

In[242]:= (**Question 2**)

Remove["Global`*"]

$$v[r_] := 4 \epsilon \left(\left(\frac{\sigma}{r} \right)^{12} - \left(\frac{\sigma}{r} \right)^6 \right);$$

(*2a*)

Solve[$\partial_r v[r] = 0$, r]

N[Solve[$\partial_r v[r] = 0$, r]]

$\partial_{\{r,2\}} v[r]$

2 SeriesCoefficient[v[r], {r, rmin, 2}]

2 SeriesCoefficient[v[r], {r, rmin, 2}] /. rmin -> $2^{1/6} \sigma$

2 N[SeriesCoefficient[v[r], {r, rmin, 2}] /. rmin -> $2^{1/6} \sigma$]

(*2b*)

Out[244]= $\left\{ \left\{ r \rightarrow -2^{1/6} \sigma \right\}, \left\{ r \rightarrow 2^{1/6} \sigma \right\}, \left\{ r \rightarrow -(-1)^{1/3} 2^{1/6} \sigma \right\}, \right.$
 $\left. \left\{ r \rightarrow (-1)^{1/3} 2^{1/6} \sigma \right\}, \left\{ r \rightarrow -(-1)^{2/3} 2^{1/6} \sigma \right\}, \left\{ r \rightarrow (-1)^{2/3} 2^{1/6} \sigma \right\} \right\}$

Out[245]= $\left\{ \left\{ r \rightarrow -1.12246 \sigma \right\}, \left\{ r \rightarrow 1.12246 \sigma \right\}, \right.$
 $\left\{ r \rightarrow (-0.561231 - 0.972081 i) \sigma \right\}, \left\{ r \rightarrow (0.561231 + 0.972081 i) \sigma \right\}, \right.$
 $\left. \left\{ r \rightarrow (0.561231 - 0.972081 i) \sigma \right\}, \left\{ r \rightarrow (-0.561231 + 0.972081 i) \sigma \right\} \right\}$

Out[246]= $4 \epsilon \left(-\frac{42 \sigma^6}{r^8} + \frac{156 \sigma^{12}}{r^{14}} \right)$

Out[247]= $2 \left(-\frac{84 \epsilon \sigma^6}{rmin^8} + \frac{312 \epsilon \sigma^{12}}{rmin^{14}} \right)$

Out[248]= $\frac{36 \times 2^{2/3} \epsilon}{\sigma^2}$

Out[249]= $\frac{57.1464 \epsilon}{\sigma^2}$

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In[3]:= (**Question 4**)
Remove["Global`*"]

r[t_, lmax_,  $\theta$ _] := Sum[(2 l + 1) Exp[- $\theta \frac{l(l+1)}{t}$ ], {l, 0, lmax}];

rclassical[t_,  $\theta$ _] :=
  Block[{l}, Integrate[(2 l + 1) Exp[- $\theta \frac{l(l+1)}{t}$ ], {l, 0,  $\infty$ }, Assumptions -> t > 0]];

(*4a*)
Print[rclassical[t, 15]]

(*4b*)
N[Table[(2 l + 1) Exp[-15  $\frac{l(l+1)}{300}$ ], {l, 0, 4}]]
Grid[Table[{l, N[r[300, l, 15]], N[rclassical[300, 15]],
  N[r[300, l, 15] / rclassical[300, 15]]}, {l, 0, 4}]]

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Out[7]= {1., 2.71451, 3.70409, 3.84168, 3.31091}

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0  1.    20.    0.05
1  3.71451 20.    0.185726
Out[8]= 2  7.4186 20.    0.37093
3  11.2603 20.    0.563014
4  14.5712 20.    0.72856

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