```
ClearAll["Global`*"]
(* problem 1a *)
kB = 1.38064852 × 10 ^ (-23) * m ^ 2 * kg / s ^ 2 / K
Troom = 300 * K
Eionization = 13.6 * eV
eV = 1.6 * 10 ^ {-19} * kg * m ^ 2 / s ^ 2
Pionization = Exp[-Eionization / kB / Troom]
```

$$\frac{1.38065 \times 10^{-23}\ \mathrm{kg}\ \mathrm{m}^2}{\mathrm{K}\ \mathrm{s}^2}$$

$$300\ \mathrm{K}$$

$$13.6\ \mathrm{eV}$$

$$\left\{\frac{1.6 \times 10^{-19}\ \mathrm{kg}\ \mathrm{m}^2}{\mathrm{s}^2}\right\}$$

$$\left\{6.92411 \times 10^{-229}\right\}$$

```
(* problem 1b *)
Erot = 10 ^ (-4) * eV
TUni = 2.7 * K
Prot = Exp[-Erot / kB / TUni]
```

$$\left\{\frac{1.6 \times 10^{-23}\ \mathrm{kg}\ \mathrm{m}^2}{\mathrm{s}^2}\right\}$$

$$2.7\ \mathrm{K}$$

$$\{0.651021\}$$

```
(* problem 2 *)
Exp[1. * eV / kB / Troom]
```

$$\left\{5.97646 \times 10^{16}\right\}$$

```
(* problem 3
 the escape velocity from gravity potential
*)
Solve[GN * M / R == v ^ 2 / 2, v]
```

$$\left\{\left\{v \to -\frac{\sqrt{2}\ \sqrt{GN}\ \sqrt{M}}{\sqrt{R}}\right\}, \left\{v \to \frac{\sqrt{2}\ \sqrt{GN}\ \sqrt{M}}{\sqrt{R}}\right\}\right\}$$

$$\text{vescape} = \frac{\sqrt{2}\ \sqrt{\text{GN}}\ \sqrt{\text{M}}}{\sqrt{\text{R}}};$$

```
GN = 6.674 * 10^(-11) * m^3 / kg / s^2;
ME = 5.9 * 10^(24) * kg;
RE = 6.371 * 10^6 * m;
MS = 1.989 * 10^(30) * kg;
RS = 695 * 10^6 * m;
vE = Assuming[kg > 0 && m > 0 && s > 0, Simplify[vescape /. {M → ME, R → RE}]]
```

$$\frac{11118.1\ \text{m}}{\text{s}}$$

```
vS = Assuming[kg > 0 && m > 0 && s > 0, Simplify[vescape /. {M → MS, R → RS}]]
```

$$\frac{618063.\ \text{m}}{\text{s}}$$

```
(* 11 km/s for Earth is correct
 part b *)

Solve[mass * v^2 / 2 == (3 / 2) * kB * T, v]
```

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

$$\left\{\left\{v \to -\frac{6.43579 \times 10^{-12}\ \sqrt{\text{kg}}\ \text{m}\ \sqrt{\text{T}}}{\sqrt{\text{K}}\ \sqrt{\text{mass}}\ \text{s}}\right\},\ \left\{v \to \frac{6.43579 \times 10^{-12}\ \sqrt{\text{kg}}\ \text{m}\ \sqrt{\text{T}}}{\sqrt{\text{K}}\ \sqrt{\text{mass}}\ \text{s}}\right\}\right\}$$

$$vT = \frac{6.43579486932267`*^{-12} \sqrt{kg}\ m \sqrt{T}}{\sqrt{K}\ \sqrt{mass}\ s}$$

```
mp = 1.6726219 × 10^(-27) * kg;
mH2 = 2 * mp;
mHe = 4 * mp;
mO2 = 2 * 16 * mp;
vTH2 = Assuming[kg > 0 && m > 0 && s > 0, Simplify[vT /. {T → 300 * K, mass -> mH2}]]
vTHe = Assuming[kg > 0 && m > 0 && s > 0, Simplify[vT /. {T → 300 * K, mass -> mHe}]]
vTO2 = Assuming[kg > 0 && m > 0 && s > 0, Simplify[vT /. {T → 300 * K, mass -> mO2}]]
```

$$\frac{6.43579 \times 10^{-12} \sqrt{kg}\ m \sqrt{T}}{\sqrt{K}\ \sqrt{mass}\ s}$$

$$\frac{1927.3\ m}{s}$$

$$\frac{1362.81\ m}{s}$$

$$\frac{481.825\ m}{s}$$

```
(* all velocities are less than escape,
but H2 and He over long time does escape, while oxigen does not *)
(* part c *)
vTH2 = Assuming[kg > 0 && m > 0 && s > 0, Simplify[vT /. {T → 6000 * K, mass -> mH2}]]
vTHe = Assuming[kg > 0 && m > 0 && s > 0, Simplify[vT /. {T → 6000 * K, mass -> mHe}]]
vTO2 = Assuming[kg > 0 && m > 0 && s > 0, Simplify[vT /. {T → 6000 * K, mass -> mO2}]]
```

$$\frac{8619.14\ m}{s}$$

$$\frac{6094.65\ m}{s}$$

$$\frac{2154.78\ m}{s}$$

```
(* none of them can escape at such T *)
```

```
(* problem 4 *)
Voffice = 3 * 4 * 5 * m^3;
T1 = (273 + 20) * K;
T2 = (273 + 24) * K;
newton = kg * m / s^2;
patm = 10^5 * newton / m^2;
N1 = patm * Voffice / kB / T1
```

1.4832×10^{27}

```
(* case a , N is the same, the total energy *)
E1 = N1 * (3 / 2) * kB * T1
```

$\frac{9. \times 10^{6}\ \text{kg}\ \text{m}^2}{\text{s}^2}$

```
(* case b, some air escape ,
but E2 is proportional to pressure so it does not change *)
N2 = patm * Voffice / kB / T2
```

1.46323×10^{27}

```
N2 - N1
```

-1.99758×10^{25}

```
(* that many molecules leave the room quitely !*)
```

```
(* Problem 5 hemispheres *)
Diam = 0.5 * m
Ar = Pi * (Diam / 2)^2
Force = patm * Ar
```

$0.5\ \text{m}$

$0.19635\ \text{m}^2$

$\frac{19\,635.\ \text{kg}\ \text{m}}{\text{s}^2}$

```
Fhorse = 100 * kg * 9.8 * m / s^2
```

$\frac{980.\ \text{kg}\ \text{m}}{\text{s}^2}$

```
Force / Fhorse
```

20.0357

```
(* so one needs 40 horses, 20 on ech side
 in Regensbourge they used 30 and could not separate! *)
```