```
(*  problem 1            *)
ClearAll["Global`*"]


Asun = 4 * Pi * Rsun ^ 2
Psun = σ * Tsun ^ 4 * Asun
Pabsorb = (1 - A) * Psun * RE ^ 2 / 4 / Distance ^ 2
Pradiated = σ * TE ^ 4 * 4 * Pi * RE ^ 2
Assuming[TE > 0 && A < 1, Solve[Pabsorb == Pradiated, TE]]
(* this means power obtained by Earth is all radiated away *)
```

$4\pi\,\mathrm{Rsun}^2$

$4\pi\,\mathrm{Rsun}^2\,\mathrm{Tsun}^4\,\sigma$

$$\frac{(1-A)\,\pi\,\mathrm{RE}^2\,\mathrm{Rsun}^2\,\mathrm{Tsun}^4\,\sigma}{\mathrm{Distance}^2}$$

$4\pi\,\mathrm{RE}^2\,\mathrm{TE}^4\,\sigma$

$$\left\{\left\{\mathrm{TE}\to-\frac{(-1)^{1/4}\,(-1+A)^{1/4}\,\sqrt{\mathrm{Rsun}}\,\mathrm{Tsun}}{\sqrt{2}\,\sqrt{\mathrm{Distance}}}\right\},\left\{\mathrm{TE}\to\frac{(-1)^{1/4}\,(-1+A)^{1/4}\,\sqrt{\mathrm{Rsun}}\,\mathrm{Tsun}}{\sqrt{2}\,\sqrt{\mathrm{Distance}}}\right\},\left\{\mathrm{TE}\to-\frac{(-1)^{3/4}\,(-1+A)^{1/4}\,\sqrt{\mathrm{Rsun}}\,\mathrm{Tsun}}{\sqrt{2}\,\sqrt{\mathrm{Distance}}}\right\},\left\{\mathrm{TE}\to\frac{(-1)^{3/4}\,(-1+A)^{1/4}\,\sqrt{\mathrm{Rsun}}\,\mathrm{Tsun}}{\sqrt{2}\,\sqrt{\mathrm{Distance}}}\right\}\right\}$$

```
Rsun = 7 * 10 ^ 8 * m;
Tsun = 6000 * K;
A = 0.2;
```

$$\mathrm{Tplanet} = \frac{(1-A)^{1/4}\,\sqrt{\mathrm{Rsun}}\,\mathrm{Tsun}}{\sqrt{2}\,\sqrt{\mathrm{Distance}}}$$

```
DistanceV = 1.1 * 10 ^ 11 * m;
DistanceE = 1.5 * 10 ^ 11 * m;
DistanceM = 2.3 * 10 ^ 11 * m;
DistanceJ = 7.8 * 10 ^ 11 * m;
```

$$\frac{1.06159\times10^{8}\,K\,\sqrt{m}}{\sqrt{\mathrm{Distance}}}$$

```
TV = Tplanet /. Distance -> DistanceV
TE = Tplanet /. Distance -> DistanceE
TM = Tplanet /. Distance -> DistanceM
TJ = Tplanet /. Distance -> DistanceJ
```

320.082 K

274.102 K

221.357 K

120.202 K

```
(*  problem 2  *)
(* flux of photons is (1/4) u c =σ T^4 *)

kB = 1.38064852 × 10 ^ (-23) * m ^ 2 * kg / s ^ 2 / K;
hbar = 6.626176 / (2 * Pi) * 10 ^ (-34) * kg * m ^ 2 / s;
c = 3 * 10 ^ 8 * m / s;
σ = Pi ^ 2 * kB ^ 4 / (60 * c ^ 2 * hbar ^ 3)
```

$$\frac{5.66225\times 10^{-8}\ \mathrm{kg}}{\mathrm{K}^4\ \mathrm{s}^3}$$

```
(* this will give energy flux, energy per quantum is about 3*kB*T
 and at Earth distance it is Psun*RE^2/4/Distance^2 *)
photons = Psun / 4 / Pi / DistanceE ^ 2 / (3 * kB * Tsun)
```

$$\frac{6.43061\times 10^{21}}{\mathrm{m}^2\ \mathrm{s}}$$

```
(* now the CMB photons *)
TCMB = 2.7 * K
fluxCMB = σ * TCMB ^ 4 / (3 * kB * TCMB)
```

2.7 K

$$\frac{2.69077\times 10^{16}}{\mathrm{m}^2\ \mathrm{s}}$$

```
(* problem 3 phonons:
   specific heat for Debye model of solid in the book (24.25)
    here xD= ThetaD/T *)
```

```
xDsalt = 320. / 300
xDdiamond = 1860. / 300
CratioSalt = (3 / xDsalt^3) * NIntegrate[x^4 * Exp[x] / (Exp[x] - 1)^2, {x, 0, xDsalt}]
CratioDia =
 (3 / xDdiamond^3) * NIntegrate[x^4 * Exp[x] / (Exp[x] - 1)^2, {x, 0, xDdiamond}]
```

1.06667

6.2

0.945344

0.248568

```
(* problem 4 gas of pions, ultrarelativistic with 3 species :
```

In[56]:=
```
ClearAll["Global`*"]
Z1 = 3 * V / Pi^2 * (1. / hbar / c / β)^3;
logZN = N * Log[Z1] - N * Log[N / E]
U = - (D[logZN, β]) /. β → 1 / (kB * T)
F = -kB * T * logZN /. β → 1 / (kB * T)
p = kB * T * D[logZN /. β → 1 / (kB * T), V]
S = (U - F) / T
Simplify[S]
```

Out[58]= $-N \operatorname{Log}\left[\frac{N}{e}\right] + N \operatorname{Log}\left[\frac{0.303964\, V}{c^3\, \text{hbar}^3\, \beta^3}\right]$

Out[59]= $3.\, kB\, N\, T$

Out[60]= $-kB\, T \left(-N \operatorname{Log}\left[\frac{N}{e}\right] + N \operatorname{Log}\left[\frac{0.303964\, kB^3\, T^3\, V}{c^3\, \text{hbar}^3}\right]\right)$

Out[61]= $\frac{1.\, kB\, N\, T}{V}$

Out[62]= $\frac{3.\, kB\, N\, T + kB\, T \left(-N \operatorname{Log}\left[\frac{N}{e}\right] + N \operatorname{Log}\left[\frac{0.303964\, kB^3\, T^3\, V}{c^3\, \text{hbar}^3}\right]\right)}{T}$

Out[63]= $kB\, N \left(2.80915 - 1.\, \operatorname{Log}[N] + \operatorname{Log}\left[\frac{kB^3\, T^3\, V}{c^3\, \text{hbar}^3}\right]\right)$

```
(* so S=const means V T^3=const,
factor 10 increase  in V means factor 10^(-1/3)  in T *)
```

In[54]:=
```
Tf = 160. / 8^(1 / 3)
Tf / 160
```

Out[54]= 80.

Out[55]= 0.5

```
(* the mean energy 3T is about 200 MeV which is not much larger than the
 pion mass, so we cannot trust ultrarelativistic approximation any more *)
```