The Quark-Gluon Plasma
at critical and supercritical couplings

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• Rethinking the QGP at $T = (1-3)T_c$: the near-critical coupling $\alpha_s \sim 1$ leads to loosely bound states and large cross sections E.Shuryak and I.Zahed,hep-ph/0307267

• A gift from the string world: AdS/CFT and a strongly coupled plasma in ($\mathcal{N}$=4 SUSY YM)

• Explaining the puzzles of the ($\mathcal{N}$=4 CFT) plasma at the supercritical coupling $g^2 N_c >> 1$: there are plenty of light bound states with a large orbital momentum (ES+IZ-03b)

• Why hydrodynamics works so well at RHIC, with about expected EoS, but surprisingly small viscous corrections?

• A gift from atomic physics: getting elliptic hydro flow from an extremely dilute cloud by shifting the resonance

Are there hadrons above T_c, in Quark-Gluon Plasma?

• **Old point of view: most hadrons (including $J/\psi = \bar{c}c$ but not $\Upsilon = \bar{b}b$) melt there.**

• **Exceptions: the pion (+sigma etc chiral multiplet) was believed to survive as resonance e.g.NJL-based papers or T.Schafer+ES, PLB 356:147,1995**

• **The issue was reconsidered recently: ES and I.Zahed,hep-ph/0307267**

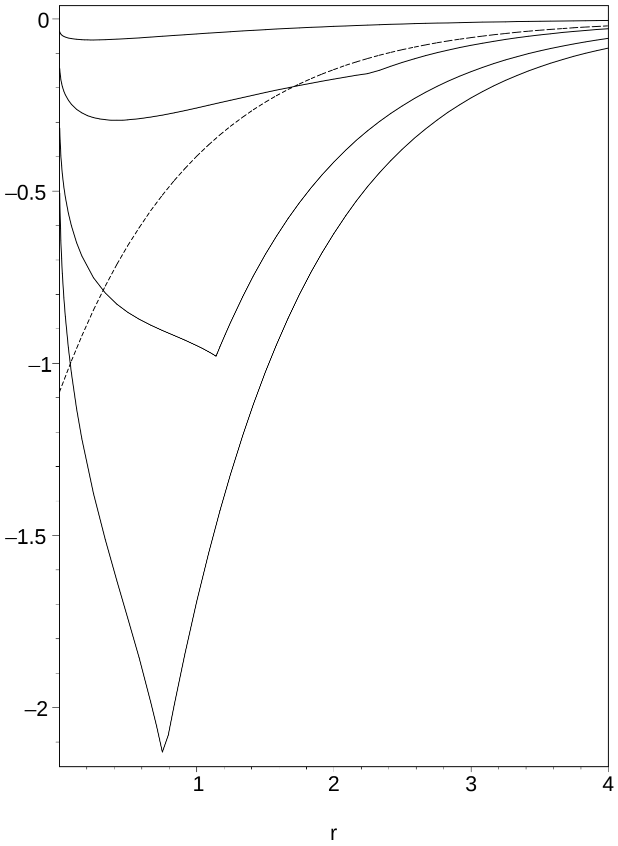
• **The loophole in the old argument: the gauge coupling constant remains frozen in the QGP (as in-vacuum charmonium potential, $\alpha_s \sim 1/3$.)**

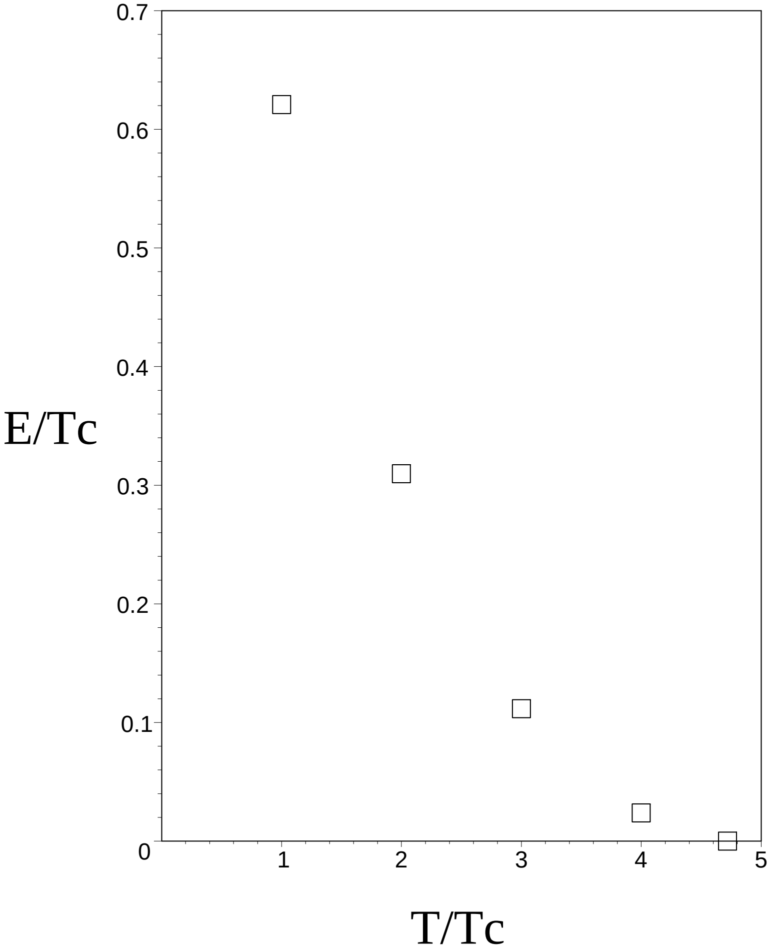
• **New idea: at $T > T_c$ the charge continues to run to larger values, stopped by the Debye screening only**

$$\frac{d^2\chi}{dx^2} + \left(\kappa^2 + \frac{4m_c}{3M_D}\frac{\alpha_s(x)}{x}e^{-x}\right)\chi = 0$$

The appearance of a level, at zero binding $E = 0, \kappa = 0$, is at $\frac{4m_c}{3M_D}\alpha_s > 1.68$

For example, using $4/3\alpha_s = 0.471$ from the Cornell potential and $m_c = 1.32\,\mathrm{GeV}$, as Satz et al did long ago, plus M_D from lattice one finds that J/ψ melts already at T_c.

