HOMEWORK 10, THERMAL PHYSICS (PHY306)

1. The noble gas Argon Ar has atomic weight $A \approx 40$ and it is present in atmosphere third after N_2, O_2. Its critical point is at $T_c \approx 150.K, p_c \approx 48.atm$.
(a) Using for its description the van der Vaals equation of state, find which volume V_c is occupied by one mole of Ar at critical conditions. Compare it to the volume occupied at normal conditions $(= 300K, p = 1\,atm)$ V_{normal}.
(b) Using relations between critical values and van der Vaals parameters a, b (26,18-20) find their values for Ar. Representing the restricted volume b (per atom, not per mole as done in the textbook!) as $(4\pi/3)R_{Ar}^3$ (the volume of a sphere), estimate the radius of it for Ar.

2. (a) Using the van der Vaals equation of state in its form normalized to the critical values (26.63), plot the isotherm $T = 135K = 0.9T_c$ on the $\tilde{p}, \tilde{V}$ plane (from $\tilde{V} \equiv V/V_c = 0.2$ to 4).
(b) for an experiment run in a compression chamber at $p = 24\,atm$, find (graphically, from the plot) all three possible values for the volume. What is the ratio of the volumes corresponding to the gas and liquid V_{gas}/V_{liquid} solutions?
(c) The Maxwellian areas (two $\int V\,dp$ integrals between points 1-2 and 2-3) apparently are not equal. Find which is larger, and in which phase Ar should be in the described experiment?
(d) Fortunately, the compression chamber allows to change the pressure (while keeping the same temperature). Using your plot and Maxwell's construction as guides, estimate (approximately) at what pressure $\tilde{p}_{coexistence}$ the gas-liquid coexistence would be observed in the chamber.

3.(a) The second virial coefficient can be expressed in terms of interatomic interaction potential $V(r)$ by (26.56)

$$B(T) = \frac{1}{2}N \int [1 - e^{-V(r)/T}]4\pi r^2 dr$$

Using potential for Argon Ar in Lenard-Jones form

$$V(r) = 4\epsilon[(\frac{r_0}{r})^{12} - (\frac{r_0}{r})^6]$$

with parameters $\epsilon/k_B = 120K, r_0 = 0.34\,nm$, plot the integrand $[1 - e^{-V(r)/T}]4\pi r^2$ at $T = 135K = 0.9T_c$. Estimate the value of $B(T)$ from the area of the plot.
(b) Compare the resulting value to what one gets from the van der Vaals equation of state, which gives (26.43) $B(T) = b - a/RT$, using the values of a, b you get in problem 1.