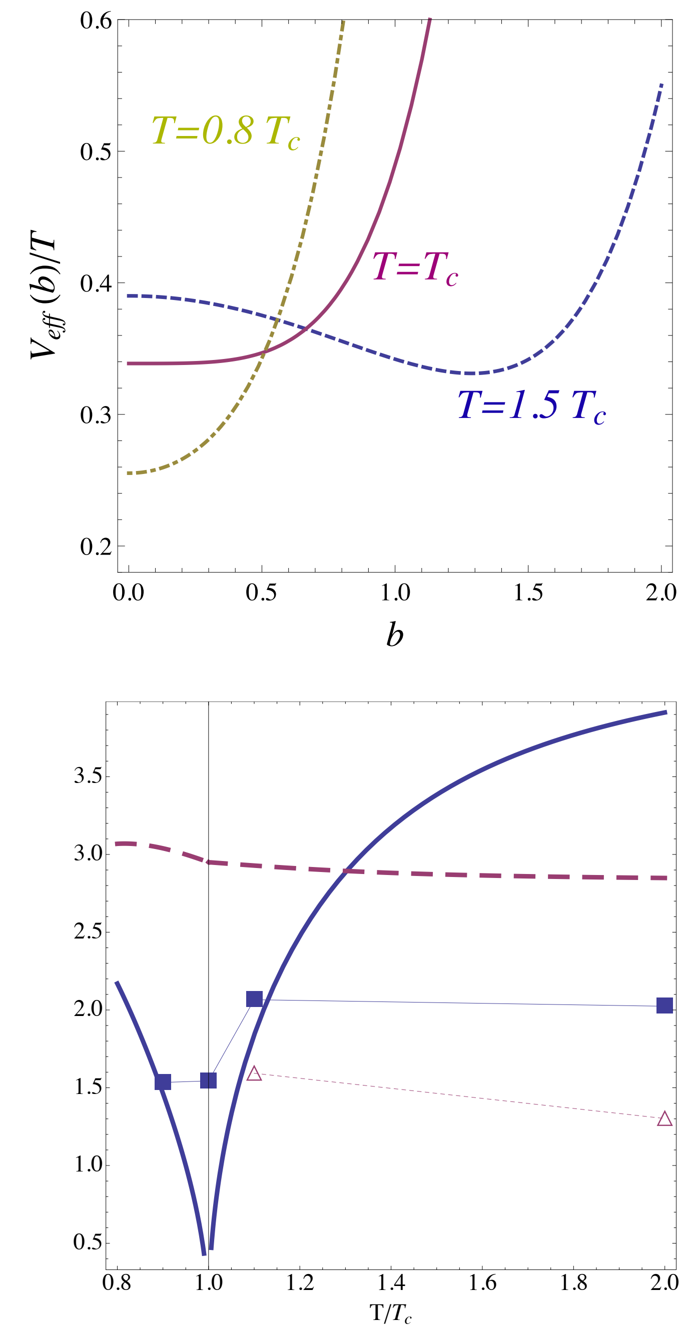
In fact the excluded volume model works well for SU(2) YM

the density is deduced from calorons
n(dyons)=(n(calorons))^(1/Nc)
and is large enough to make second-order in density term do its work

the only parameter A is fixed from known Tc and has a reasonable size (including the Coulomb enhancement)

electric and magnetic screening masses are even factor 2 **too large** as compared to those from lattice propagators:

their ratio ME/MM is well reproduced



m_E

m_M

FIG. 2: The upper plot shows the effective potential $V_{eft}(b)/T$ (13) for $T/T_c = 0.8, 1, 1.5$ shown by the dashed,solid and dot-dashed lines, respectively. The plot shows electric m_E/T and magnetic m_M/T screening masses versus temperature, indicated by the solid and dashed lines, respectively. Thick lines are our model, the data points are from lattice propagators [26], the lines connecting data points are shown simply for their identification.

predictions: densities of the M and L dyons

crosses: "unidentified topological objects", an upper limit

circles: identified M

L dyon size is very small and
measuring <P> at its center
is hard, as well as E and M charges: not done yet

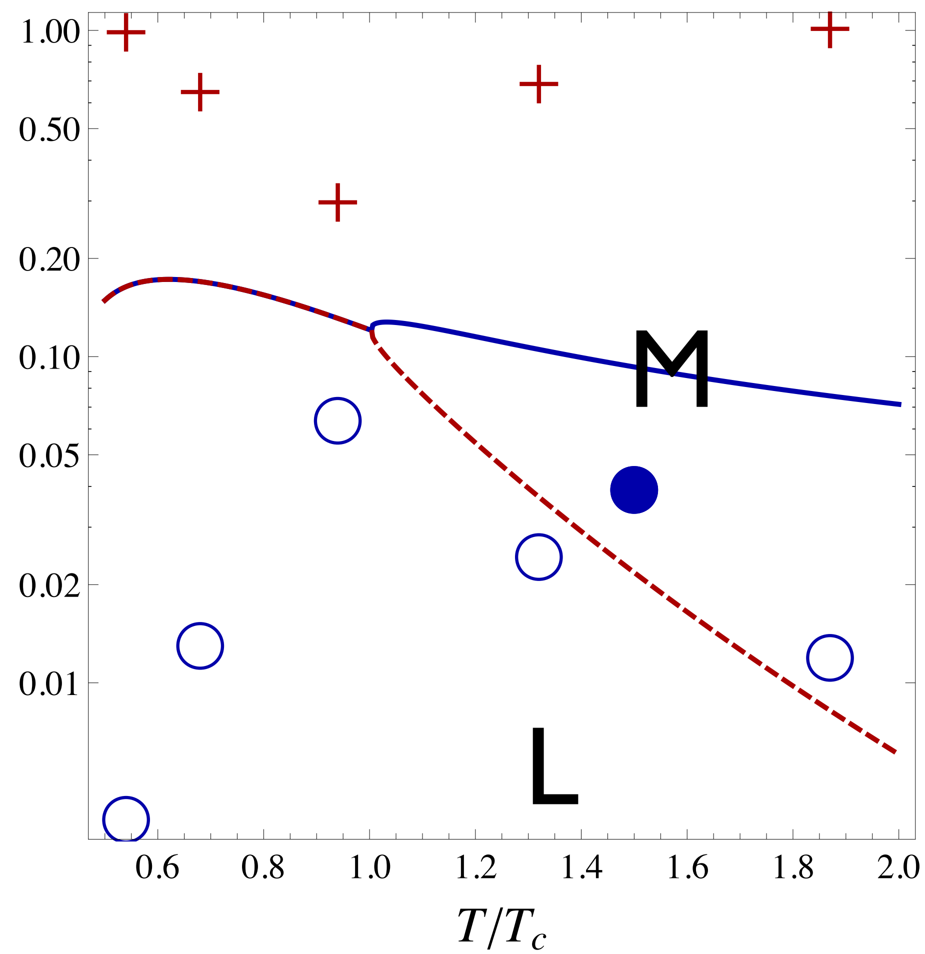


FIG. 3: Prediction of the model for the temperature dependence of the density of the instanton-dyons are shown by the lines, those with solid and dashed lines are for M, L type dyons, respectively. Open (filled) circles show identified M-type dyons from ref. [19] ([20]). The crosses show "unidentified topological objects" from [19]. Circles and crosses provide the lower and the upper bound for the dyon density.

Classical interactions of the instanton-dyons with antidyons

Rasmus Larsen and Edward Shuryak
Department of Physics and Astronomy, Stony Brook Univesity, Stony Brook NY 11794-3800, USA

Instanton-dyons, also known as instanton-monopoles or instanton-quarks, are topological constituents of the instantons at nonzero temperature and holonomy. While the interaction between instanton-dyons have been calculated to one-loop order by a number of authors, that for dyon-antidyon pairs remains unknown even at the classical level. In this work we are filling this gap, by performing gradient flow calculations on a 3d lattice. We start with two separated and unmodified objects, following through the so called "streamline" set of configurations, till their collapse to perturbative fields.

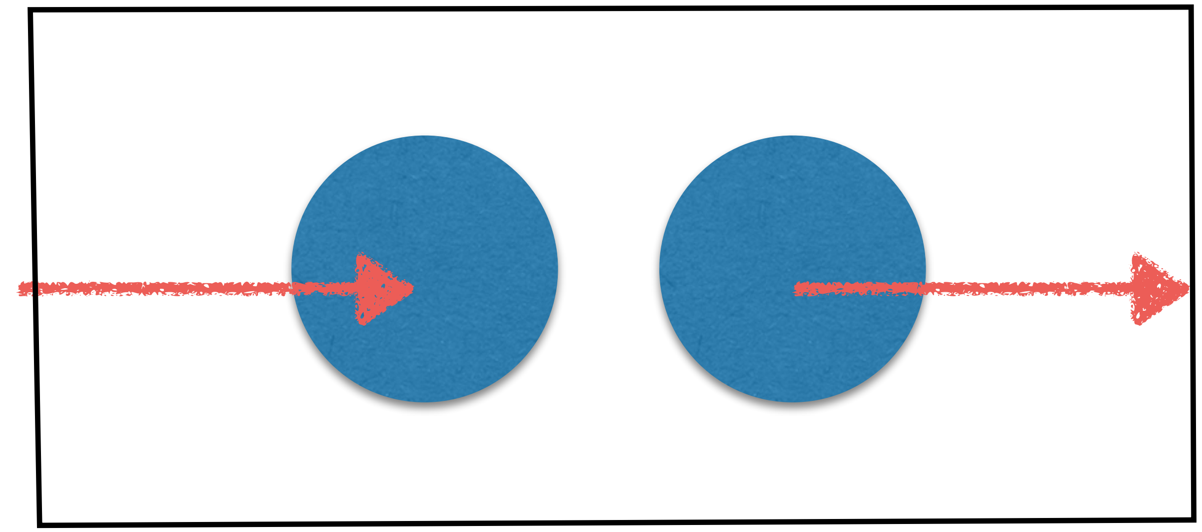
M Mbar pair on a 3d lattice (not periodic)
start with a "combed" sum ansatz and then do action gradient flow
=>"streamline configurations" found,
total magnetic charge =0
=> only Dirac string is left
total electric charge =2 (unlike instanton-antiinstanton pair)
=> massive charged gluons leave the box

Let us remind that the gauge action can be expressed in terms of the 3-dimensional action

$$S = \frac{1}{g^2}\int_0^{1/T} dx_4 S_3 = \frac{S_3}{g^2 T} \qquad (25)$$

which itself scales as $S_3 \sim v$: thus the M dyon action is $\sim v/T$. We do not care about T and the gauge coupling g since it is just an overall factor in the action, and work with the S_3 itself. Furthermore, since our classical 3d theory is invariant under the transformation $A_\mu \to vA_\mu$ and $r \to vr$, the absolute units are unimportant and we can work with $v = 1$.

The gradient flow process was found to proceed via the following stages:
(i) *near initiation*: starting from relatively arbitrary ansatz one finds rapid disappearance of artifacts and convergence toward the streamline set
(ii) following the *streamline itself*. The action decrease at this stage is small and steady. The dyons basically approach each other, with relatively small deformations: thus the concept of an interaction potential between them makes sense at this stage
(iii) a *metastable state* at the streamline's end: the action remains constant, evolution is very slow and consists of internal deformation of the dyons rather than further approach
(iv) *rapid collapse* into the perturbative fields plus some (pure gauge) remnants



Dirac strings setting

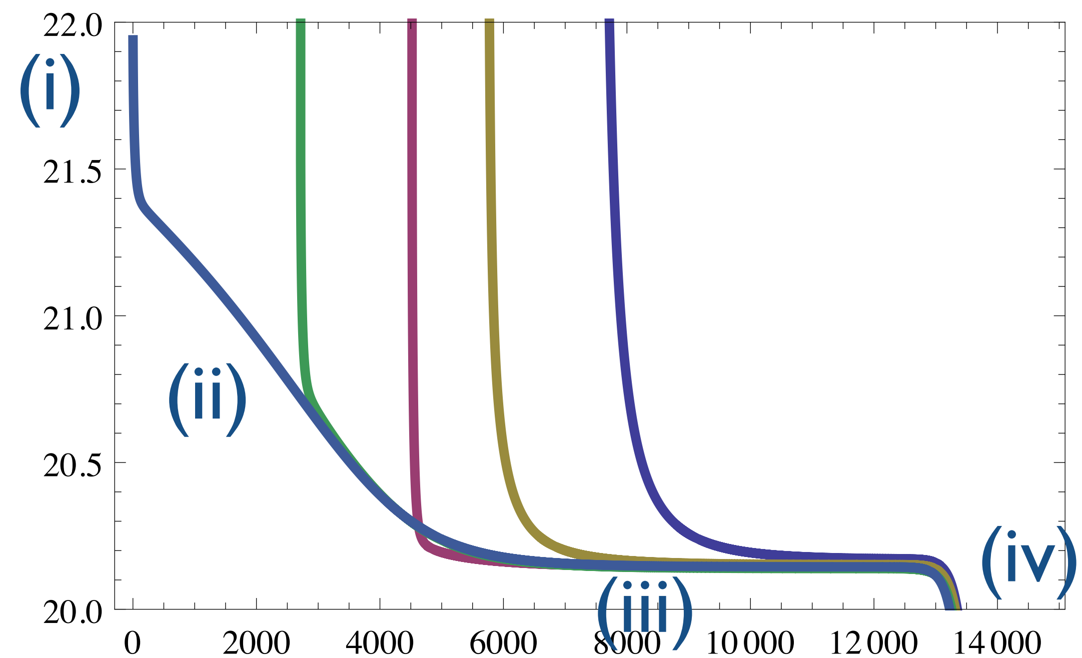


FIG. 2: Action for $v = 1$ as a function of computer time (in units of iterations of all links) for a separation $|r_M - r_{\bar{M}}|v = 0$, 2.5, 5, 7.5, 10 between the M and $\bar{M}$ dyon from right to left in the graph. The action of two well separated dyons is 23.88.