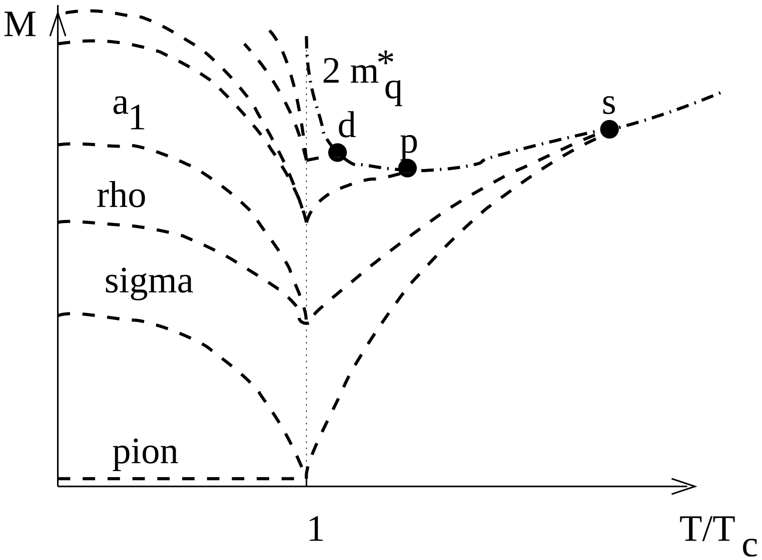


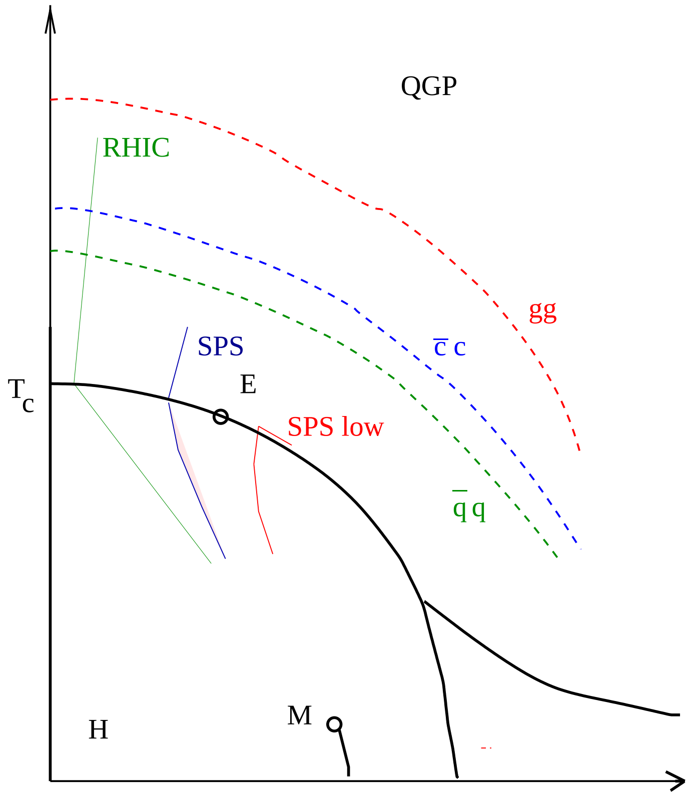


The combination $(M_c/M_D)rV(r)$, versus the distance rM_D.
The dashed line corresponds to $4\alpha_s/3 = 0.471$, while the solid lines correspond to the running coupling constant, for $T = 1; 1.5; 3; 10T_c$, from bottom up. The cusps occur when α_s reaches 1.

- But, if the running of the QCD coupling continues till the screening length, we calcualted that charmonium remains bound till $T_{cc} = 1.62 T_c$.
- Recent lattice results (Karsch etal found bound η_c at $1.5 T_c$, very recently Asakawa-Hatsuda (hep-lat/0309001) have shown the debinding point is in between 1.6 and 1.9 T_c.
- so this idea passed the first test

• **If so, there are also weakly bound states of $\bar{q}q$ and gg type, the latter persisted till $T \sim 500\, MeV$ or so, plus colored composites.**

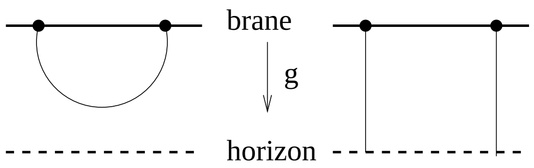




• **Why is it important? Because weakly bound state (like deuteron) means huge "unitarity limited" scattering cross sections.**

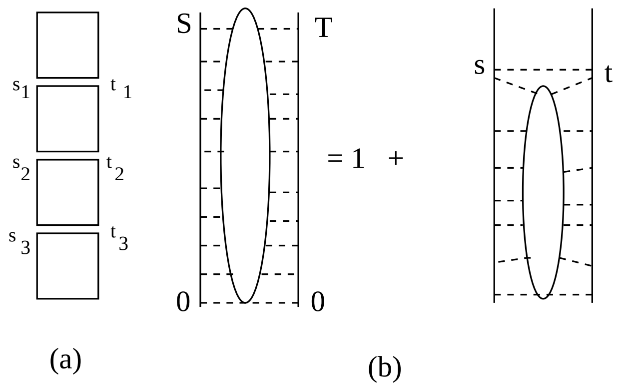
A plasma phase at a supercritical coupling

