

HOMEWORK 5, THERMAL PHYSICS (PHY306)

1. Using the density of states of a gas in volume $g(k)dk = Vk^2dk/2\pi^2$ defined in chapter 21, calculate total number of states below some energy E_{max} , $\Omega(E_{max}) = \int_{E < E_{max}} g(k)dk$. Using it find the temperature from its microcanonical definition $\frac{1}{T} = \frac{d \log(\Omega(E))}{dE}$ and compare it what we get from canonical (Boltzmann) distribution.

2. A gas of rubidium-87 atoms is kept in laser/magnetic trapped at low density $n = 10^{21} m^{-3}$, and then slowly cooled by evaporation. The experiment aims at such low T that the density will be equal to quantum concentration $n_{quantum} = (mk_B T / 2\pi\hbar^2)^{3/2}$. Which temperature would be needed to reach such conditions?

3. D is the heavy isotope of hydrogen, its mass is twice that of H because of extra neutron in the nucleus. To a good approximation electrons do not notice extra neutron, so the electronic ground state energies of molecules H_2, D_2, HD are all the same.

(a) Assuming that r.m.s. distance between atoms in all three cases is the same R , compare their moments of inertia relative to that of H_2 .

(b) Yet the rotational and vibrational (next problem) partition functions are not the same. The proton, a nucleus of H is a fermion, and thus the wave functions should be antisymmetric (multiplied by (-1)) when two H are interchanged. $D = pn$ nuclei is a boson with spin 1. So their interchange should be symmetric (multiplied by (+1)). The rotational wave function after rotation by angle $\pi = 180^\circ$ obtain a factor $(-1)^J$. The nuclear spin part of the wave function have different cases: Out of $2 \times 2 = 4$ spin states of H_2 , 3 are symmetric and 1 antisymmetric under interchange of protons. Out of $3 \times 3 = 9$ spin states of D_2 , 6 are symmetric and 3 antisymmetric. Only odd or even J are allowed by quantum statistics. Using this information, calculate rotational partition functions for all three cases, at $k_B T = (1/2)\hbar^2/2I_{H_2}$.

4.(a) Using condition of maximizing Z for the mixture of H_2, D_2, HD molecules. In doing do note that the reaction between these diatomic molecules $H_2 + D_2 \leftrightarrow 2HD$ conserve number of H and D atoms, so there are two conditions

$$2N_{H_2} + N_{HD} = const, \quad 2N_{D_2} + N_{HD} = const$$

(b) Calculate the combination $(N_{HD})^2/N_{H_2}N_{D_2}$ at temperature T such that

$$\frac{\hbar^2}{2I} \ll k_B T \ll \hbar\omega$$

high compared to rotational parameters for all species (so that classical rotational partition function can be used) and that vibration excitations can be ignored. However vibrational

zero point energy $\hbar\omega/2$ is different for three species. Assuming the oscillator constant of harmonic potential is the same and only the effective mass is different, find the difference in zero point energies and thus the T -dependence of this combination. (Hint: relative motion is described by momenta of each atom adding to zero, resulting in the effective “reduced mass” $1/M = 1/m_1 + 1/m_2$ in kinetic energy.)