HOMEWORK 9, THERMAL PHYSICS (PHY306)

1. Consider non-relativistic Bose gas is $two - dimensions$, with the nonzero momenta p_x and p_y (no p_z). Its sum over states reads, in polar coordinates in momenta

$$N = \int \frac{2\pi p dp V_2}{(2\pi\hbar)^2} \frac{1}{exp\big[(p^2/2m - \mu)/k_B T\big] - 1}$$

where V_2 is the 2-dimensional volume (area). Show that, in contrast to the three-dimensional case we studied before, it does *not* have Bose-Einstein condensation at any temperatures. (Hint: consider the integrand at small momenta.)

2.N Bose atoms are placed in a spherically symmetric harmonic three-dimensional trap, in which their classical particle energy is

$$E(p,r) = \frac{p^2}{2m} + \frac{m\omega^2}{2} r^2$$

and the sum over states can be approximated by integral over the "phase space" (momenta $\vec{p}$ and coordinates $\vec{r}$ with the Bose occupation factor

$$N = \int\int \frac{d^3p d^3r}{(2\pi\hbar)^3} \frac{1}{exp\big[(E(p,r) - \mu)/k_B T\big] - 1}$$

Find the critical temperature for Bose-Einstein condensation. (Hint: rescale momenta and coordinates p, r in a way making the dimensionless double integral. Estimate it using expansion of the denominator of the type $1/[exp(f) - 1] = e^{-f} + e^{-2f} + ...$ with each term being a product of Gaussian integrals. Calculate the critical number N_c if the $\omega = 10^3\, rad/s$ and $T = 0.5 * 10^{-6} K$.

3.(a) Using for kinetic and gravitational energies of the white dwarf star simplified expressions

$$E_K \sim N \frac{\hbar^2 (N/V)^{2/3}}{2m_e}, \;\; E_G \sim -\frac{G_N M_{star}^2}{R}$$

where m_e is the mass of the electron and $V = (4\pi/3)R^3$ is the star volume. Find the star radius R_{min} at which the total energy $E_K + E_G$ is minimal.

(b) Sirius B is the second white dwarf discovered, with the mass close to that of the Sun $M_{Sun} \approx 2 * 10^{30} kg$. Evaluate the number of protons N (assuming Sun is made of only hydrogen, no neutrons, so N is also the number of electrons), its radius R and particle number density N/V. Compare it to the Sun's radius $R_{Sun} = 695 \cdot 10^6\, m$ and the Sun's mean number density N/V_{Sun}.

(c) Similar treatment as in problem 3 for white dwarfs is possible for the neutron star, with the obvious substitution $m_{electron} \to m_{neutron}$ in the kinetic energy. What would be the radius of the neutron star with the same mass $M = M_{Sun}$? Compare the neutron star density N/V to that of nuclei, $n_{nucl} \sim 0.16\, fm^{-3}$.