```
(*  problem 1     Bose gas in 2 dimensions   ************   *)
ClearAll["Global`*"]
(* the number of non-condensate particles is given by the given integral with μ=
 0. In D=3 it is converegent, and all other particles go into the condensate.
   In D=2 the integral diverges at small p. As suggested in the hint
   it looks at small p as pdp/(p^2/(2m kB T))
   so integral is proporional to log(p_min)→
  Infinity and thus any N can be accomodated without the condensate. *)



(*  problem 2     Bose gas in harmonic trap  *****************
  rescaled momenta and coordinates I call pp and rr
  pp=p/Sqrt[2*mass*kB*T], rr=r*Sqrt[mass*ω^2/2/kB/T]
   the expression at μ=0 gives the critical number Nc
 *)
Nc = Assuming[mass > 0 && T > 0 && kB > 0 && ω > 0,
  Simplify[(4 * Pi / (2 * Pi * hbar) ^ 3) * (2 * mass * kB * T) ^ (3 / 2) *
    (4 * Pi) * (mass * ω ^ 2 / 2 / kB / T) ^ (-3 / 2) * nintegral]]
```

$$\frac{16\, kB^3\, nintegral\, T^3}{hbar^3\, \pi\, \omega^3}$$

```
nintegral =
 NIntegrate[pp^2 * rr^2 / (Exp[pp^2 + rr^2] - 1), {pp, 0, Infinity}, {rr, 0, Infinity}]
0.236023

(* that was the numerical value of the integral,
let me now do the proposed expansion which allows
 to split the integral into the product of two Gaussians *)

nintegral1 =
 NIntegrate[pp^2 * rr^2 / (Exp[pp^2 + rr^2]), {pp, 0, Infinity}, {rr, 0, Infinity}]
nintegral2 = NIntegrate[pp^2 * rr^2 * (Exp[-pp^2 - rr^2] + Exp[-2 * pp^2 - 2 * rr^2]),
  {pp, 0, Infinity}, {rr, 0, Infinity}]
nintegral3 = NIntegrate[pp^2 * rr^2 *
   (Exp[-pp^2 - rr^2] + Exp[-2 * pp^2 - 2 * rr^2] + Exp[-3 * pp^2 - 3 * rr^2]),
  {pp, 0, Infinity}, {rr, 0, Infinity}]
0.19635

0.220893

0.228165
```

```
0.2281
(* good converegence, now the numerical value *)
```

0.2281

```
hbar = 6.626176 / (2 * Pi) * 10 ^ (-34) * kg * m^2 / s;
kB = 1.38064852 × 10 ^ (-23) * m^2 * kg / s^2 / K;
T = 0.5 * 10 ^ (-6) * K;
ω = 10^3 / s;
Nc
```

337 159.

```
(* indeed of the order of number of atoms used,
start with 10^6 and cool by evaporation  *)
```

```
(*  problem 3    white dwarf *****************
ClearAll["Global`*"]
Kin = Ntot^(1 + 2 / 3) * hbar^2 / 2 / me / (4 / 3 * Pi * R^3)^(2 / 3)
Pot = -GN * Ntot^2 * mp^2 / R
DTot = Assuming[R > 0, Simplify[D[Kin + Pot, R]]]
```

$$\frac{3^{2/3}\,\text{hbar}^2\,\text{Ntot}^{5/3}}{4\times 2^{1/3}\,\text{me}\,\pi^{2/3}\,\left(R^3\right)^{2/3}}$$

$$-\frac{\text{GN}\,\text{mp}^2\,\text{Ntot}^2}{R}$$

$$-\frac{\text{hbar}^2\,\text{Ntot}^{5/3}\left(\frac{6}{\pi}\right)^{2/3}-4\,\text{GN}\,\text{me}\,\text{mp}^2\,\text{Ntot}^2\,R}{4\,\text{me}\,R^3}$$

```
Solve[DTot == 0, R]
```

$$\left\{\left\{R\to\frac{3^{2/3}\,\text{hbar}^2}{2\times 2^{1/3}\,\text{GN}\,\text{me}\,\text{mp}^2\,\text{Ntot}^{1/3}\,\pi^{2/3}}\right\}\right\}$$

```
hbar = 6.626176 / (2 * Pi) * 10 ^ (-34) * kg * m ^ 2 / s;
me = 9.1 * 10 ^ (-31) * kg;
mp = 1.67 * 10 ^ (-27) * kg;
GN = 6.67 * 10 ^ (-11) * (kg * m / s ^ 2) * m ^ 2 / kg ^ 2;
MSun = 1.989 * 10 ^ (30) * kg;
RSun = 695 * 10 ^ 6 * m
Ntot = MSun / mp
```

695 000 000 m

1.19102×10^{57}

$$\text{Radius} = \frac{3^{2/3}\,\text{hbar}^2}{2 \times 2^{1/3}\,\text{GN}\,\text{me}\,\text{mp}^2\,\text{Ntot}^{1/3}\,\pi^{2/3}}$$

2.38525×10^{6} m

```
density = Ntot / (4 * Pi / 3 * Radius ^ 3)
densSun = Ntot / (4 * Pi / 3 * RSun ^ 3)
```

$$\frac{2.09521 \times 10^{37}}{\text{m}^3}$$

$$\frac{8.46984 \times 10^{29}}{\text{m}^3}$$

```
(* compare it to Sun radius  it is a real dwarf! *)


(* problem 4, neutron star
 the only difference is that Fermi energy
 is not that of electrons but neutrons, so ne→np   *)
```

$$\text{RadiusNS} = \frac{3^{2/3}\,\text{hbar}^2}{2 \times 2^{1/3}\,\text{GN}\,\text{mp}^3\,\text{Ntot}^{1/3}\,\pi^{2/3}}$$

1299.75 m

```
(* radius smaller by mp/me=1800, density larger by 1800^3
 the formulae used are crude, the real radius is about 10 times that
*)
```