- The $\mathcal{N}=4$ SUSY Yang Mills gauge theory is conformal (CFT) (the coupling does not run). At finite T it is in a plasma phase at ANY coupling, including very strong $\lambda = g^2 N_c \gg 1$.
- Unexpected help from the string theory AdS/CFT correspondence by Maldacena turned the strongly coupled gauge theories to a classical problem in gravity albeit in 10 dimensions
- Example: a modified Coulomb's law (by Maldacena) $V(L) = -\frac{4\pi^2}{\Gamma(1/4)^4}\frac{\sqrt{\lambda}}{L}$



- Free energy at large λ is $F = (3/4 + O(1/\lambda^{3/2}))F_{free}(T)$

G. Semenoff and K.Zarembo,hep-th/0202156. have shown that a modified Coulomb law can be understood by the resummed gluonic ladder



$\Gamma(S,T) = 1 + \frac{\lambda}{4\pi^2} \int_0^S ds \int_0^T dt \frac{1}{(s-t)^2+L^2} \Gamma(s,t)$

$\frac{\partial^2 \Gamma}{\partial S\, \partial T} = \frac{\lambda/4\pi^2}{(S-T)^2+L^2} \Gamma(S,T)$

Change variables to $x = (S-T)/L$ **and** $y = (S+T)/L$

Expansion of the kernel in the first power of $(S-T)^2$ **leads to an oscillator potential and the problem is easily solved**

$\Gamma(x,y) =\approx C_0\, e^{-\sqrt{\lambda}\, x^2/4\pi}\, e^{\sqrt{\lambda}\, y/2\pi}$

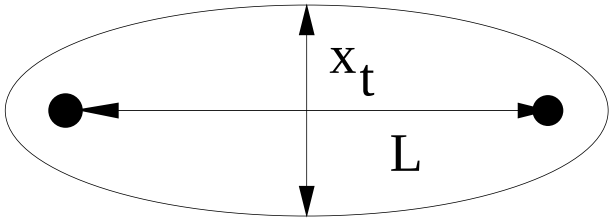
$V_{\mathrm{lad}}(L) = -\lim_{T\to+\infty} \frac{1}{T} \Gamma\,(T,T) = -\frac{\sqrt{\lambda}/\pi}{L}$

the same parametric form but different coefficient: $1/\pi = 0.318$ **while the exact Maldacena value is 0.228.**

• We observed that the exchanged gluons move with the superluminal speed $v \sim \delta t/L \sim \lambda^{1/4} \gg 1$

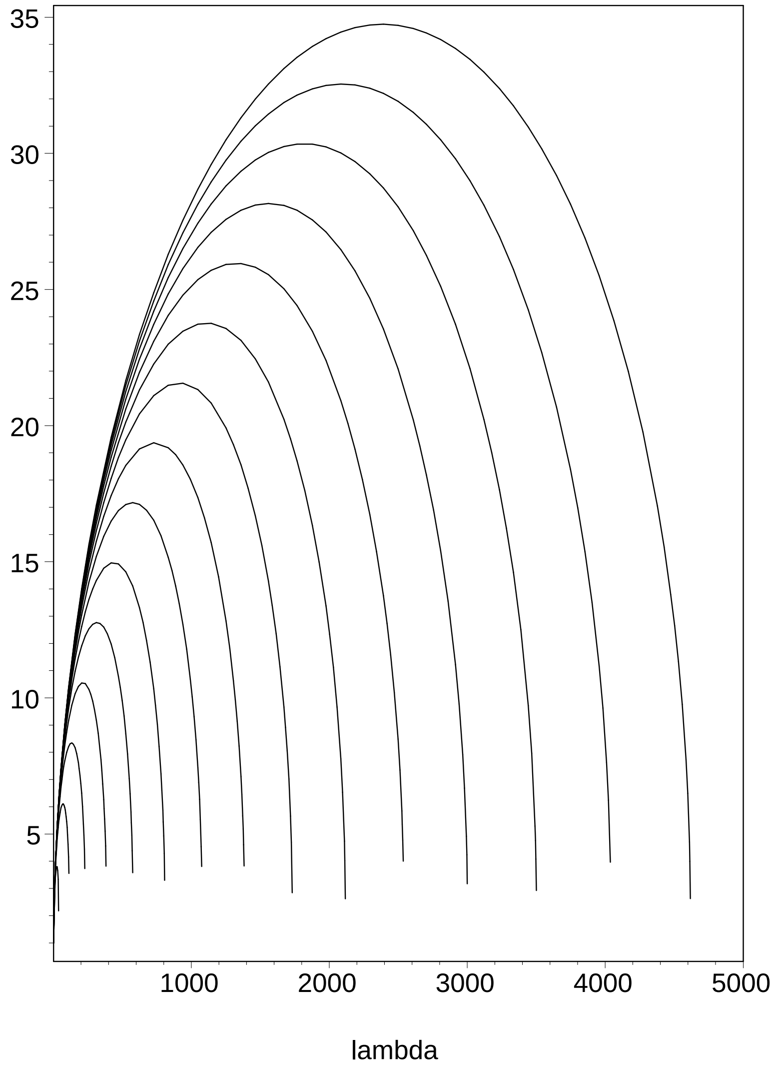
• Some higher order diagrams have extra $\lambda d^4 x \sim O(1)$: multi-body Bethe-Salpeter

• vertices distributed in a quasi-string regime



- **The main idea: the modified Coulomb law can be used even for relativistic bound states, with $v \sim 1$.**

