Short History of Nuclear Many Body Problems

Gerry Brown in memory

Stony Brook November 24, 2013

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- I find history of physics (almost) as interesting as physics itself. History involves ideas and people behind the ideas often not found in textbooks or published papers. The real physics is of course what experiments reveal to us. The human brain seeks to understand the phenomena and that is what theorists are trying. Theory is the subject of my talk.
- When does history of nuclear many body problem start? One of the greatest discoveries was the nuclear shell-model. Liquid drop was the picture theorists had in mind. Experiments showed nuclear spectra looking like atomic. So how can one explain the shell-model? Another problem: nuclear saturation. An explanation: N-N interaction repulsive at short distances, Jastrow. But how reconcile the strong interactions with a shell-model. These were the problems some 60 years ago.

I first met Gerry at least 55 years ago but I never worked directly with him but interacted with him in various ways over the years. In 1959 he was my opponent at my PHD defense in Uppsala. Last time we met was, I believe, at Osnes' retirement in Oslo (2008). He told me after my talk that I should have "spruced it up". I'll try today.

He was a good friend.

Gerry had many collaborators not only among his own students. He was always able to make others interested in problems he considered important. That was one of his strengths.
Most of his publications were with co-authors.
It is not possible to cover more than a small fraction of his work on many body physics in a short talk.

One of the great discoveries in Nuclear Theory was the Nuclear Shell Model. (Nobel 1963)
How could it be understood knowing that the NN-forces are strong, consistent with Liquid drop models.
Another unsolved problem: Nuclear Saturation.

The stage was set for someone to come up with a many-body theory of nuclear structure.
Gerry Brown’s (and other’s) nuclear structure work was based on the Brueckner theory.
I will review this theory briefly.

Related to the shell-model is the optical model from the 50's, which pictured nucleons moving in a mean field. It was successfully explained by Watson as a $\boxed{\text{multiple scattering problem with elementary scatterings being via T-matrices.}}$

This idea was picked up by Brueckner. Maybe a nuclear many body theory for bound states could be built on the T-matrix, instead of a NN-potential interactions.

But the T-matrix is complex

$$\boxed{T = v + v\frac{1}{k^2 - {k'}^2 + i\eta}T \sim e^{i\delta}\sin\delta}$$

It seemed to make sense to instead use the Reactance matrix (R-matrix) which implies a principal value integration

$$\boxed{R \sim \tan\delta}$$

replacing the interaction potential with an "effective" interaction

$$\boxed{V(k) \sim \tan\delta(k)}$$

This idea had some degree of success.
BUT, the R-matrix refers to a scattering problem with boundary problems different from that of a bound state. It is fairly easy to show that putting two particles in a box, square or Harmonic oscillator (Busch) the binding energy is

$$\boxed{B.E. \sim \delta}$$

$$not \sim \tan\delta$$

In the scattering problem one has a continuum set of states but in the bound state problem one has a discrete set of states.

“Infinite” nuclear matter still implies a bound state problem. Summation over a $\boxed{\text{discrete}}$ set of states no matter how dense is different from integration over a $\boxed{\text{continuum}}$. (de Witt, Watson, Newton,1956).

The difference between δ and $\tan\delta$ is of course small for small δ. With large scattering lengths and $\delta = \pi/2$ it does make a big difference.

The Busch formula expresses the binding energy of two nucleons in an oscillator well in terms of phase-shifts.I recently showed that the SHIFT in energy in this case is also given by δ. (Arxiv 2011)

Two Particles in a Trap

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Abstract

The Busch-formula relates the energy-spectrum of two point-like particles interacting in a 3-D isotropic Harmonic Oscillator trap to the free scattering phase-shifts of the particles. This formula is used to find an expression for the *shift* in the spectrum from the unperturbed (non-interacting) spectrum rather than the spectrum itself. This shift is shown to be approximately $\Delta = -\delta(k)/\pi \times dE$, where dE is the spacing between unperturbed energy levels. The resulting difference from the Busch-formula is typically $< \frac{1}{2}\%$ except for the lowest energy-state and small scattering length when it is 3%. It goes to zero when the scattering length $\rightarrow \pm\infty$.

The energy shift Δ is familiar from a related problem, that of two particles in a spherical infinite square-well trap of radius R in the limit $R \rightarrow \infty$. The approximation is however as large as 30% for finite values of R, a situation quite different from the Harmonic Oscillator case.

The square-well results for $R \rightarrow \infty$ led to the use of in-medium (effective) interactions in nuclear matter calculations that were $\propto \Delta$ and known as the *phase shift approximation* . Our results indicate that the validity of this approximation depends on the trap itself, a problem already discussed by DeWitt more than 50 years ago for a cubical vs spherical trap.

1 Introduction

The $\delta -$ (phase-shift) approximation of the effective interaction is good if medium effects can be neglected.

This is true at low density AND for 'weak' interactions for example large angular momenta, $l \cong 4$ or larger.