a stream, a pool, and then waterfall observed

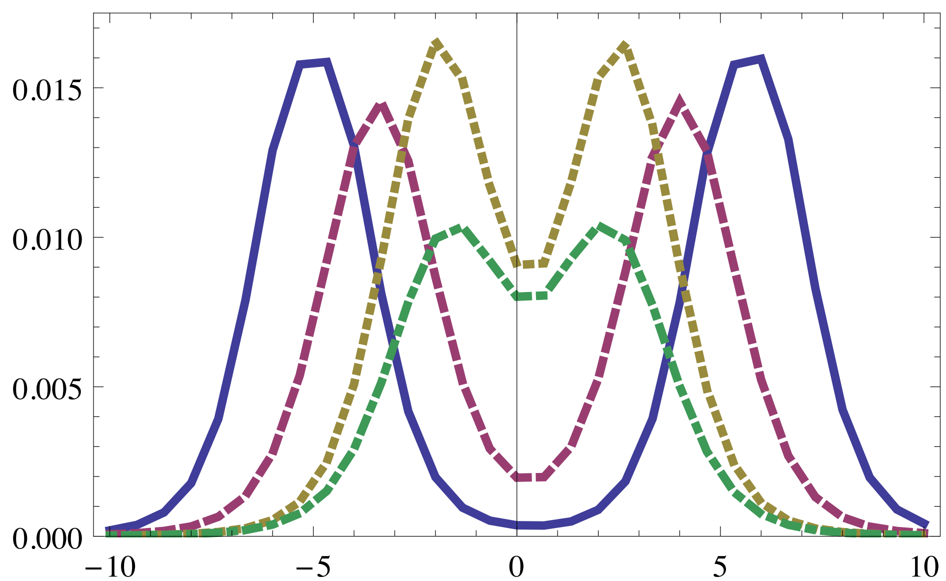


FIG. 5: Action density along the z axis in natural units for a separation $|r_M - r_{\bar{M}}|v = 10$ between the center of the 2 dyons. The configuration with the maximums furthest from each other is the start configuration. After 3000 steps it has moved further towards the center. At 12000 steps the configuration has reached the metastable configuration with a separation between the maximums of around 4. At 13700 the configuration has collapsed around halfway, and will continue to shrink until the action is 0. Times are as shown in Fig. 2.

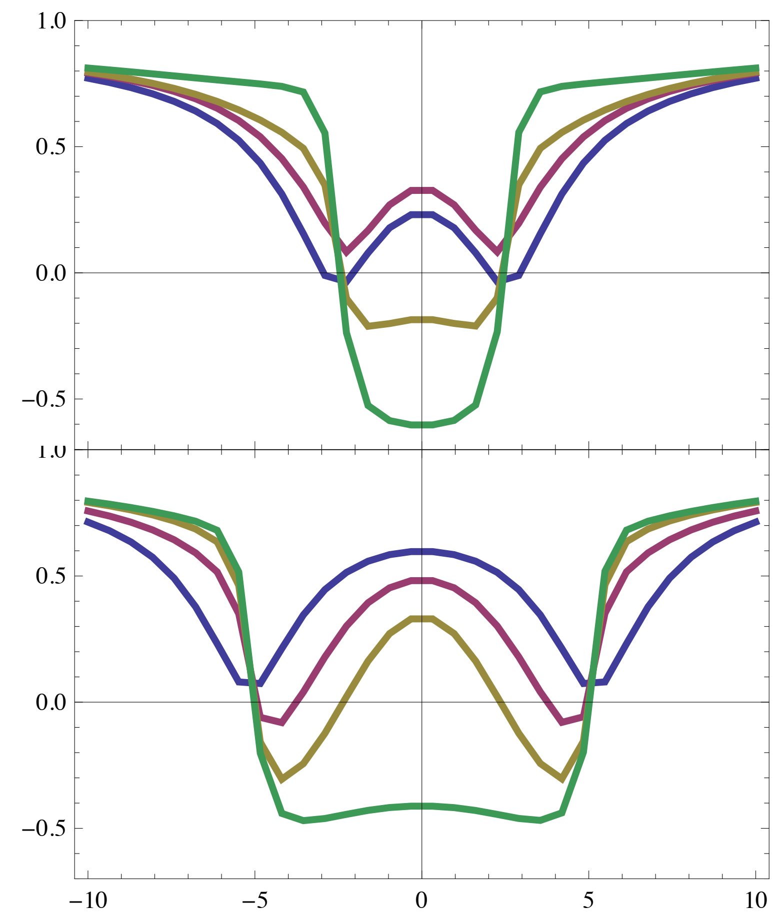


FIG. 6: Subsequent snapshots of A_4^3 along the z axis in natural units for a separation of 5 (a) and 10 (b) between the center of the 2 dyons. (a) The configuration which is smallest at the edges is the start configuration. After 5000 iterations the gradient flow has raised the minimums slightly, but is overall the same shape. After 9400 iterations the configuration has started collapsing. At 10000 the configuration has collapsed completely. (b) The configuration which is smallest at the edges is the start configuration. After 3000 iterations the minimums have moved slightly towards the middle and the minimums have become smaller. At 10000 the configuration has reached the stable almost flat area in the action. At 14000 the configuration has collapsed completely.

Electric and magnetic charges

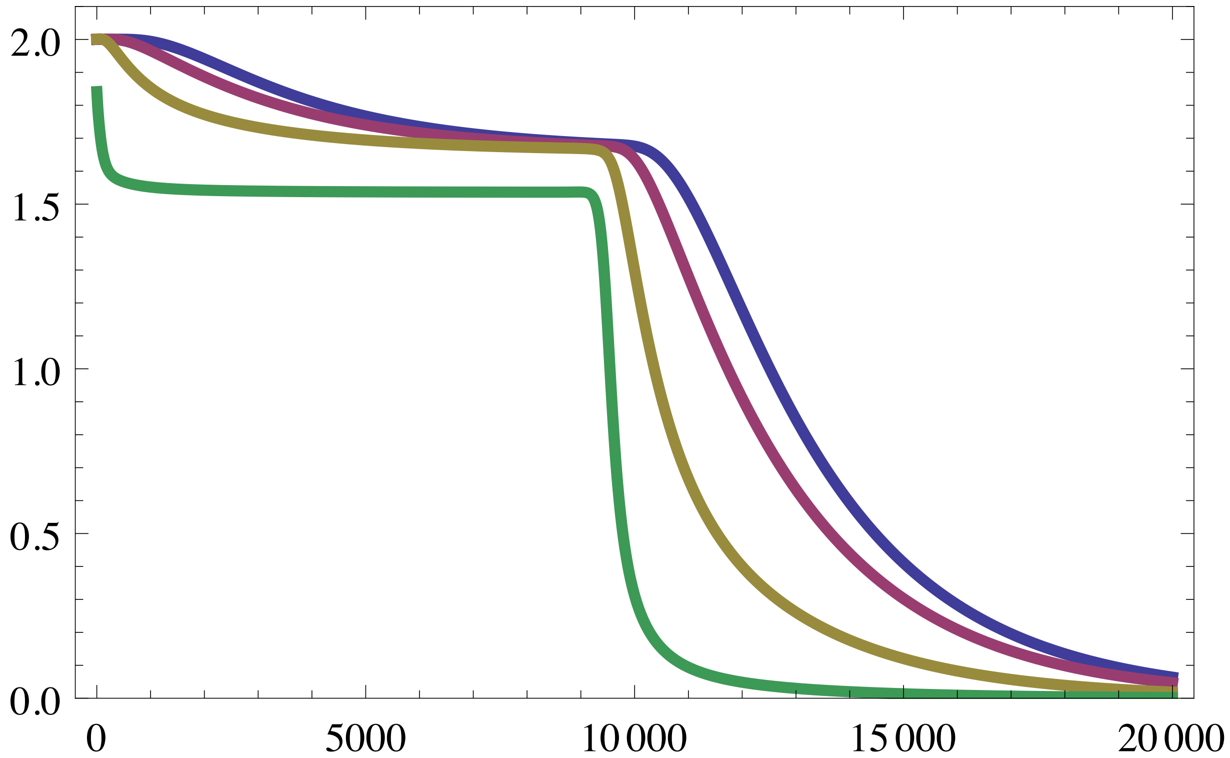


FIG. 7: Electric charge for $v = 1$ as a function of computer time (in units of iterations of all links) for a separation $|r_M - r_{\bar{M}}|v = 5$ between the dyons. The electric charge is found from a box of size 62^3, 46^3, 30^3 and 14^3 points each centered at the center. The electric charge is biggest for 62^3. Total lattice size is 64^3 points.

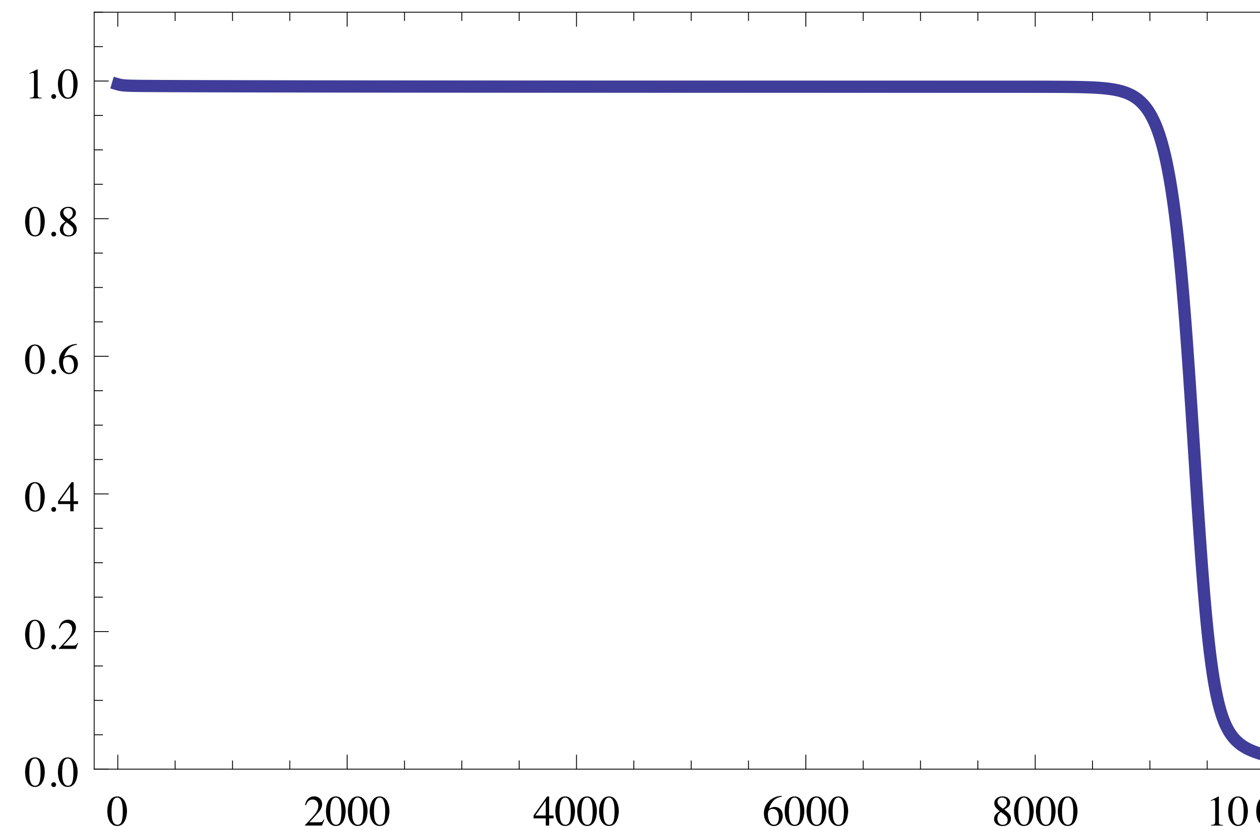


FIG. 8: Magnetic charge for $v = 1$ as a function of compu time (in units of iterations of all links) for a separation $|r$ $r_{\bar{M}}|v = 5$ between the dyons. The magnetic charge is fo from a box that goes from the center in z and to the e The drop happens at the same time as the drop in action

Interacting Ensemble of the Instanton-dyons and Confinement in SU(2) Gauge Theory

Rasmus Larsen and Edward Shuryak
Department of Physics and Astronomy, Stony Brook Univesity, Stony Brook NY 11794-3800, USA

Instanton-dyons, also known as instanton-monopoles or instanton-quarks, are topological constituents of the instantons at nonzero temperature and holonomy. We perform numerical simulations of the ensemble of interacting dyons for SU(2) pure gauge theory, and calculate its free energy as a function of the holonomy and the dyon density. We observe that at the dyon density grows, its minimum moves from zero to a value corresponding to confinement.