- **Using a Klein-Gordon eqn $(E - V)^2 - m^2 = p^2, V = -C/r$ with a Coulomb potential, by WKB or exactly, one can find the spectrum. (Known from about 1930).**



$$E_{nl} = m\left[1+\left(\frac{C}{n+1/2+\sqrt{(l+1/2)^2-C^2}}\right)^2\right]^{-1/2}$$

Small C - nonrelat. atoms, New regime at large $C >> 1$: families of relativistic deeply bound states, in which large orbital momentum balances the supercritical Coulomb

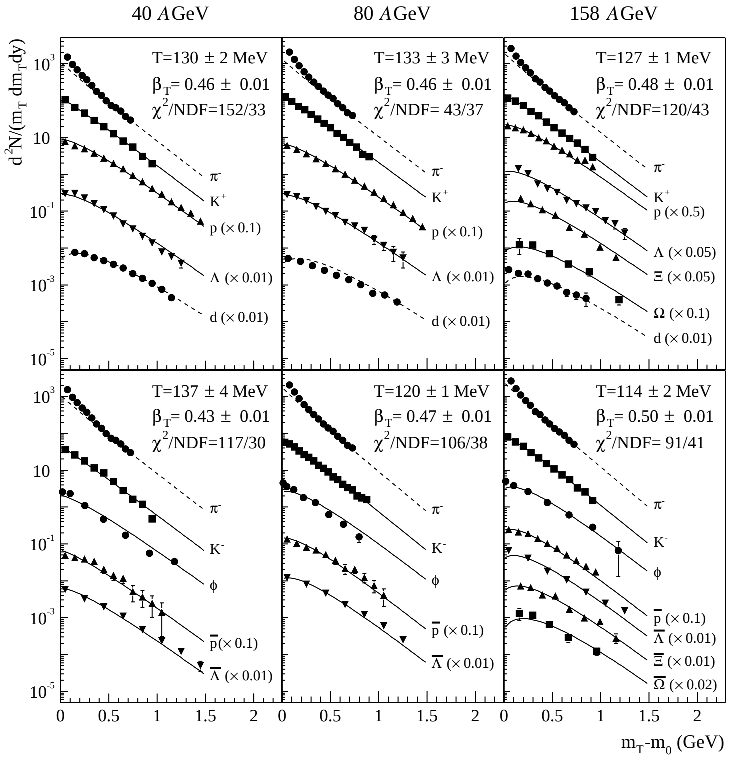
• These states in fact dominate the matter: their masses are $\sim T$ much smaller than those of original quasiparticles $\sim \lambda^{1/2} T$.

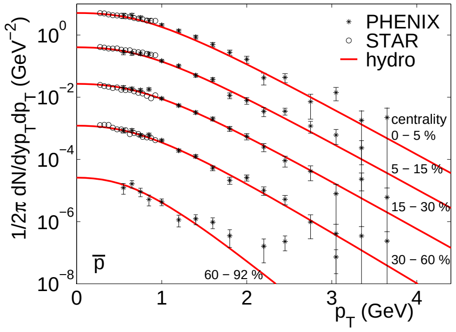
At strong coupling only those compound states are thermally excited, • Furthermore, the density of states can be shown not to depend on λ.

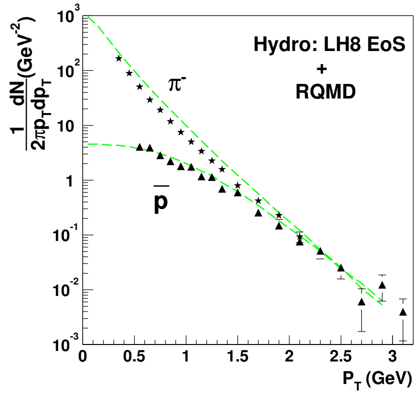
The composites carry large angular momentum $l \approx \sqrt{\lambda} + 1/\sqrt{\lambda}$ and $\int_{l_{min}}^{l_{max}} dl^2 = l_{max}^2 - l_{min}^2 \approx \lambda^0$ thus the free energy $\mathrm{F} = \mathrm{C} N_c^2 T^4$ and without dependence on λ

• (The oscillator-kind spectrum $M \sim constnT$ some unexplained states have been inferred by Starinets and also Teaney before, by a direct calculation of the stress tensor correlator – via graviton propagator in AdS metrics.)

Hydrodynamics and radial flow at SPS/ RHIC



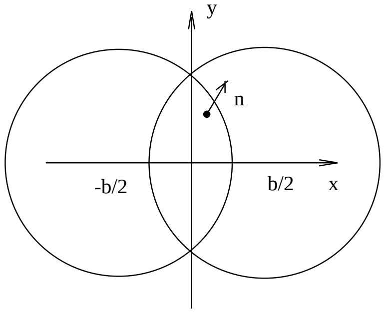




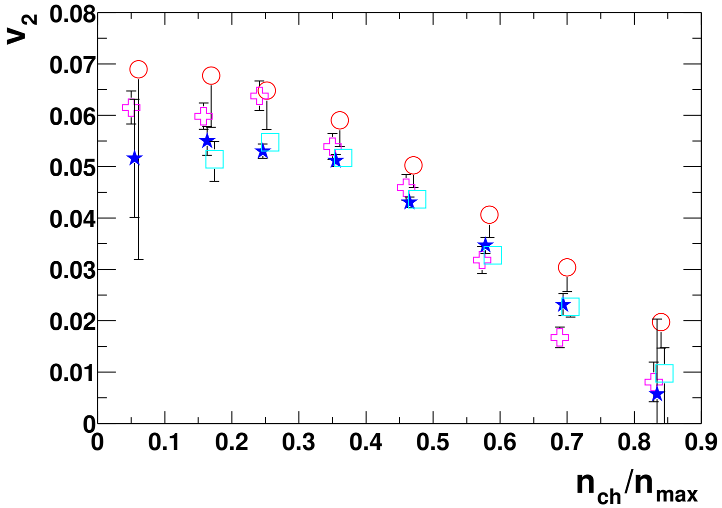
The SPS (NA49) and RHIC spectra: **all species till d** can be well fitted with the same velocity and T_f

