HOMEWORK 2, THERMAL PHYSICS (PHY306)

1. (a) The binding energy of an electron in hydrogen atom is 13.6 eV. What fraction of atoms would be ionized at room temperature $T = 300K$ and in the Sun's photosphere $T_S = 6000K$?
(b) The energy of rotational state of certain diatomic molecule is $10^{-4}\,eV$. What is the probability of its excitation at temperature of microwave background radiation filling the Universe, $T = 2.7K$?

2.A photon of energy 1 eV is absorbed by a room's wall which is at room temperature and its energy is equilibrated among the wall's atoms. By what factor does the number of states of the wall Ω_{wall} has changed?

3. The masses and radii of the Earth and the Sun are

$$M_E = 5.972 \times 10^{24} kg,\;\; R_E = 6.371 \times 10^6 m\;\; M_S = 1.989 \times 10^{30} kg,\;\; R_S = 695.508 \times 10^6 m$$

(a) Using Newton's gravitational potential and energy conservation, calculate the escape velocities from Earth and Sun, v_E, v_S.
(b) For molecular hydrogen (H_2), atomic helium (^4He) and molecular oxygen (O_2) at room temperature $T = 300K$ calculate their r.m.s. velocities and compare them to the escape velocities for Earth found in (a)
(c) Repeat the same for the Sun, using its surface temperature $T_S = 6000K$.

4. You are in the office of size $3 \times 4 \times 5\,m^3$ at $T = 20^oC$ and $p = 10^5\,Pa$. Feeling cold, you switched a heater on and raised the air temperature to $T = 24^oC$. What was the number of atoms and their total kinetic energy in the room before heating N_{before}, E_{before} and is after the heating N_{after}, E_{after}?
Consider two cases: (a) the room's door is tightly closed, so no air can escape; (b) the room's door is slightly open, so that the pressure remains constant (same as outside $p = 10^5\,Pa$).

5.The so called Magdeburg hemispheres are a pair of large copper hemispheres, with mating rims. They were used to demonstrate the power of atmospheric pressure, the demonstration was 8 May 1654 in Regensburg, in front of German emperor's court. When the rims were sealed with grease and the air was pumped out, the sphere contained a vacuum and could not be pulled apart by two teams of many horses. Assuming that the diameter of the sphere was $d = 0.5\,m$ and that pressure applies to corresponding disk (inside the rim) and atmospheric pressure, calculate how many horses N_{horses} were needed to pull them apart. Assume that one horse can produce maximal force which can pull a weight of $M = 100\,kg$ vertically (say a heavy person or large bucket of water from the well). Horses were divided symmetrically, half and half on each side.

1. (a) The binding energy of an electron in hydrogen atom is 13.6 eV. What fraction of atoms would be ionized at room temperature $T = 300K$ and in the Sun's photosphere $T_S = 6000K$?
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1a.) $$\frac{P(\varepsilon_1)}{P(\varepsilon_2)} = \frac{e^{-\frac{\varepsilon_1}{T}}}{e^{-\frac{\varepsilon_2}{T}}} = e^{\frac{\varepsilon_2 - \varepsilon_1}{T}}$$

bound: $\varepsilon_2 = -13.6\ eV$

ion: $\varepsilon_1 = 0\ eV$

So $$\frac{P(\varepsilon_1)}{P(\varepsilon_2)} = e^{\frac{-13.6eV}{T}}$$

total fraction would be

$$f(T) = \frac{P(\varepsilon_1)}{\sum_i P(\varepsilon_i)} = \frac{P(\varepsilon_1)}{P(\varepsilon_1) + P(\varepsilon_2)}$$

$$= \frac{\frac{P(\varepsilon_1)}{P(\varepsilon_2)}}{\frac{P(\varepsilon_1)}{P(\varepsilon_2)} + 1} = \frac{e^{\frac{-13.6eV}{T}}}{e^{\frac{-13.6eV}{T}} + 1}$$

$$= \frac{1}{1 + e^{\frac{13.6eV}{T}}}$$

for $T = T_E = 300K = 0.025852 eV$

$$f(300K) = 3.39 \cdot 10^{-229}$$

for $T = T_S = 6000K = 0.51704 eV$

$$f(6000K) = 3.77 \cdot 10^{-12}$$

b.) The fraction of atoms at this state is

$$f(T) = \frac{1}{e^{\frac{10^{-4} eV}{T}} + 1} = 0.3943$$

$$T = 2.7K = 2.33 \cdot 10^{-4} eV$$

this is also the probability of a particle will be rotating so if we have N particles