The instanton-dyons are placed on the 3-dimensional sphere S^3 and their coordinates are updated with the Metropolis algorithm.

new element is the inclusion of the leading order dyon-antidyon interaction

new account for Debye screening

evaluation of the total free energy, including $P_{GPY}(v)$

$$\Delta S_{D\bar{D}} = -\frac{\nu 102.25}{g^2}\frac{(x-0.907)^2}{x^3+15.795} \tag{2}$$

$$x = 2\pi T \nu r \tag{3}$$

for distances larger than $x > 4$. For x smaller than 4 we have a core which we describe by

$$\Delta S_{D\bar{D}} = \frac{\nu V_0}{1+\exp(-(x-4))} \tag{4}$$

$$-2\int d^3r \frac{e^{-M_D r_1}}{r_1}\frac{e^{-M_D r_2}}{r_2} = -\frac{2\pi}{M_D}e^{-2M_D r_{12}}$$

preliminary result show we do have

confinement!

high T
low density
nu=0

$$< P >= cos(\pi\nu) = 0$$

below Tc
high density
nu=1/2

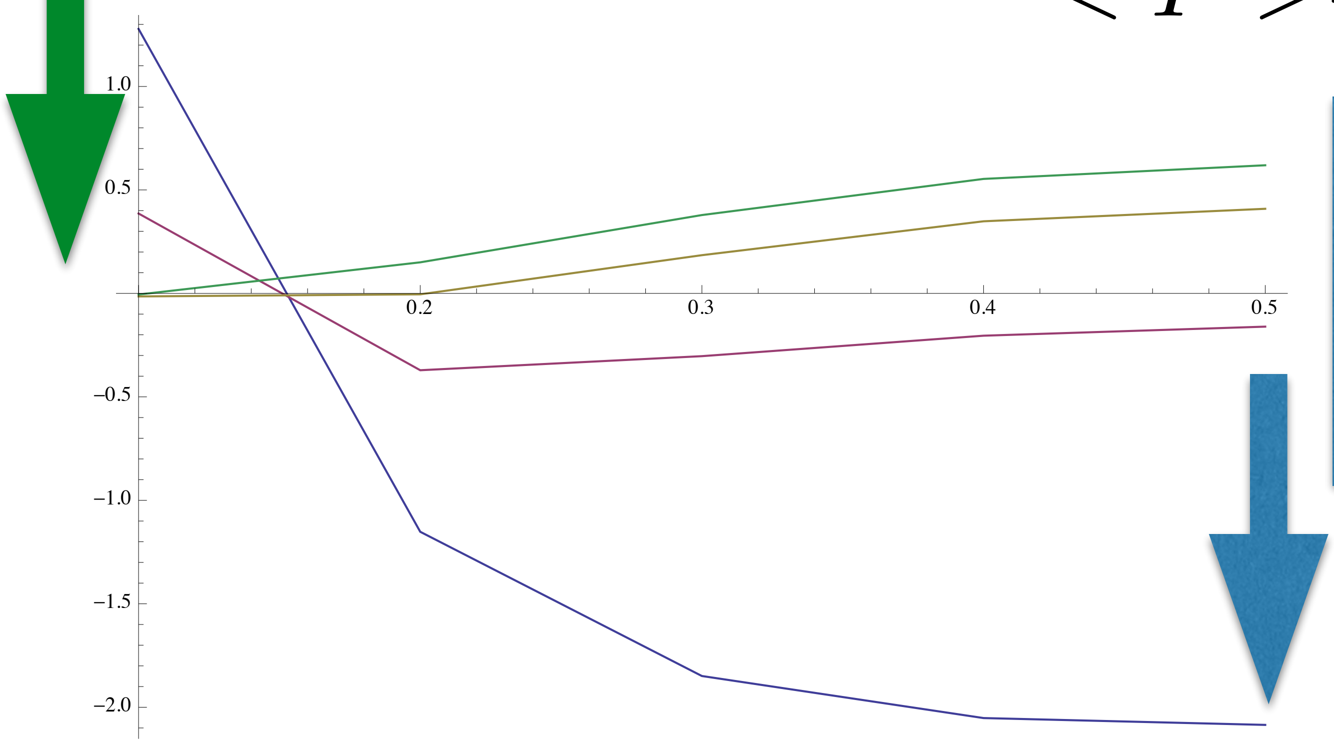
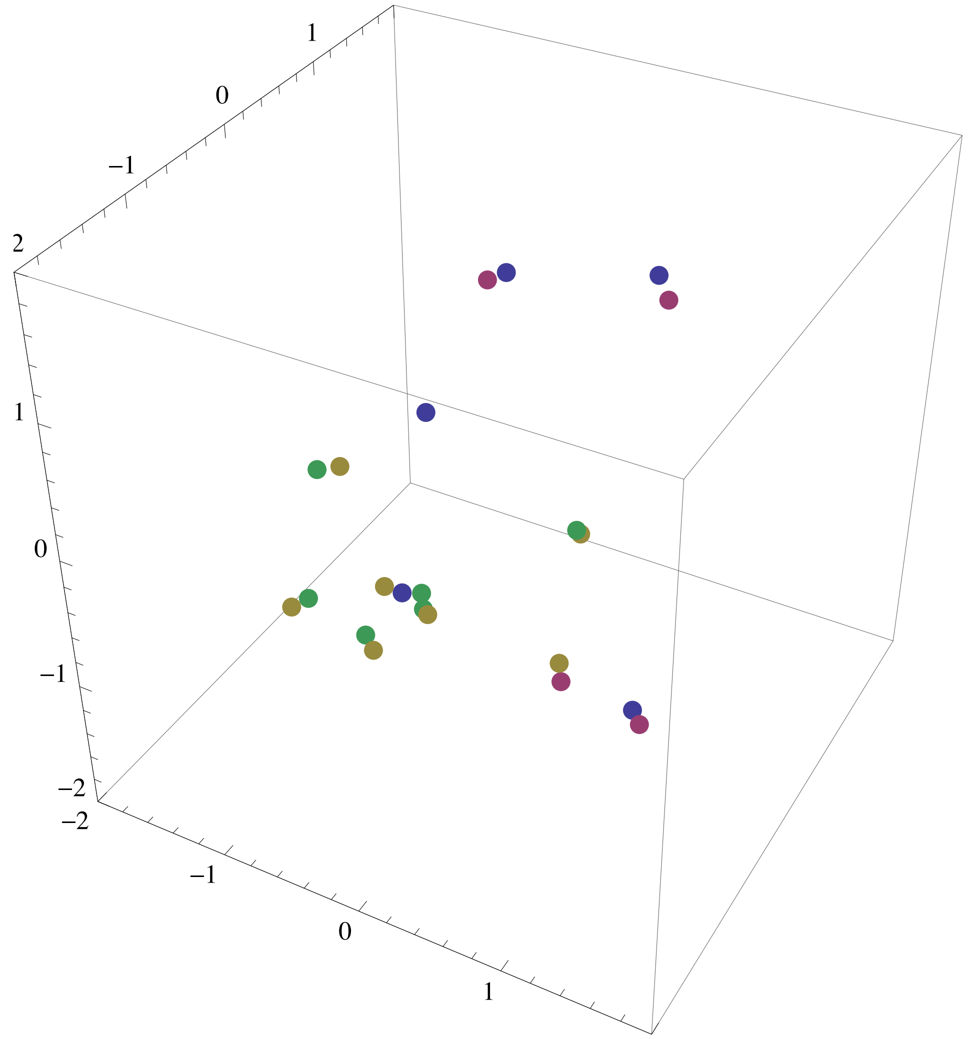
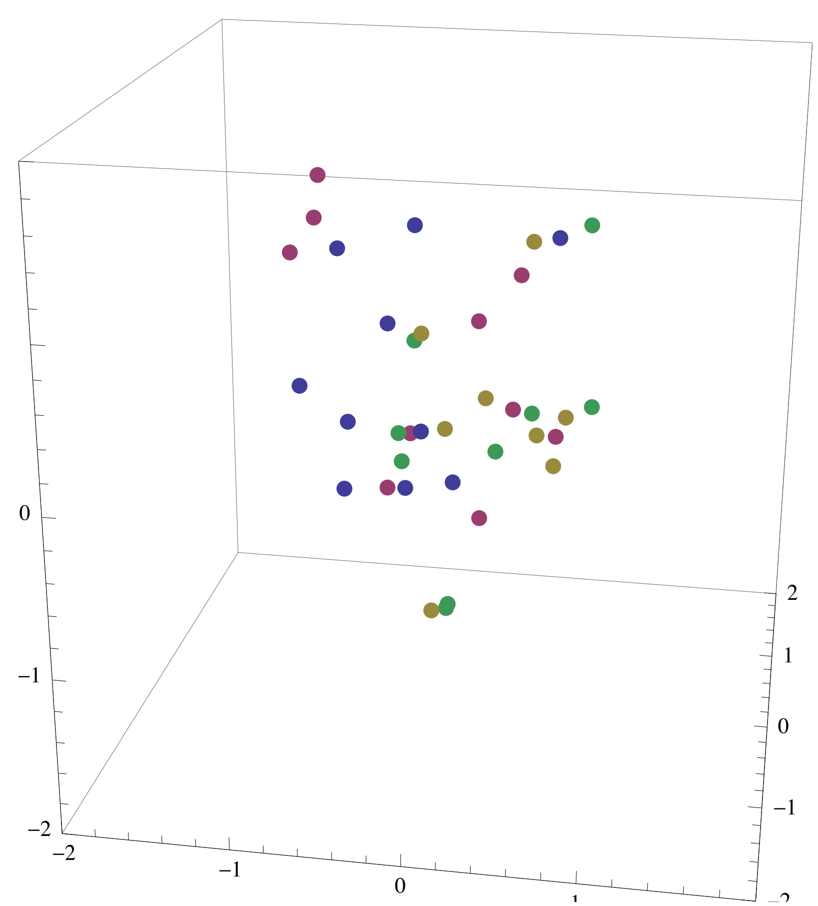


FIG. 1: Total free energy as a function of holonomy ν. Green, brown, red and blue (top to bottom at the r.h.s.) curves are for increasing density of the dyons, $n_d = 0.605, 0.207, 0.0944, 0.0507$ respectively.

projections: 1/2 S_3 sphere



low density pairs (calorons) are visible

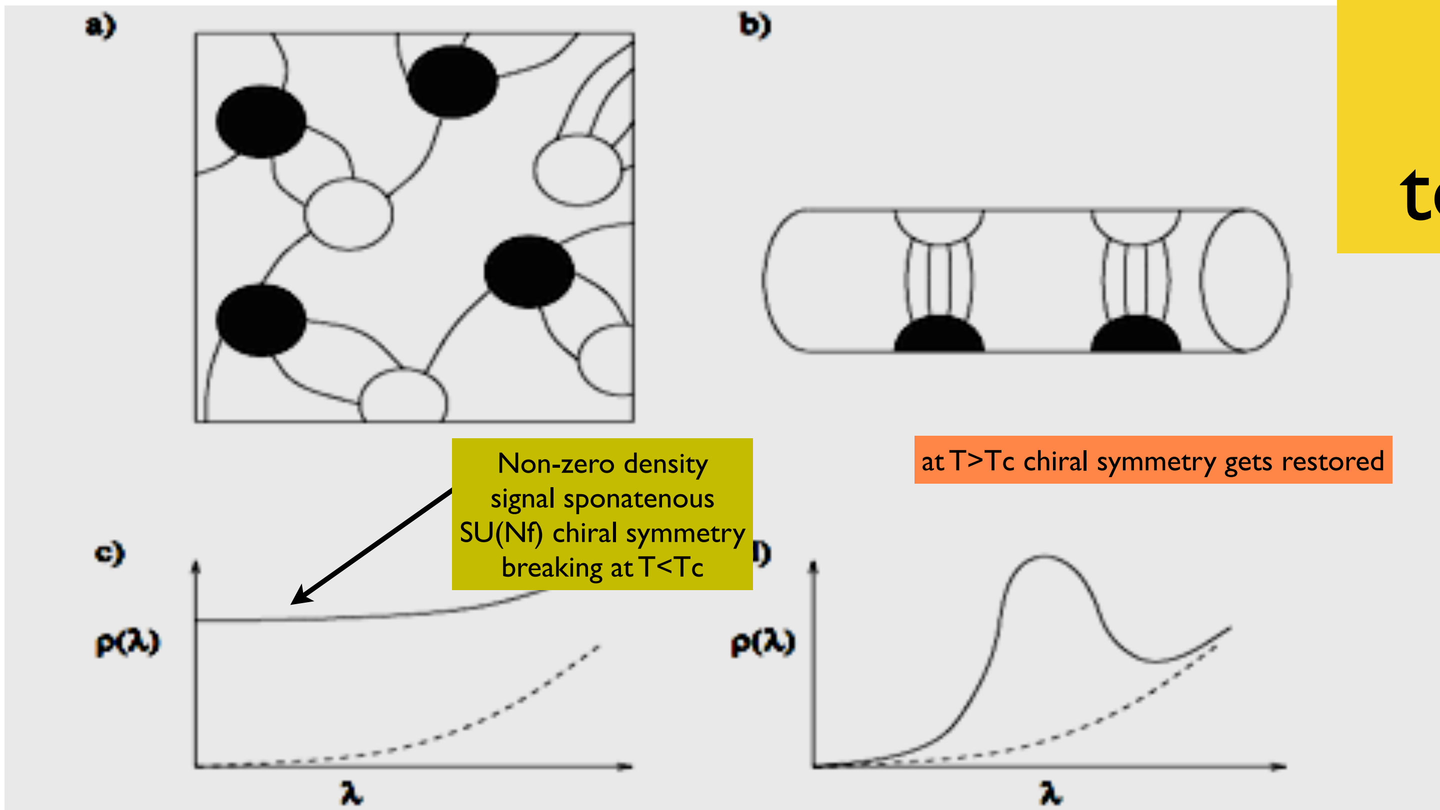


high density

Chiral symmetry breaking and ZMZ

Instanton liquid at T=0 and T>Tc (schematic pictures)