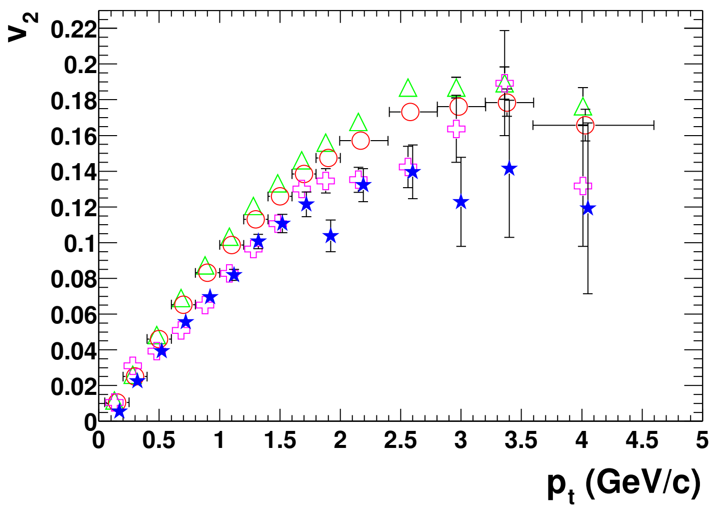
at RHIC more $\bar{p}$ are observed than π^- for $p_t > 2GeV$, exactly as hydro boosts predicts!

Elliptic flow starts with the “initial almond” (for non-central collisions) and is described by Fourier harmonics

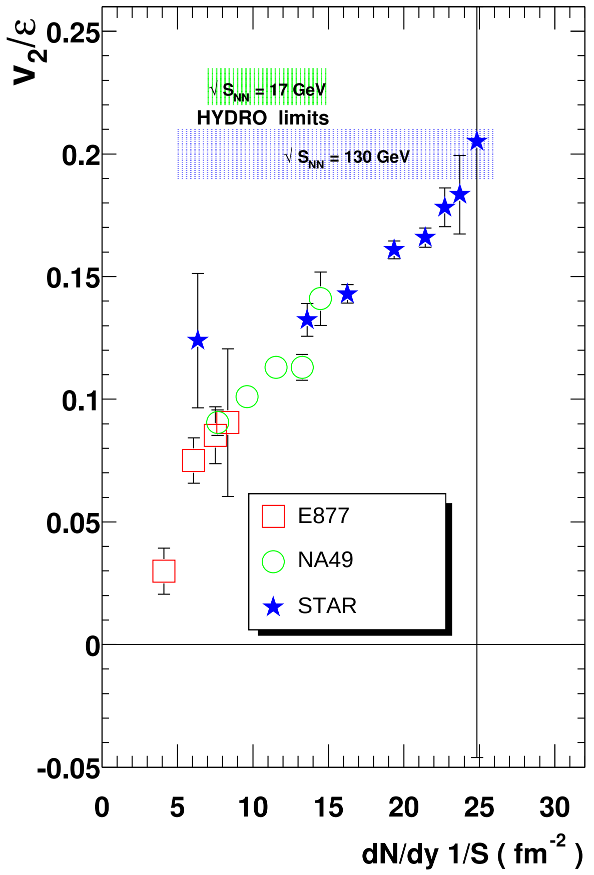


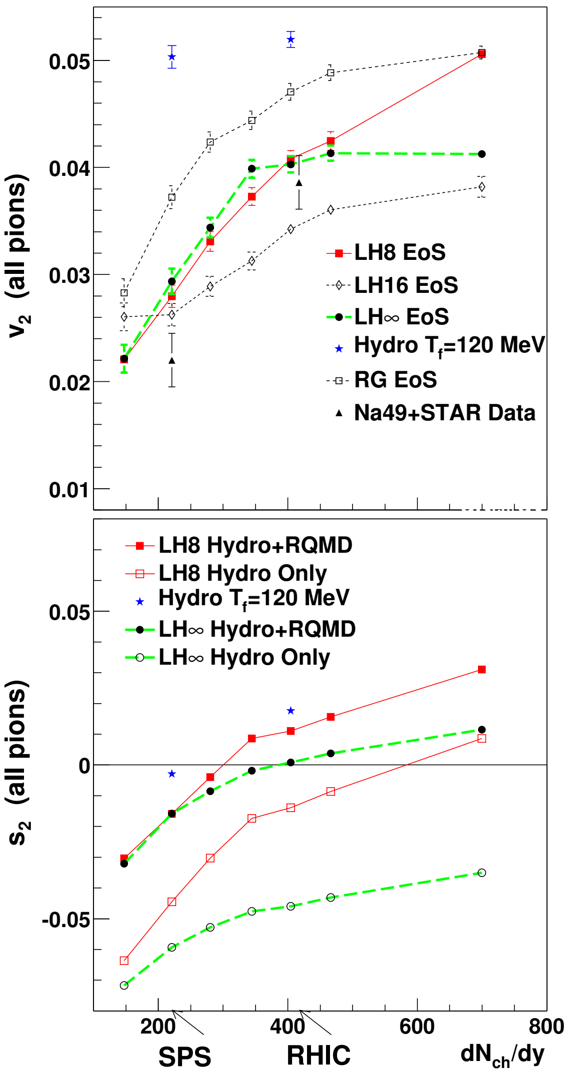
$$\frac{dN}{d\phi} = \frac{v_2}{2\pi} + \frac{v_2}{\pi}\cos(2\phi) + \frac{v_4}{\pi}\cos(4\phi) + \cdots$$





Elliptic flow versus (a) centrality and (b) p_t for Au + Au at $\sqrt{s_{NN}} = 130$ GeV: several methods of v_2 extraction, all from STAR at RHIC.

