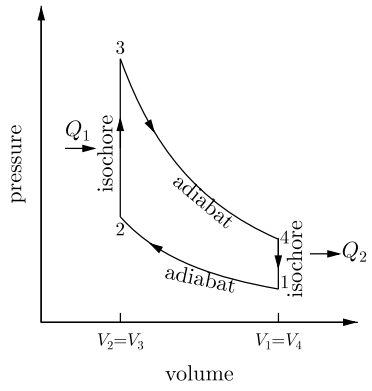
HOMEWORK 6, THERMAL PHYSICS (PHY306)

1. We had shown before that maximizing the entropy

$$S = -k_B \sum_i P_i log(P_i)$$

with 2 conditions ($\sum_i P_i = 1, \sum_i E_i P_i = E$) adding them with Lagrange multipliers, one finds Boltzmann distribution. If we now add the third condition of known particle number $\sum_i N_i P_i = N$ with Lagrange multiplier μ, show that it leads to distribution in a grand canonical ensemble.

2.Weakly salted water – about 1 percent of NaCl by weight – has the same osmotic pressure as blood. (Skin cuts which hurt in fresh water do not do so in a sea.) Estimate its absolute magnitude p_{NaCl} and compare to normal atmosheric pressure $P_0 = 10^5\, Pa$.



3.The Otto cycle is the one used in internal combustion engines of cars: it consists of two adiabates and two isochores (V=const) lines. Consider the volumes V_1, V_2 (and thus their ratio $R_{12} \equiv V_1/V_2$) are known, as well as p_1. Another input is the ratio $p_4/p_1 \equiv R_{14}$
(a) Assuming that the cycle is done with an ideal gas with adiabatic index $\gamma = 5/3$, express temperatures $T_i, i = 1, 2, 3, 4$ and entropies $S_i, , i = 1, 2, 3, 4$ at all corners of the cycle in terms on the input parameters.
(b) The efficiency η is defined, as usual, as work over heat entering $\eta = \frac{W}{Q_{in}}$. Using the results from (a), show that

$$\eta = 1 - R_{12}^{1-\gamma}$$