The Distribution should be near Binomial

$\Rightarrow$ the Probability of X Particles rotating is

$$P(X) = \binom{N}{X}\left(f(T)\right)^X\left(1-f(T)\right)^{N-X}$$

If all particles in the Universe are Rotating. the Probability of such an event is

$$P(N) = \binom{N}{N}\left(f(T)\right)^N$$

$$= \left(f(T)\right)^N = (0.3943)^N$$

2.A photon of energy 1 eV is absorbed by a room's wall which is at room temperature and its energy is equilibrated among the wall's atoms. By what factor does the number of states of the wall Ω_{wall} has changed?

$$S = \ln \Omega$$

$$dE = T dS - P dV$$

if $dV \to 0$ then

$$dE = T dS$$

$\to \Delta E = T \Delta S$ if T is constant

$$\Rightarrow \Delta S = \frac{\Delta E}{T}$$

$\Delta E = 1\,eV$, $T = 300\,K$
$= 0.026\,eV$

so $\Delta S = \frac{1\,eV}{0.026\,eV} = 38.\overline{461538}$

$$\Delta S = \ln\left(\frac{\Omega}{\Omega_0}\right)$$

$$\Rightarrow \quad e^{\Delta S} = \frac{\Omega}{\Omega_0} = 5.05 \cdot 10^{16}$$

3. The masses and radii of the Earth and the Sun are

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3a.) $$\frac{1}{2} m v^2 - \frac{GMm}{R} = 0$$

$$\Rightarrow \quad v = \sqrt{\frac{GM}{R}}$$

$$v_E = \sqrt{\frac{GM_E}{R_E}} = 11{,}180 \ \frac{m}{s}$$

$$= 11.18 \ km/s$$

$$v_S = \sqrt{\frac{GM_S}{R_S}} = 617{,}700 \ \frac{m}{s}$$

$$= 617.6 \ km/s$$

3b/c.) $\frac{3}{2}T = \frac{1}{2}mv^2$

$$\Rightarrow v = \sqrt{\frac{3T}{m}}$$

V_{rms}	Earth	Sun
H_2	1,927 m/s	8,616 $\frac{m}{s}$
He	1,367 m/s	6,114 $\frac{m}{s}$
O_2	484 m/s	2,163 $\frac{m}{s}$

there is about 1-2 order of magnitude diff. between rms of Earth versus escape on Earth

there is about a 2 magnitude diff. between rms of Sun and escape velocity of Sun

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a.) $E = \frac{3}{2} NT = \frac{3}{2} PV$

so

$$dE = \frac{3}{2} T \cancel{dN}^{\,0} + \frac{3}{2} N dT$$

$$\Rightarrow dE = \frac{3}{2} N dT \Rightarrow \Delta E = \frac{3}{2} N \Delta T$$