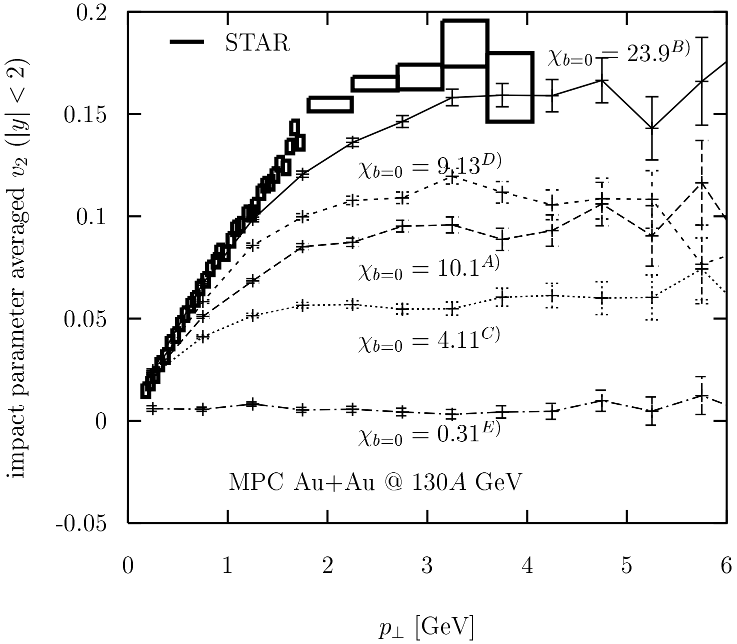




From Teaney, Lauret, ES (shown already at QM99, so it is well documented prediction). The red curve has the latent heat (LH) =800 MeV/fm^3, it is is the closest to the lattice EoS is also the best fit to all flow data at SPS and RHIC

It is only possible to get elliptic flow if the quasiparticle rescattering is increased by big factor ~ 50 relative to the pQCD expectations.

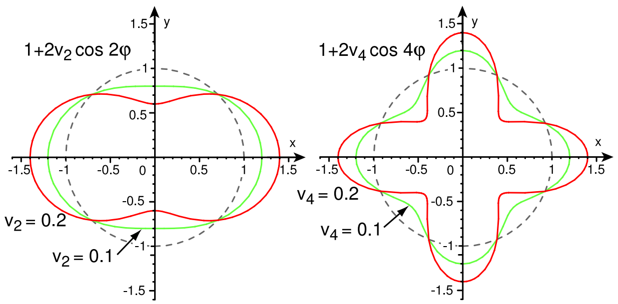
The Figure (from Gyulassy and Molnar) shows how the measured effect (boxes) can be reached if matter opacity of matter grows. The smallest value, roughly corresponding to gg scattering in pQCD, shows no collective effects whatsoever, and the rescattering cross section should be boosted by a big factor before the effect appears.

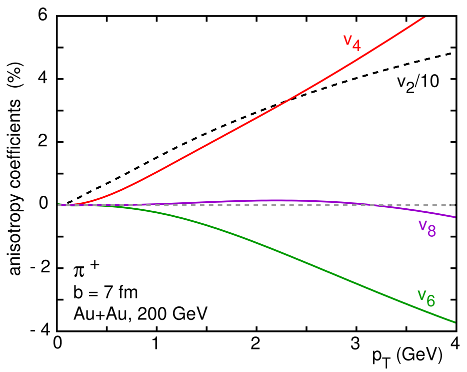


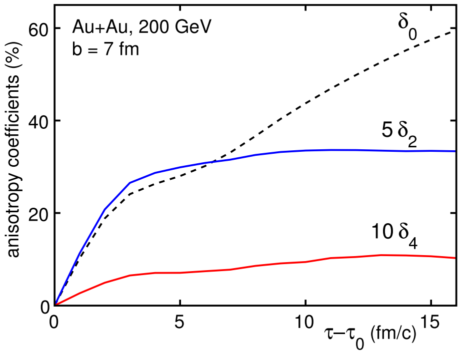
(Gluon elliptic flow for Au+Au at $\sqrt{s} = 130A$ GeV with transport opacities $\chi_{b=0} = 0.31$, 4.11, 9.13, 10.1 and 23.9.)

The next moment v_4

- D.Teaney+ES, PRL 1999, The unusual “nutcracker” solution at $b > 8\,fm$ which is more square than elliptic
- Recently P.Kolb 2003 calculated $v_n(b, p_t)$ in a systematic way







Rumors are this is seen by STAR, in good agreement with hydro once again...

The viscous corrections to hydro

• Ideal hydrodynamics assumes that all the dynamics is local, it ignores the finiteness of the mean free path l (or other correlation lengths).

• Including the first $O(l/L)$ dissipative terms, e.g. into boost-inv. (Bjorken) and axially symmetric hydro eqn:

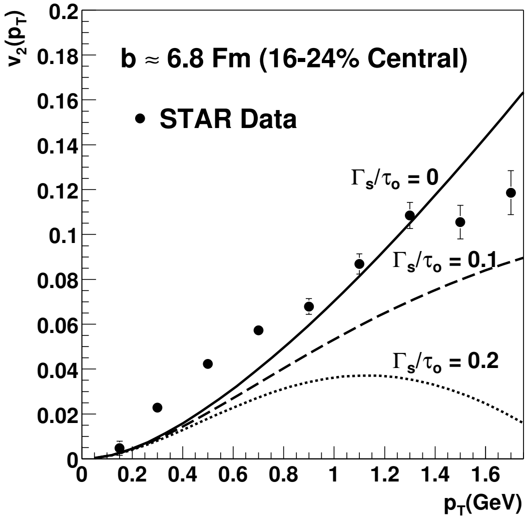
$$\frac{1}{\epsilon+p}\frac{d\epsilon}{d\tau}=\frac{1}{s}\frac{ds}{d\tau}=-\frac{1}{\tau}\left(1-\frac{\Gamma_s}{\tau}\right)$$

one finds in the r.h.s. exactly the combination which also appears in the sound attenuation length $\Gamma_s=\frac{4}{3}\eta/(\epsilon+p)$ which tells us at which times one cannot neglect the viscosity

• Following Derek Teaney, hep-ph/0301099, one may try to extract it from the data, by looking at deviations from ideal hydro predictions.