fundamental concept:
ZMZ,
a collectivized set of
topological zero modes



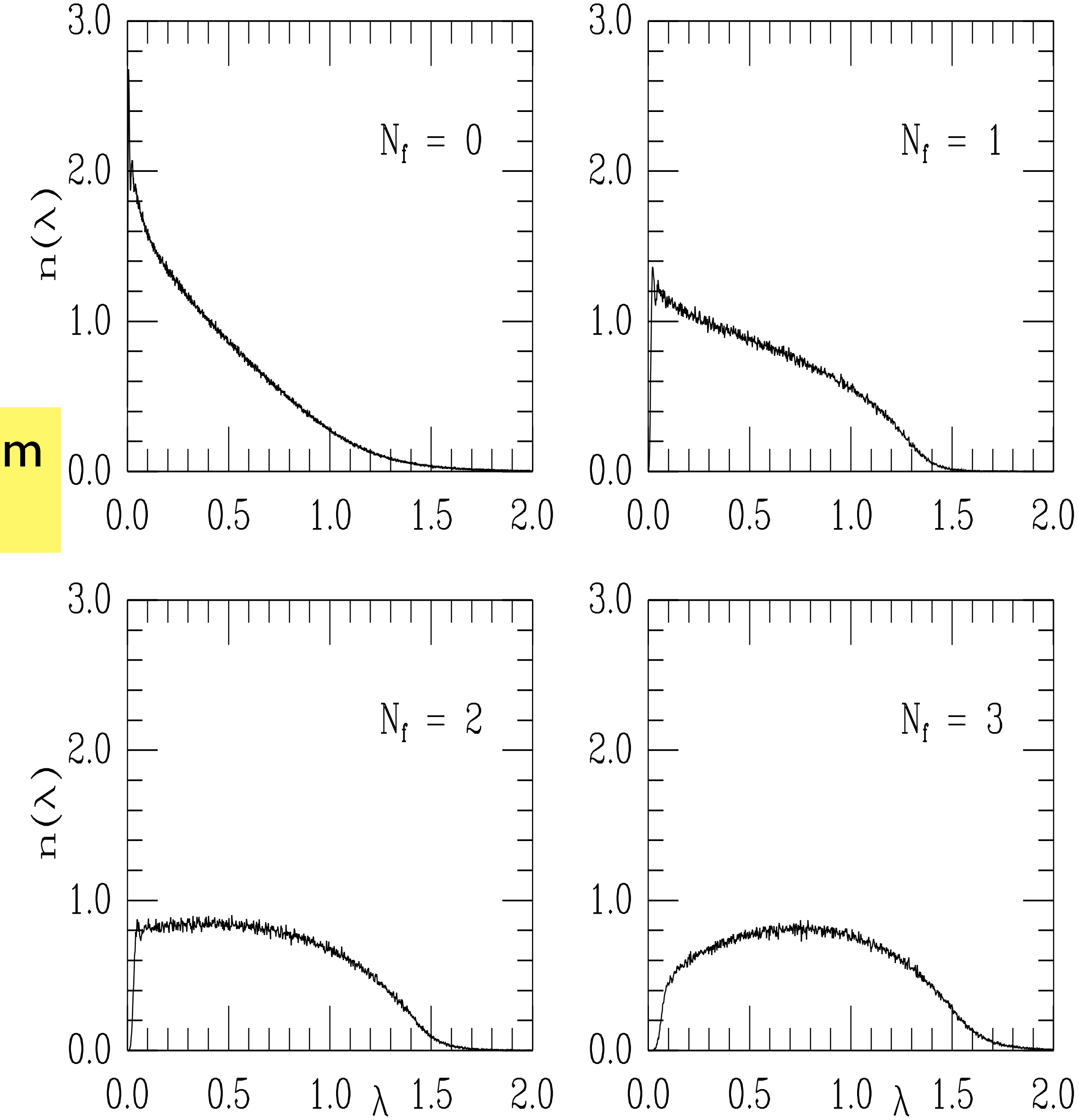
$$D_\mu \gamma_\mu \psi_\lambda = \lambda \psi_\lambda$$

density of states (0) =>
nonzero quark condensate
``conductor'' at low T

zero density of states (0) =>
zero quark condensate
``insulator'' at high T

chiral symmetry transition is thus understood in a ``single-body'' language as conductor-insulator transition in 4d

The spectrum of the Dirac eigenvalues

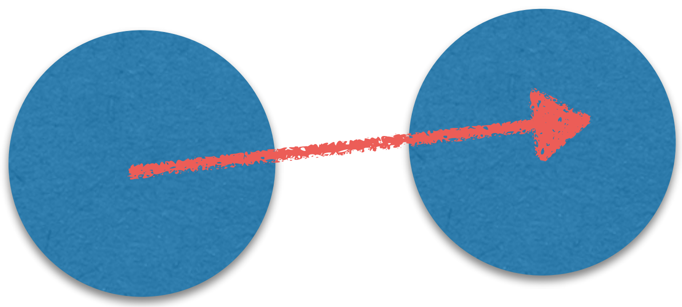


Smilga-Stern theorem
-|lambda|(Nf-2)

the width of the ZMZ is surprisingly small

the magnitude of the hopping from one instanton to the next can be estimated as

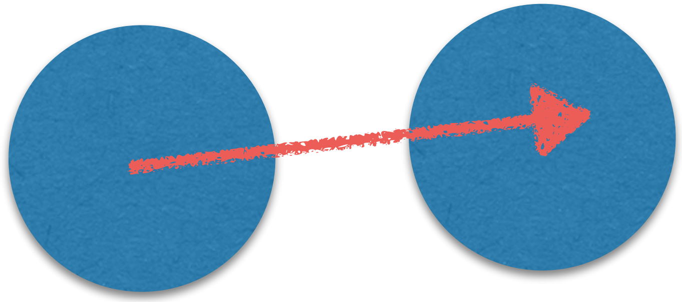
$$T_{I\bar{I}} \sim \frac{\rho^2}{R^3} \sim \frac{(0.3fm)^2}{(1fm)^3} \sim 20MeV$$



the width of the ZMZ is surprisingly small

the magnitude of the hopping from one instanton to the next can be estimated as

$$T_{I\bar{I}} \sim \frac{\rho^2}{R^3} \sim \frac{(0.3fm)^2}{(1fm)^3} \sim 20MeV$$



- that is why quark mass dependence is nontrivial when m is of this order, and chiral perturbation extrapolations are not as good as people hoped!

recently the opposite exercise was done by the Graz group

Symmetries of hadrons after unbreaking the chiral symmetry

L. Ya. Glozman,* C. B. Lang,† and M. Schröck‡
Institut für Physik, FB Theoretische Physik, Universität Graz, A–8010 Graz, Austria

By eliminating ZMZ strip with width sigma (about 50 modes or 10^-4 of all) one changes masses O(1): near-perfect chiral pairs are left

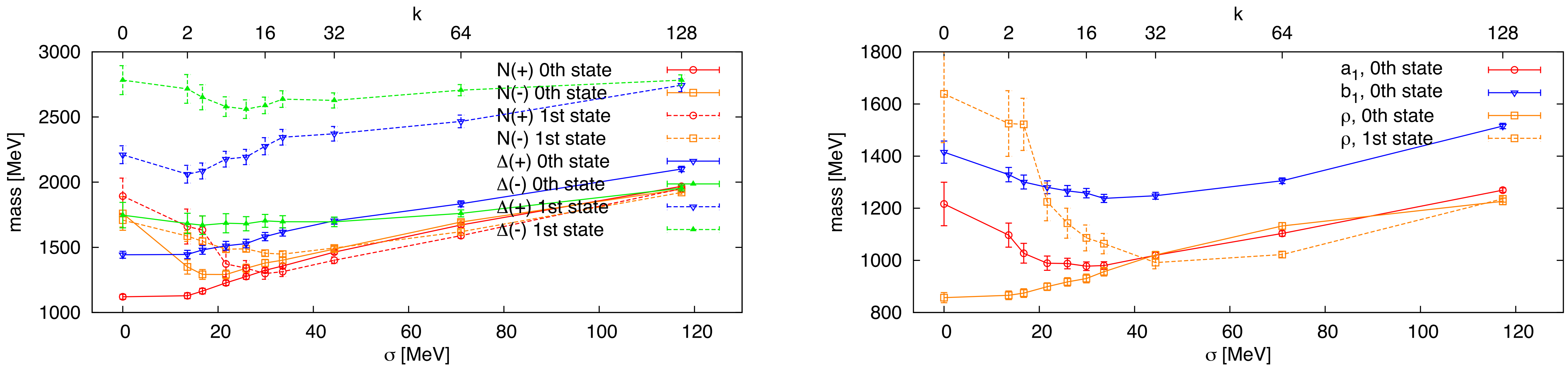


FIG. 13. Summary plots: Baryon (l.h.s.) and meson (r.h.s.) masses as a function of the truncation level.

recently the opposite exercise was done by the Graz group

Symmetries of hadrons after unbreaking the chiral symmetry

L. Ya. Glozman,* C. B. Lang,† and M. Schröck‡
Institut für Physik, FB Theoretische Physik, Universität Graz, A–8010 Graz, Austria

By eliminating ZMZ strip with width sigma
(about 50 modes or 10^-4 of all)
one changes masses O(1): near-perfect chiral pairs are left

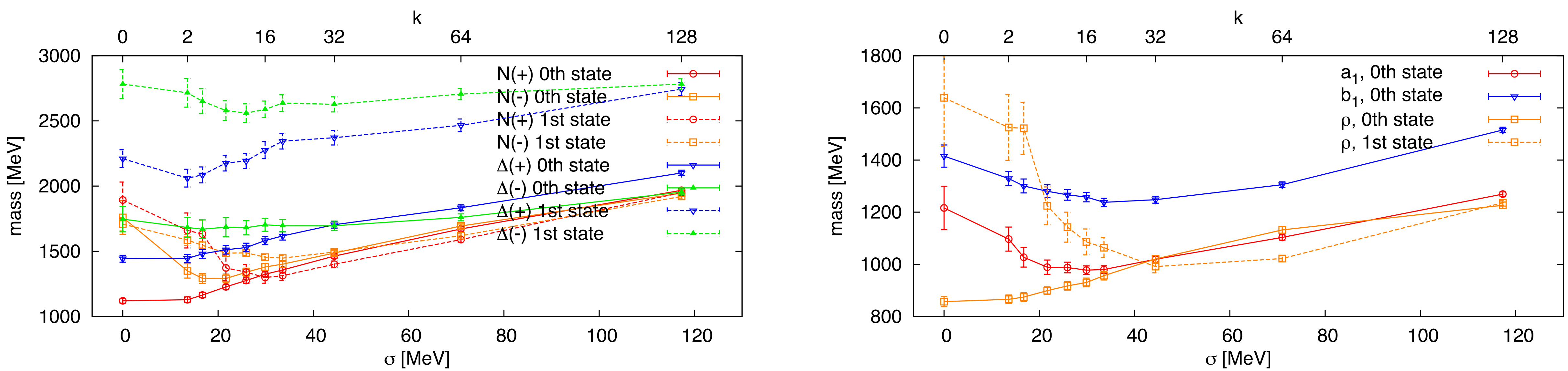


FIG. 13. Summary plots: Baryon (l.h.s.) and meson (r.h.s.) masses as a function of the truncation level.

Comment for lattice practitioners: the ZMZ states are also responsible for most of the statistical noise in simulations with dynamical fermions: ZMZ needs attention!