$$N_{before} = N_{after} = 2.2 \cdot 10^{28}$$

$$E_{after} - E_{Before} = \frac{3}{2} N (\Delta T)$$

$$N_{after} = N_{Before} = \frac{P_0 V_0}{T_0} = 36{,}559.8 \text{ moles}$$

$$E_{after} - E_{Before} = \frac{3}{2} \frac{P_0 V_0}{T_0} \Delta T$$

$$= \frac{3}{2} \frac{(10^5 \cdot 60)J \cdot 4K}{293.2K}$$

$= 122{,}783 \text{ J}$

$E_{Before} = 9 \cdot 10^{6} \text{ J}$

$E_{after} = 9.12 \cdot 10^{6} \text{ J}$

b.) Pressure is constant and the Volume is constant

So $PV = NT$

$\Rightarrow N_{Before} T_{Before} = N_{after} T_{after}$

$$\Rightarrow \frac{N_{after}}{N_{Before}} = \frac{T_{Before}}{T_{after}}$$

$$= 1.004$$

$N_{Before} = 2.2 \cdot 10^{28}$

$= 36{,}559.8$ moles

$N_{after} = 2.193 \cdot 10^{28}$

$= 36{,}414$ moles

Since $E = \frac{3}{2} NT = \frac{3}{2} PV$

Where P, V are constant

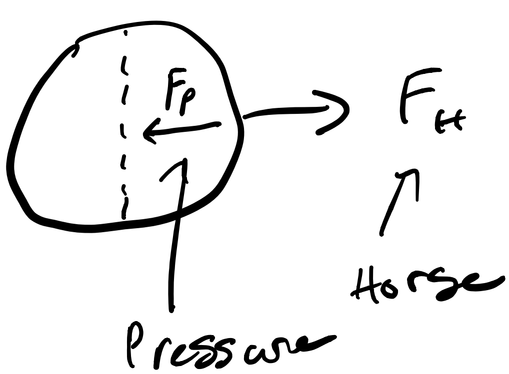
then

$E_{Before} = E_{after} = 9 \cdot 10^6 J$

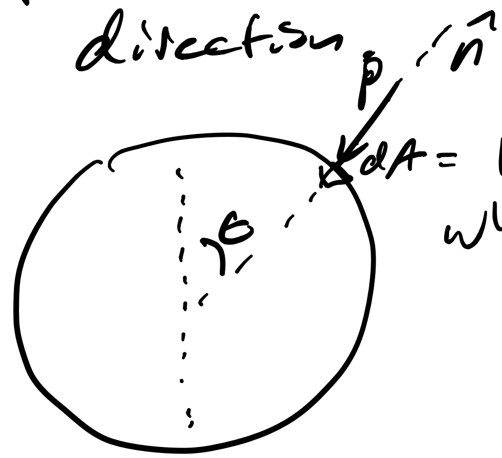
5.The so called Magdeburg hemispheres are a pair of large copper hemispheres, with mating rims. They were used to demonstrate the power of atmospheric pressure, the demonstration was 8 May 1654 in Regensburg, in front of German emperor's court. When the rims were sealed with grease and the air was pumped out, the sphere contained a vacuum and could not be pulled apart by two teams of many horses. Assuming that the diameter of the sphere was $d = 0.5\,m$ and that pressure applies to corresponding disk (inside the rim) and atmospheric pressure, calculate how many horses N_{horses} were needed to pull them apart. Assume that one horse can produce maximal force which can pull a weight of $M = 100\,kg$ vertically (say a heavy person or large bucket of water from the well). Horses were divided symmetrically, half and half on each side.

The horse can provide a force

of $100 kg \cdot 9.81 \frac{m}{s^2} = 981 N$



F_p is the force of pressure in the x direction



$dA = R^2 d\Omega$ where $R = \frac{d}{2}$

So $\vec{p} = -p\hat{n}$

$\hat{n} = (\cos\phi \sin\theta, \sin\phi \sin\theta, \cos\theta)$

So the pressure force on the Half sphere in the x-direction

is $\left| \iint_{\text{Half sphere}} p \cos\phi \sin\theta \, d\Omega \right|$

$$= \left| pR^2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\phi \cos\phi \int_0^{\pi} d\theta \sin^2\theta \right|$$

$$= \left| R^2 P\left(-1-(+1)\right)\left(\frac{\pi}{2} + \frac{1}{2}\cancelto{0}{\int_0^{\pi} \cos(2\theta)\,d\theta}\right)\right|$$

$$= P\pi R^2 = \frac{\pi}{4} P d^2$$

alternatively we can call the z-direction instead of x-direction

$$\text{So } \left| P R^2 \int_0^{2\pi} d\phi \int_0^{\frac{\pi}{2}} d\theta \, \sin\theta\cos\theta \right|$$

$$= \left| P R^2 \cdot \pi \int_0^{\frac{\pi}{2}} d\theta \, \sin(2\theta) \right|$$

$$= \left| P R^2 \cdot \frac{\pi}{2} \int_0^{\pi} du \, \sin u \right|$$

$$= \left| P R^2 \cdot \frac{\pi}{2} \cdot (-1 + -1) \right|$$

$$= P\pi R^2 = \frac{\pi}{4} P d^2$$

(Pressure times crossection)

So $F_p = \frac{\pi}{4} P d^2$

$\Rightarrow F_p = 19{,}900\,N$

So $F_H > F_p$

$\Rightarrow F_H > 19{,}900\,N$

So $\frac{F_H}{981\,N} \gtrsim 20.29$

So you need about
41 Horses since
you need another
set of horses
to keep the Ball
stationary, or
21 horses and
strong rope attached to
the Ball and immovable

Wall