• the viscous correction to particle distribution is completely determined $f = f_o(1 + \frac{\eta}{s} p^\alpha p^\beta \left\langle \nabla_\alpha u_\beta \right\rangle)$. Consequence: Corrections are $\sim \frac{\Gamma_s}{4\tau} \left(\frac{p_T}{T}\right)^2$

• We need QGP viscosity at earlier times.: thus we look at elliptic flow. the fireball is spatially anisotropic only for $\tau < 3 fm/c$, when matter is in the QGP phase.



Elliptic flow v_2 as a function of p_T for different values of Γ_s/τ_o. The data points are four particle cummulant data from the STAR collaboration [?]. Only statistical errors are shown. The results for different Γ_s/τ is shown The experimental data deviate from ideal hydro curve at $p_\perp \approx 1.6\, GeV$ which indicates $\Gamma_s/\tau \sim 0.05$ or so. Substituting here the relevant time $\tau \sim 3 fm/c$ we get surprisingly small $\Gamma_s \sim .15 fm$.

The theory of viscosity

• Developed for long time in the weak coupling perturbative framework: large, $\eta/T^3 \sim const/g^4 log(1/g) >>> 1, const \sim 100$ at small $g << 1$

If so, $l \sim few\, fm$ and no hydro at RHIC ! (Recall the Gyulassy-Molnar plot here)

• However, in the strongly coupled plasma ($\mathcal{N} = \triangle$ SUSY Yang-Mills or CFT) Polycastro, Son, Starinets, Phys. Rev. Lett. 87 (2001) 081601 the value for the sound attenuation length

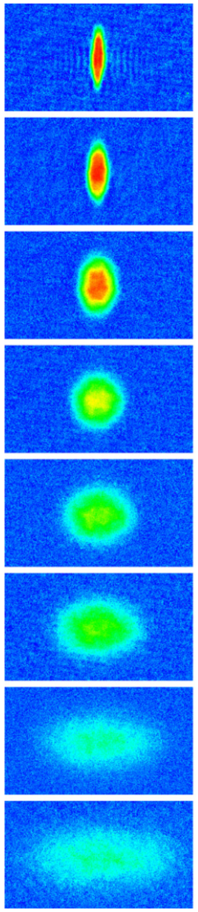
$$\Gamma_s = \frac{4}{3}\frac{\eta}{(\epsilon + p)} = \frac{1}{3\pi T} \sim .1\, fm$$

If so, excellent hydro

Elliptic flow with trapped Li^6 atoms:

K.M.O'Hara et al, Science 298,2179, 2002

T.Bourdel et al, PRL 91 020402 , July 11 2003



100 μs
200 μs
400 μs
600 μs
800 μs
1000 μs
1500 μs
2000 μs

Magnetic field $B \sim 800 Gauss$ shifts the Feschbach resonance and makes the zero energy state, or infinite scattering length a.
Normally $ak_f \ll 1$ and gas expands in collisionless way Isotropically
But in the strong coupling regime $ak_f \gg 1$ it explodes hydrodynamically !, see the figure
However the EoS hardly changes by 10%

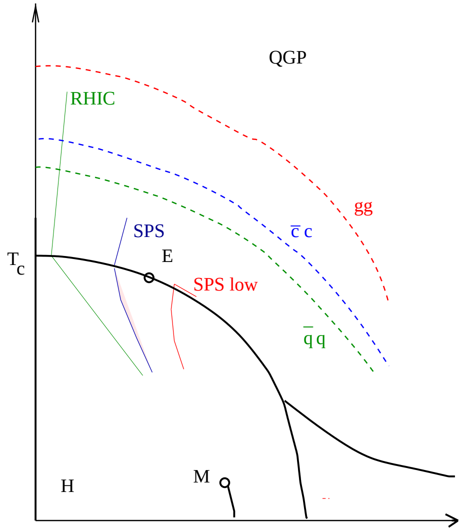
Adiabatic capture

- Very recent important discovery

J.Cubizolles et al, cond-mat/0308018, K.Strecker et al,cond-mat/0308318

- If one change magnetic field so that the molecular level moves from unbound to bound, with high efficiency (~ 85 percents) atoms are turned into Li_2 molecules, all of course in the same relative state.
- The phenomenon is reversible
- Going further into the bound region one finds that binding energy goes into heating the gas
- The adiabatic path in heavy ion collision also crosses the zero biding line in this direction. So a "hadronization" may probably happen at this line, not at $T = T_c$

Summary



- All s-wave mesons and glueballs do not melt at T_c, but at around 1.5-3 T_c, which generates strong (unitarity limited) rescattering

- Viscosity corrections in QGP (for large b and at early time relevant for v_2) seem to be small $\Gamma_s/\tau < .1$
- This disagrees with pQCD or weak coupling but agrees better with the strong coupling regime
- New connection to strongly coupled trapped Li^6 atoms
- New connection to string world, via Ads/CFT correspondence by Maldacena, to strongly coupled QGP. The puzzles of the CFT plasma at the supercritical coupling are explained by the multiple deeply bound highly rotational states with masses $M \sim T$

Viscosity: an overview

• Transport in gases is done by atomic motion

$$\eta \sim Dmn \sim \frac{(mT)^{1/2}}{\sigma}$$

where D, m, n are diffusion coeff. mass and density, σ is the atomic cross section.