Isoscalar mesons upon unbreaking of chiral symmetry

M. Denissenya,* L.Ya. Glozman,† and C.B. Lang‡
Institute of Physics, University of Graz, A–8010 Graz, Austria

In a dynamical lattice simulation with the overlap Dirac operator and $N_f = 2$ mass degenerate quarks we study all possible J=0 and J=1 correlators upon exclusion of the low lying "quasi-zero" modes from the valence quark propagators. After subtraction of a small amount of such Dirac eigenmodes all disconnected contributions vanish and all possible point-to-point J=0 correlators with different quantum numbers become identical, signaling a restoration of the $SU(2)_L \times SU(2)_R \times U(1)_A$. The original ground state of the π meson does not survive this truncation, however. In contrast, in the I=0 and I=1 channels for the J=1 correlators the ground states have a very clean exponential decay. All possible chiral multiplets for the J=1 mesons become degenerate, indicating a restoration of an $SU(4)$ symmetry of the dynamical QCD-like string.

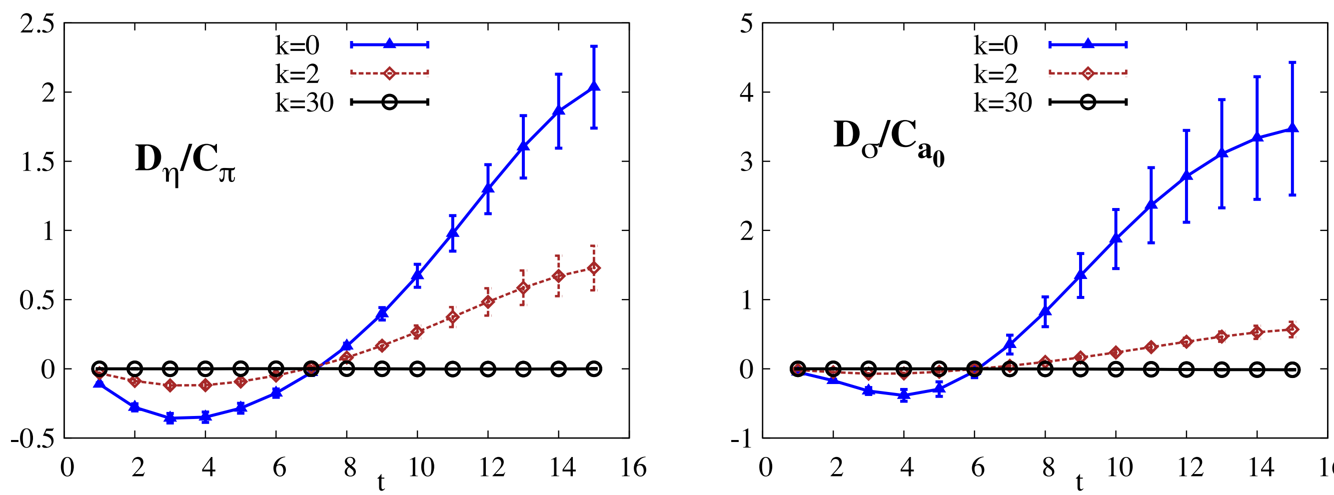


FIG. 4: Ratios of disconnected and connected $J = 0$ correlators, $k = 0, 2, 30$.

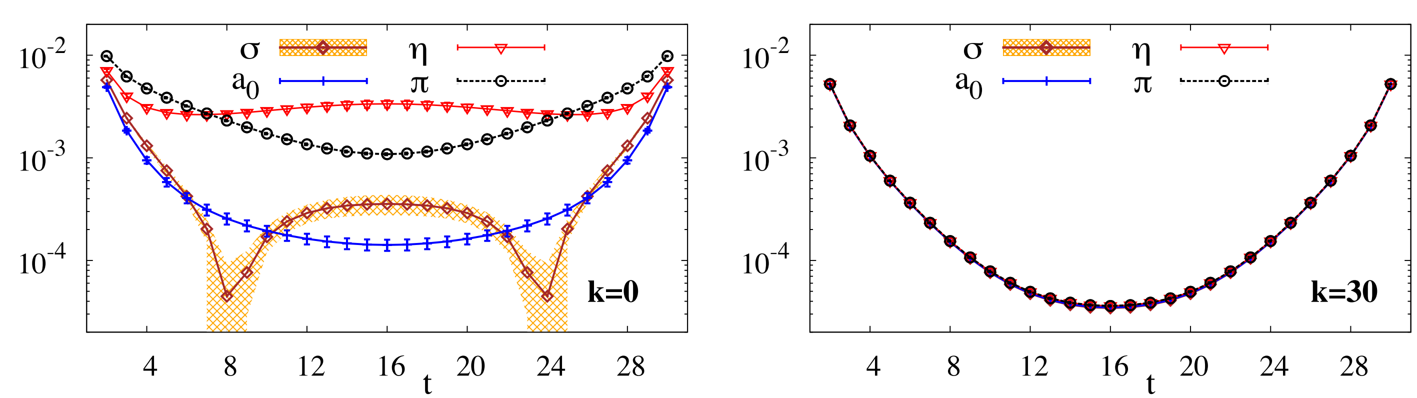


FIG. 5: π, σ, a_0, η correlators upon exclusion of the near-zero modes, $k = 0, 30$.

all Ua(1) breaking disappear after ZMZ is eliminated

density

The first statistical simulations
64 dyons on S^3,
Faccioli+ES

PHYSICAL REVIEW D **87,** 074009 (2013)

QCD topology at finite temperature: Statistical mechanics of self-dual dyons

Pietro Faccioli[1,2] and Edward Shuryak[3]
[1]*Physics Department, Trento University, Via Sommarive 14, Povo, Trento I-38100, Italy*
[2]*Gruppo Collegato di Trento, Istituto Nazionale di Fisica Nucleare, Via Sommarive 14, Povo, Trento I-38100, Italy*
[3]*Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York 11794, USA*
(Received 27 January 2013; published 9 April 2013)

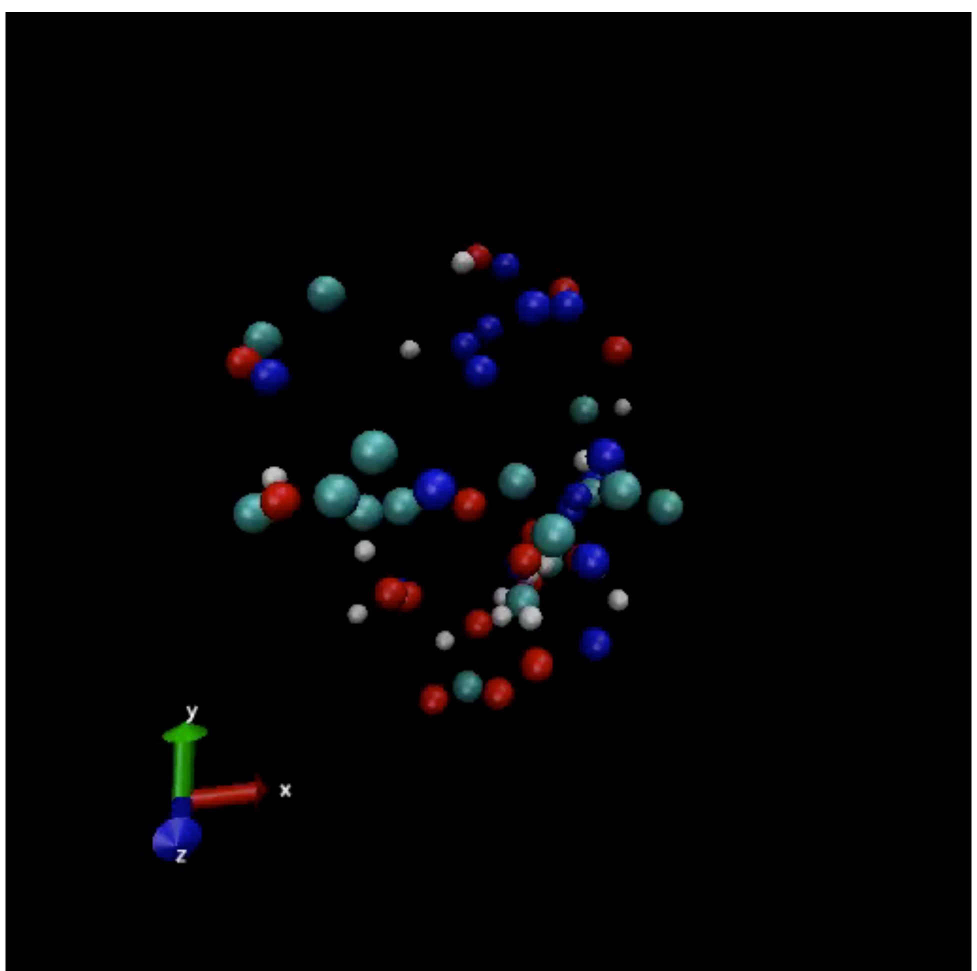
Topological phenomena in gauge theories have long been recognized as the driving force for chiral symmetry breaking and confinement. These phenomena can be conveniently investigated in the semiclassical picture, in which the topological charge is entirely carried by (anti-)self-dual gauge configurations. In such an approach, it has been shown that near the critical temperature, the nonzero expectation value of the Polyakov loop (holonomy) triggers the "Higgsing" of the color group, generating the splitting of instantons into N_c self-dual dyons. A number of lattice simulations have provided some evidence for such dyons, and traced their relation with specific observables, such as the Dirac eigenvalue spectrum. In this work, we formulate a model, based on one-loop partition function and including Coulomb interaction, screening and fermion zero modes. We then perform the first numerical Monte Carlo simulations of a statistical ensemble of self-dual dyons, as a function of their density, quark mass and the number of flavors. We study different dyonic two-point correlation functions and we compute the Dirac spectrum, as a function of the ensemble diluteness and the number of quark flavors.

Coulomb ++

Coulomb +-

Nf

fermions



density

The first statistical simulations
64 dyons on S^3,
Faccioli+ES

PHYSICAL REVIEW D **87,** 074009 (2013)

QCD topology at finite temperature: Statistical mechanics of self-dual dyons

Pietro Faccioli[1,2] and Edward Shuryak[3]

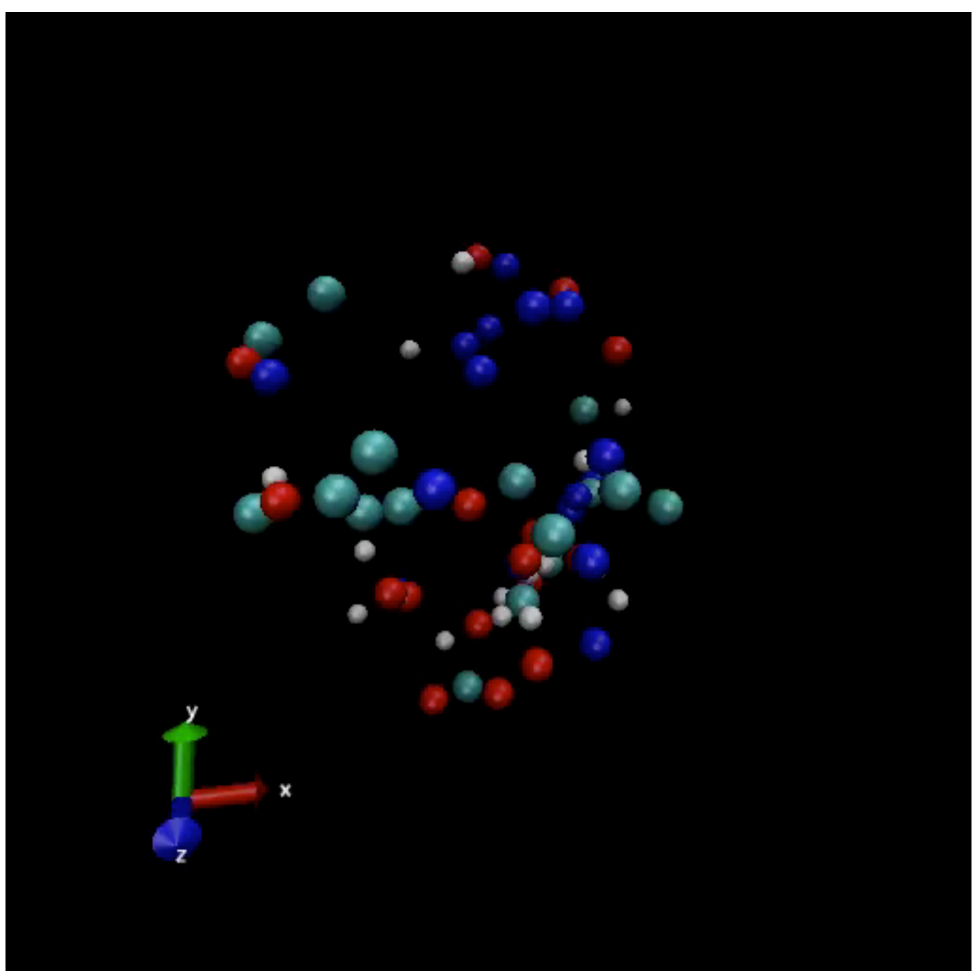
[1]*Physics Department, Trento University, Via Sommarive 14, Povo, Trento I-38100, Italy*
[2]*Gruppo Collegato di Trento, Istituto Nazionale di Fisica Nucleare, Via Sommarive 14, Povo, Trento I-38100, Italy*
[3]*Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York 11794, USA*
(Received 27 January 2013; published 9 April 2013)

Topological phenomena in gauge theories have long been recognized as the driving force for chiral symmetry breaking and confinement. These phenomena can be conveniently investigated in the semiclassical picture, in which the topological charge is entirely carried by (anti-)self-dual gauge configurations. In such an approach, it has been shown that near the critical temperature, the nonzero expectation value of the Polyakov loop (holonomy) triggers the "Higgsing" of the color group, generating the splitting of instantons into N_c self-dual dyons. A number of lattice simulations have provided some evidence for such dyons, and traced their relation with specific observables, such as the Dirac eigenvalue spectrum. In this work, we formulate a model, based on one-loop partition function and including Coulomb interaction, screening and fermion zero modes. We then perform the first numerical Monte Carlo simulations of a statistical ensemble of self-dual dyons, as a function of their density, quark mass and the number of flavors. We study different dyonic two-point correlation functions and we compute the Dirac spectrum, as a function of the ensemble diluteness and the number of quark flavors.

Coulomb ++

Coulomb +-

Nf



fermions

density

Nf

PHYSICAL REVIEW D **87,** 074009 (2013)

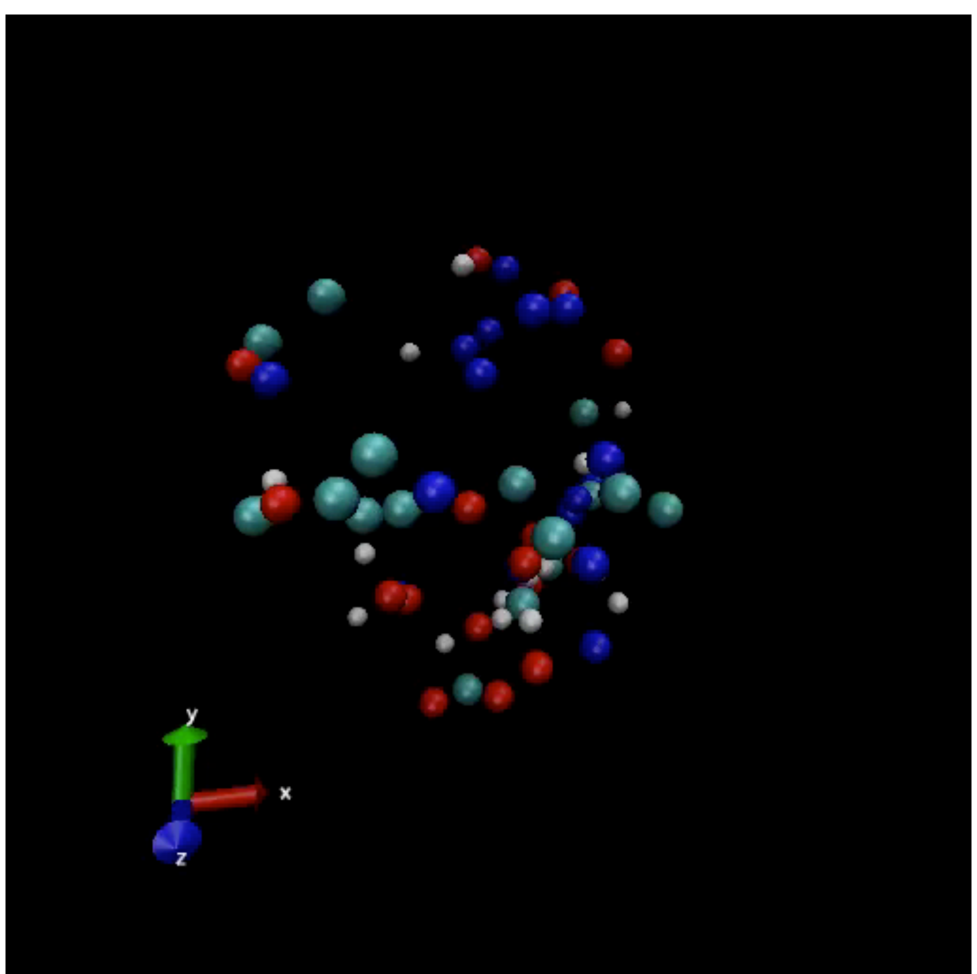
QCD topology at finite temperature: Statistical mechanics of self-dual dyons

Pietro Faccioli[1,2] and Edward Shuryak[3]

[1]*Physics Department, Trento University, Via Sommarive 14, Povo, Trento I-38100, Italy*
[2]*Gruppo Collegato di Trento, Istituto Nazionale di Fisica Nucleare, Via Sommarive 14, Povo, Trento I-38100, Italy*
[3]*Department of Physics and Astronomy, Stony Brook University, Stony Brook, New York 11794, USA*

(Received 27 January 2013; published 9 April 2013)

Topological phenomena in gauge theories have long been recognized as the driving force for chiral symmetry breaking and confinement. These phenomena can be conveniently investigated in the semiclassical picture, in which the topological charge is entirely carried by (anti-)self-dual gauge configurations. In such an approach, it has been shown that near the critical temperature, the nonzero expectation value of the Polyakov loop (holonomy) triggers the "Higgsing" of the color group, generating the splitting of instantons into N_c self-dual dyons. A number of lattice simulations have provided some evidence for such dyons, and traced their relation with specific observables, such as the Dirac eigenvalue spectrum. In this work, we formulate a model, based on one-loop partition function and including Coulomb interaction, screening and fermion zero modes. We then perform the first numerical Monte Carlo simulations of a statistical ensemble of self-dual dyons, as a function of their density, quark mass and the number of flavors. We study different dyonic two-point correlation functions and we compute the Dirac spectrum, as a function of the ensemble diluteness and the number of quark flavors.



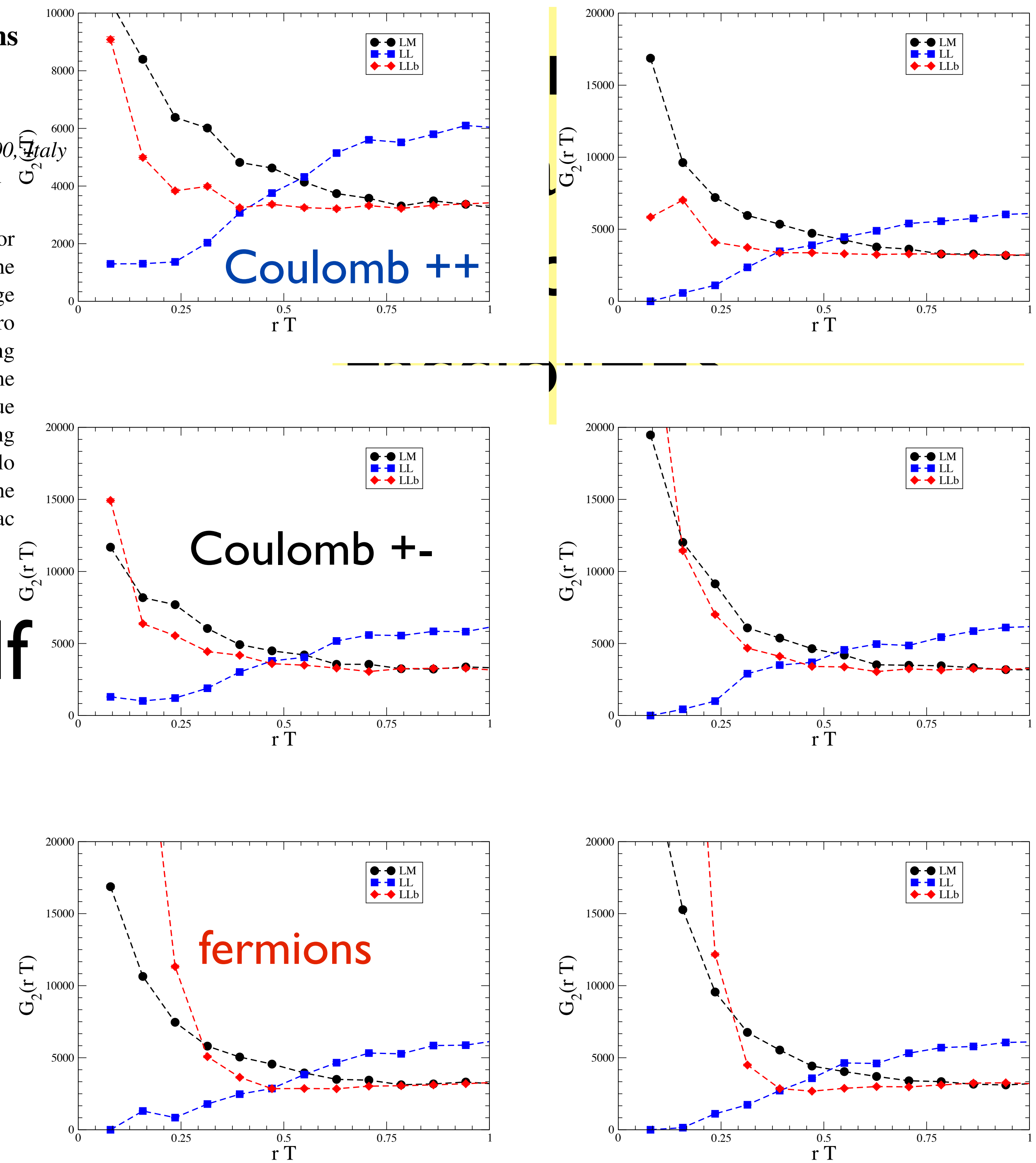
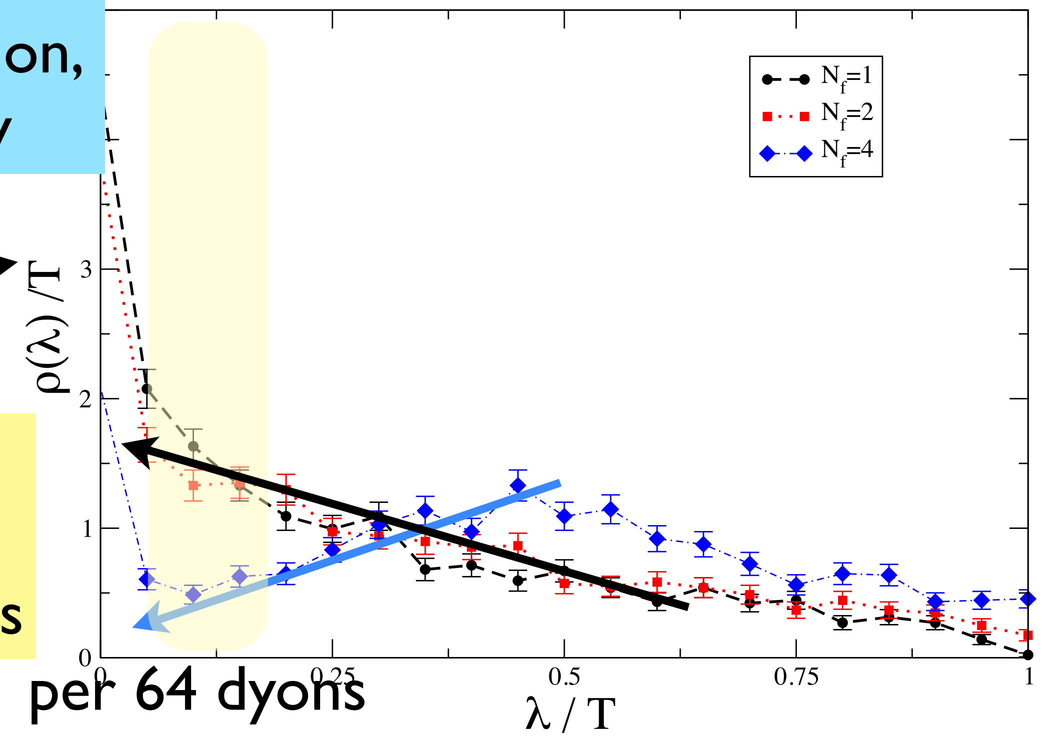


FIG. 2: The correlation function for LM, LL and $L\bar{L}$ dyons versus distance, normalized to the volume available. From top to bottom we show $N_f = 1, 2, 4$, respectively. Left/right columns are for the volumes per dyon $VT^3 = 0.31, 1.04$.

dyons	$R(S^3)T$	$VT^3/dyon$
64	4.5	28.
64	3.0	8.3
64	2.5	4.8
64	2.2	3.28
64	1.5	1.04
64	1.2	0.53
64	1.	0.31