• Transport of energy/momentum in liquids is like in solids and done by transfer of atomic oscillations – phonons.

Frenkel, 1926: *"The fact to be explained is not the viscosity but fluidity"*.

Large viscosity corresponds to small diffusion, he found

$$\eta \sim \frac{T}{aD} \sim exp(+W/T + p/p0)$$

where a is atomic size and W is some activation energy needed to move an atom and p is pressure.

• **Especially well known special case is liquid He^4 at low T, where phonons and rotons are the well defined quasiparticles, with small cross section: it is a gas once again. Some exp. had mean free path of $\sim 1cm$ scale, so no hydro for "normal component" but a "ballistic regime" when quasiparticles just fly from their source by straight lines.**

The QGP quasiparticles beyond quarks and gluons

- **RHIC QGP is at** $T = 170 - 350 MeV$ **and effectively** $\lambda_{tHooft} = g^2 N_c \sim 12 >> 1$, **pQCD series are not converging... So we have a relatively strongly coupled plasma: what is it?**

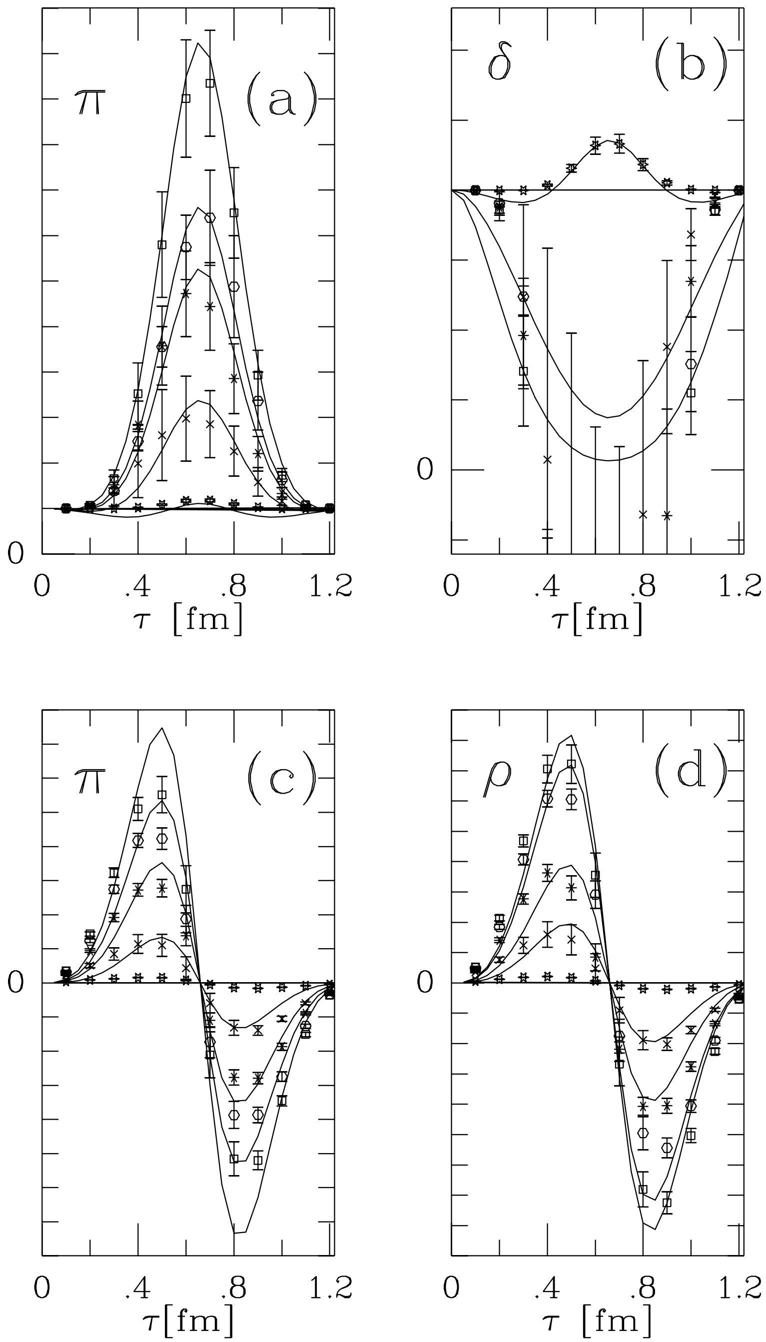
- **Lattice: e.g. Karsch hep-lat/0110111 gives effective** $g \sim 1.6 - 3$, **from potential, and also plasma frequencies**

$$\omega_q(p = 0) \approx \omega_g(p = 0) \approx 3T \sim .5 - 1 GeV$$

- **The sound: phonons are massless and still have large mean free path at small k**

- **The pions+sigma quartet (for 2 flavors): in QGP no longer Goldstone particles, but still exist**

The correlators (ratio to thermal $\bar{q}q$ propagators) Schafer, ES:CAN HADRONS SURVIVE THE CHIRAL PHASE TRANSITION? Phys.Lett.B356:147-152,1995 hep-ph/9506282 Instanton liquid above T_c breaks into finite clusters with top.Q=0



Also there are lattice results starting from DeTar who found small "screening masses" $< 2\pi T$ for pion/sigma in QGP.

- Instead of conclusions, a question:

why those lightest mode do not generate large viscosity? small velocity close to T_c ? or small mean free paths? Not studied yet...