Example of the Dirac eigenvalue distribution, high dyon density



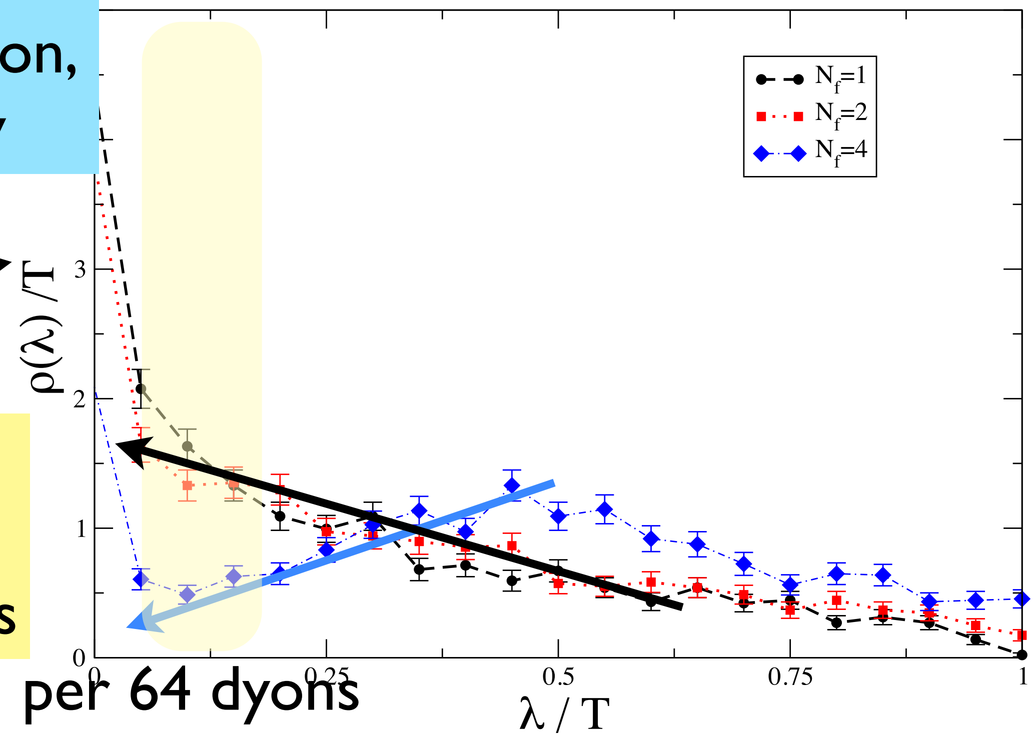
at small Dirac eigenvalues one finds known finite volume effects

per 64 dyons

per 64 dyons

dyons	$R(S^3)T$	$VT^3/dyon$
64	4.5	28.
64	3.0	8.3
64	2.5	4.8
64	2.2	3.28
64	1.5	1.04
64	1.2	0.53
64	1.	0.31

Example of the Dirac eigenvalue distribution, high dyon density



at small Dirac eigenvalues one finds known finite volume effects

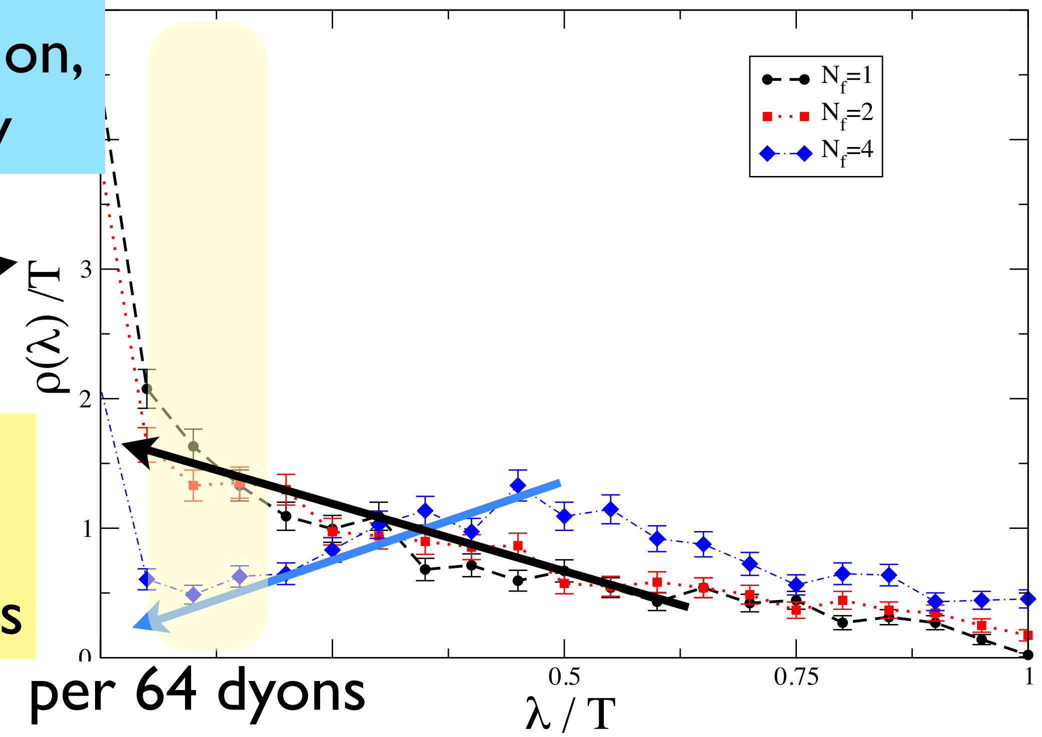
per 64 dyons

for Nf=4 the chiral symmetry is broken only at very high density => low T as is indeed the case on the lattice

per 64 dyons

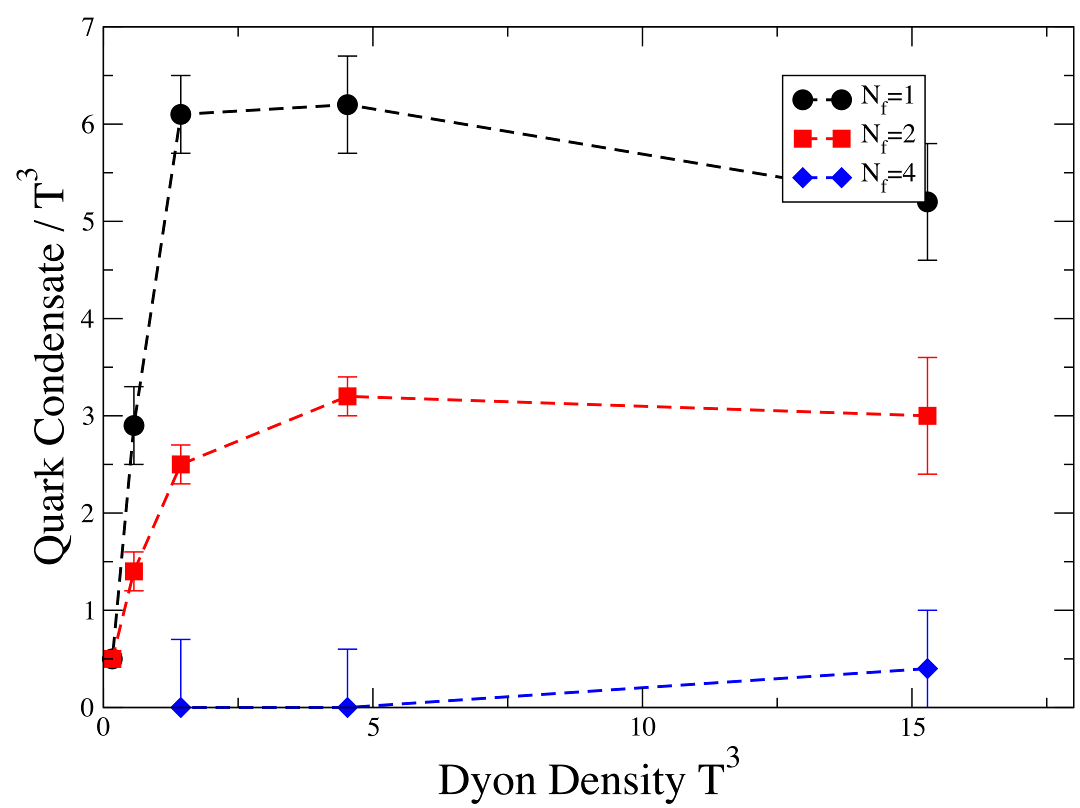
dyons	$R(S^3)T$	$VT^3/dyon$
64	4.5	28.
64	3.0	8.3
64	2.5	4.8
64	2.2	3.28
64	1.5	1.04
64	1.2	0.53
64	1.	0.31

Example of the Dirac eigenvalue distribution, high dyon density



at small Dirac eigenvalues one finds known finite volume effects

per 64 dyons



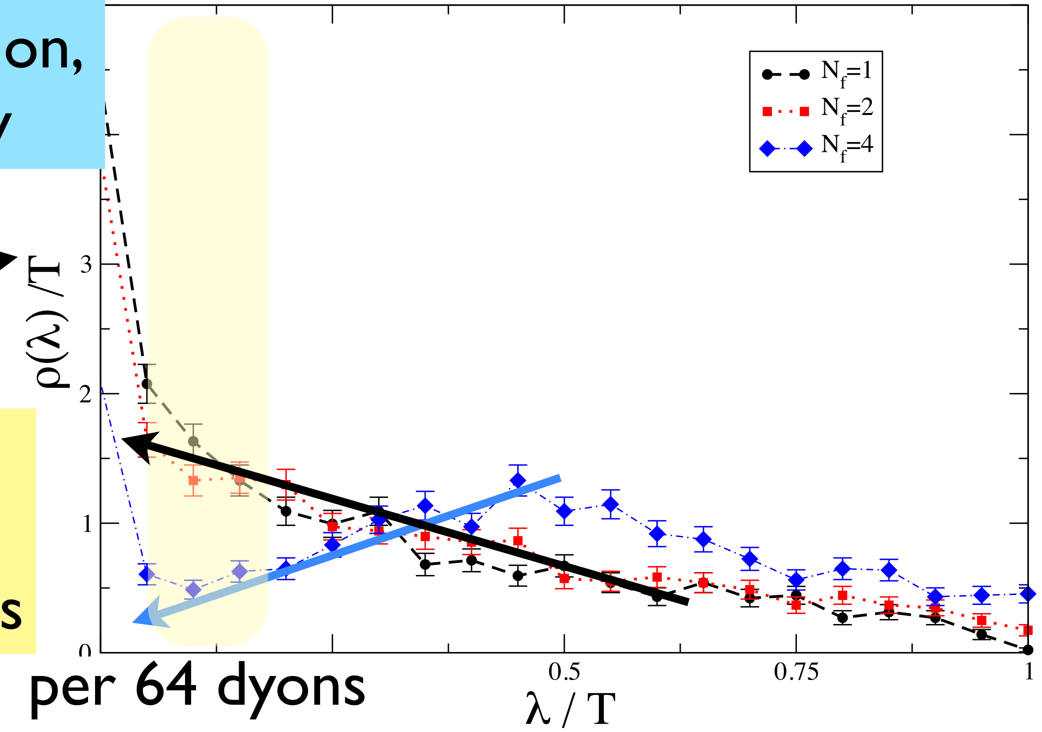
for Nf=4 the chiral symmetry is broken only at very high density => low T as is indeed the case on the lattice

per 64 dyons

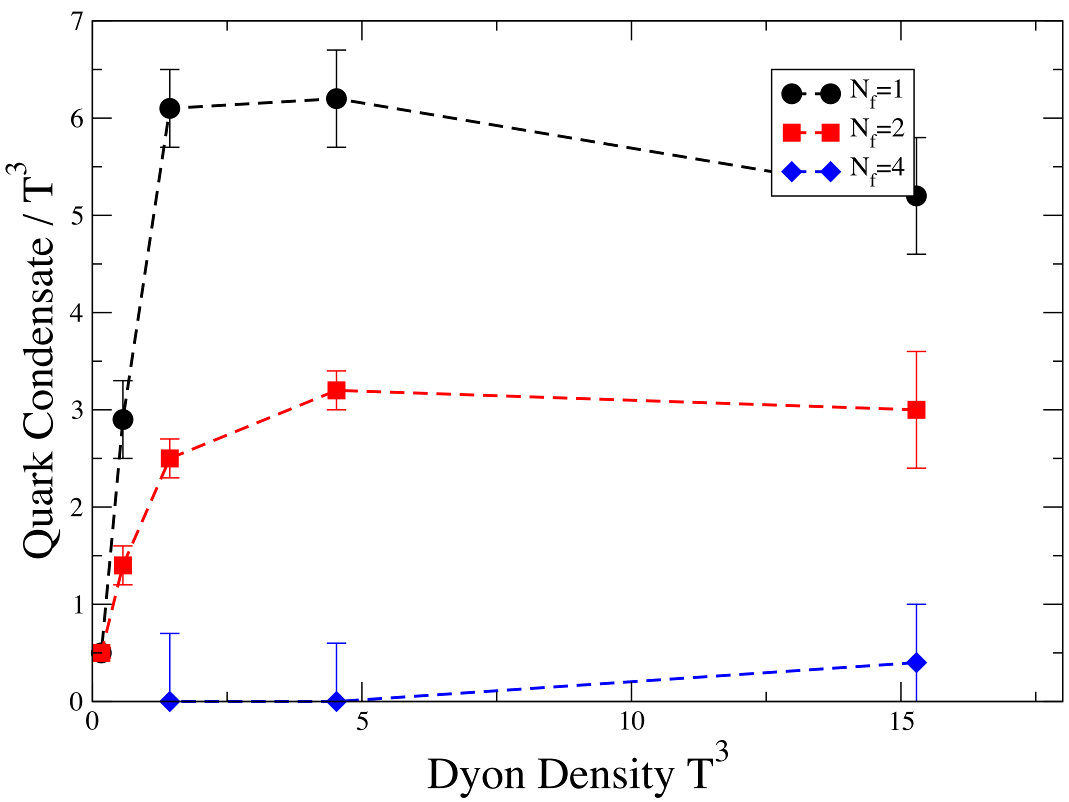
dyons	$R(S^3)T$	$VT^3/dyon$
64	4.5	28.
64	3.0	8.3
64	2.5	4.8
64	2.2	3.28
64	1.5	1.04
64	1.2	0.53
64	1.	0.31

Example of the Dirac eigenvalue distribution, high dyon density

at small Dirac eigenvalues one finds known finite volume effects



per 64 dyons



per 64 dyons

for Nf=4 the chiral symmetry is broken only at very high density => low T as is indeed the case on the lattice

at small density => high T, chiral symmetry gets restored

Summary

Summary

- Nonzero <Polyakov loop> => instantons split into Nc instanton-dyons (van Baal) which have electric and magnetic charges=> Coulomb plasma

Summary

- Nonzero <Polyakov loop> => instantons split into Nc instanton-dyons (van Baal) which have electric and magnetic charges=> Coulomb plasma
- **dion-antidyon repulsion => back reaction on holonomy =>L-M symmetry=> confinement**

Summary

- Nonzero <Polyakov loop> => instantons split into Nc instanton-dyons (van Baal) which have electric and magnetic charges=> Coulomb plasma
- **dion-antidyon repulsion => back reaction on holonomy =>L-M symmetry=> confinement**
- first simulation done, total free energy dyons+P_{GPY}

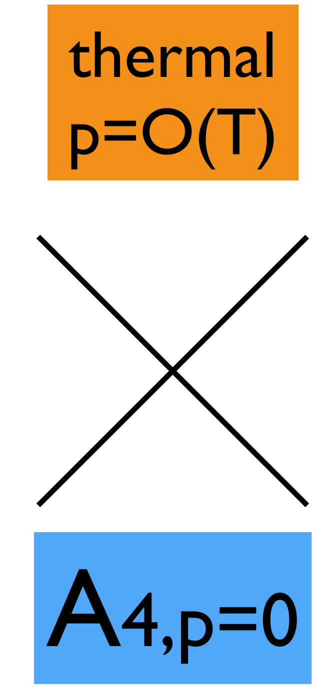
Summary

- Nonzero <Polyakov loop> => instantons split into N_c instanton-dyons (van Baal) which have electric and magnetic charges=> Coulomb plasma
- **dion-antidyon repulsion =>** **back reaction on holonomy =>L-M symmetry=>** **confinement**
- first simulation done, total free energy dyons+P_{GPY}
- dyonic ZMZ and **chiral restoration also calculated for N_f=0..4**

Summary

- Nonzero <Polyakov loop> => instantons split into Nc instanton-dyons (van Baal) which have electric and magnetic charges=> Coulomb plasma
- **dion-antidyon repulsion => back reaction on holonomy =>L-M symmetry=> confinement**
- first simulation done, total free energy dyons+P_{GPY}
- dyonic ZMZ and **chiral restoration also calculated for N_f=0..4**
- **so we understand now why both needs large density of instanton-dyons, and why it grows with Nf (so confinement shifts to stronger coupling, lower T etc)**

“near-confinement” of the instanton-quarks (Diakonov et al)



In SU(2): thermal quanta in QGP scatter on the instanton and generate linear potential

$$V_{12} \sim \langle (A_4)^2 \rangle = \int d^3x \left| \frac{1}{r_L} - \frac{1}{r_M} \right|^2 = 4\pi \; r_{LM}$$

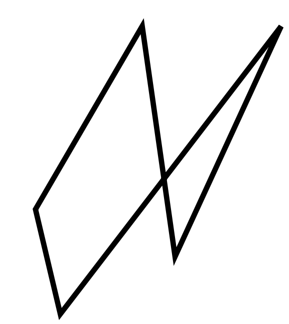
We further note that the form (14) can be obtained directly by the instanton screening term calculated by Pisarski and Yaffe [30] by recalling that the instanton size ρ and the $L - M$ separation are related by the expression

$$\pi \rho^2 T = r_{ML}. \qquad (17)$$

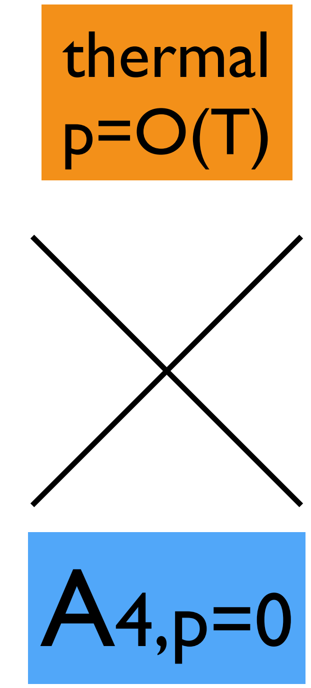
which relates the “4-d dipole” of the instanton field to the “3-d dipole” of the dyon LM pair made of opposite charges.

In SU(Nc): instanton=baryon linear potential => perimeter of a polygon

$$V \sim M_D^2 \sum_{i=1,N_c} |r_{i,i+1}|$$



“near-confinement” of the instanton-quarks (Diakonov et al)



In SU(2): thermal quanta in QGP scatter on the instanton and generate linear potential

$$V_{12} \sim \langle (A_4)^2 \rangle = \int d^3x \left| \frac{1}{r_L} - \frac{1}{r_M} \right|^2 = 4\pi\ r_{LM}$$

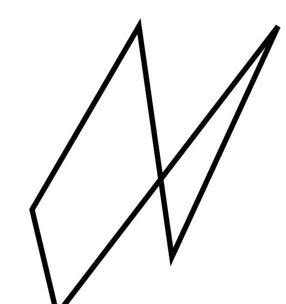
We further note that the form (14) can be obtained directly by the instanton screening term calculated by Pisarski and Yaffe [30] by recalling that the instanton size ρ and the $L - M$ separation are related by the expression

$$\pi\rho^2 T = r_{ML}. \qquad (17)$$

which relates the “4-d dipole” of the instanton field to the “3-d dipole” of the dyon LM pair made of opposite charges.

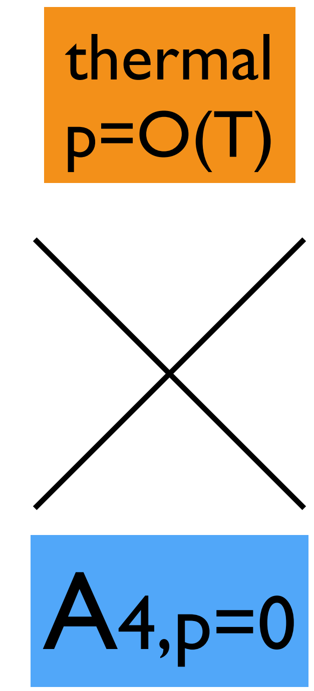
In SU(Nc): instanton=baryon
linear potential => perimeter of a polygon

$$V \sim M_D^2 \sum_{i=1,N_c} |r_{i,i+1}|$$



not really a confinement as A0 is massive,

“near-confinement” of the instanton-quarks (Diakonov et al)



In SU(2): thermal quanta in QGP scatter on the instanton and generate linear potential

$$V_{12} \sim \langle (A_4)^2 \rangle = \int d^3x \left| \frac{1}{r_L} - \frac{1}{r_M} \right|^2 = 4\pi \ r_{LM}$$

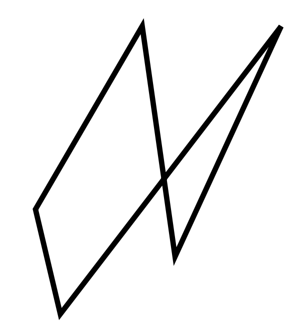
We further note that the form (14) can be obtained directly by the instanton screening term calculated by Pisarski and Yaffe [30] by recalling that the instanton size ρ and the $L - M$ separation are related by the expression

$$\pi \rho^2 T = r_{ML}. \tag{17}$$

which relates the “4-d dipole” of the instanton field to the “3-d dipole” of the dyon LM pair made of opposite charges.

In SU(Nc): instanton=baryon linear potential => perimeter of a polygon

$$V \sim M_D^2 \sum_{i=1,N_c} |r_{i,i+1}| \ \ e^{-M_D r}$$



not really a confinement as A0 is massive,

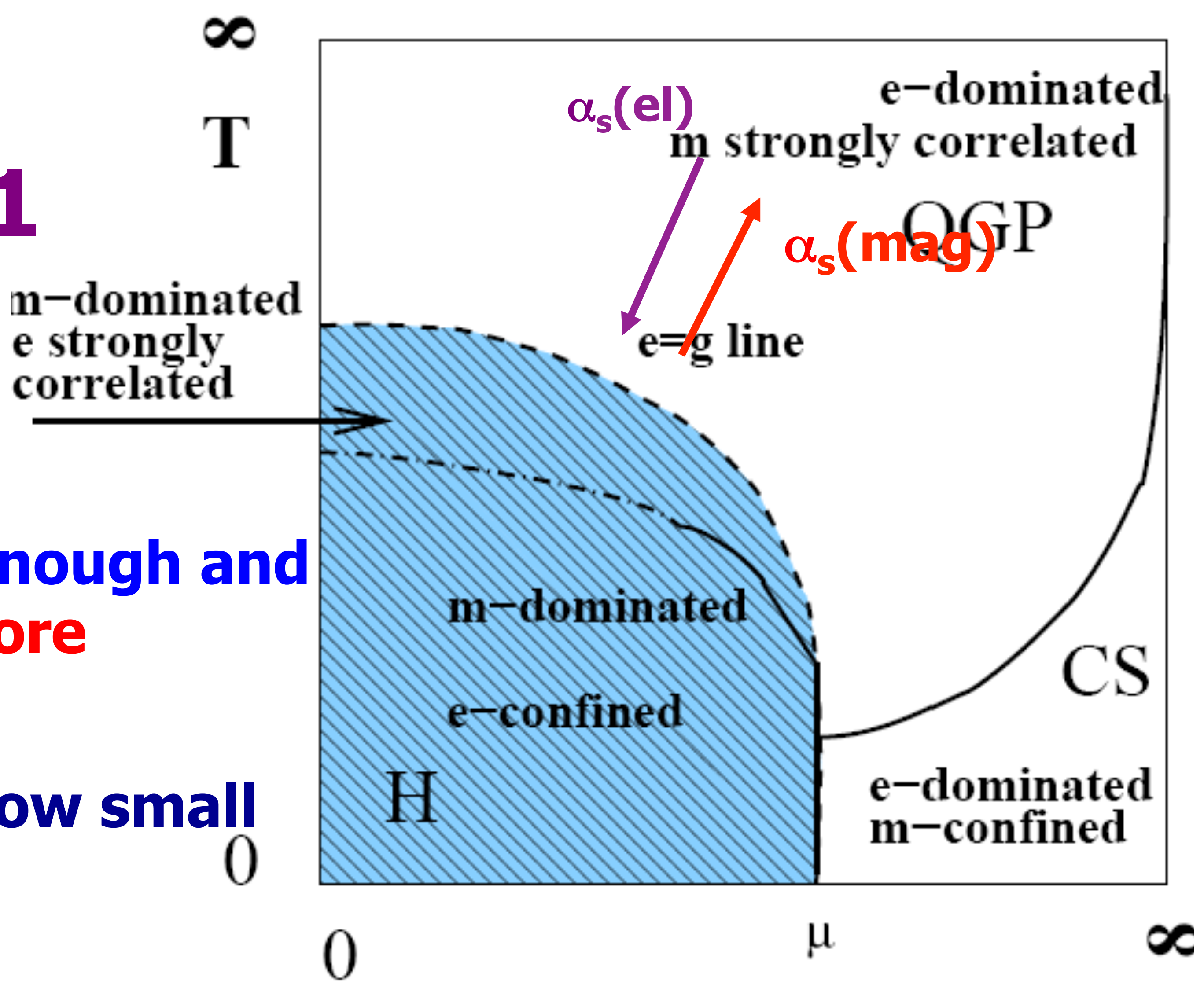
``magnetic scenario”: Liao,ES hep-ph/0611131,Chernodub+Zakharov

Old good Dirac condition

α_s(electric) α_s(magnetic)=1

=>electric/magnetic couplings (e/g) must run in the opposite directions!

the “equilibrium line”
α_s(el)= α_s(mag) =1
needs to be in the plasma phase



monopoles should be dense enough and sufficiently weakly coupled before deconfinement to get BEC

=>α_s(mag) < α_s(el): how small can α_s(mag) be?

lattice puzzle

(which worried me from around 2000)

lattice puzzle

(which worried me from around 2000)

- (Gattringer et al): while quenched (pure YM) gauge ensembles show chiral restoration at $T>T_c$ for **antiperiodic** quarks,
- and yet, it is **not so for periodic quarks!**
(not physical but need to be understood anyway. One can do arbitrary periodicity angle as well, and see a gradual transition as well)
- an instanton has **one zero mode, whatever fermions one uses!**
- let me repeat, the ensemble is quenched, so no back reaction. It is the same gauge fields, and this makes the puzzle harder to solve