What about medium, many body effects. We deal with a fermion-system. The summation over intermediate states cannot include occupied states. So modified effective interaction:

$$\boxed{K = v + \sum v \frac{Q}{k^2 - k'^2} K}$$

This was the **second** Brueckner approximation.
Note that K now is real. No integration over a pole.
No discrete-continuum controversy.
(Problem at fermi-surface. BCS.)

Mean Field Correction

But Brueckner then realized that nucleons move in a mean field U(k),

consistent with the shell-model,

so that energies would be not

$e(k) = k^2$

but rather

$$\boxed{e(k) = k^2 + U(k)}$$

Result: $\boxed{\text{Brueckner Reaction Matrix:}}$

$$\boxed{K = v + \sum v \frac{Q}{e(k)-e(k')} K}$$

Total energy (first order):

$$\boxed{E_T = \sum k^2 + \frac{1}{2} \sum K}$$

Mean field:

$$\boxed{U(k) = \sum K}$$

$\boxed{\text{Brueckner self-consistency.}}$

What has been achieved?
The interaction $\boxed{v,}$ with a strong short-ranged repulsion has been replaced by a 'smooth' effective interaction, the Reaction matrix $\boxed{K.}$

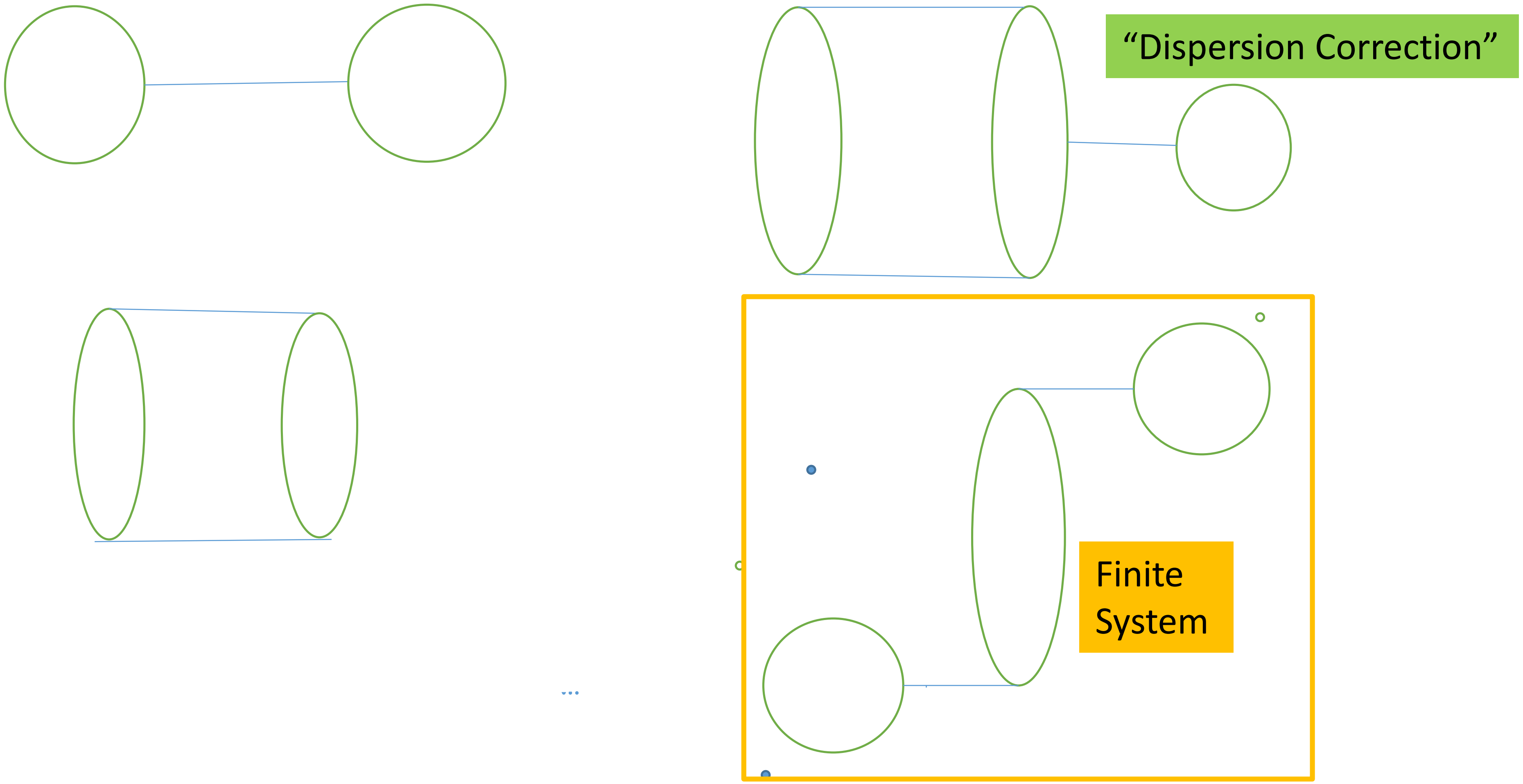
Two modifications of the T-matrix were made
1. Pauli-operator
2. Mean field.

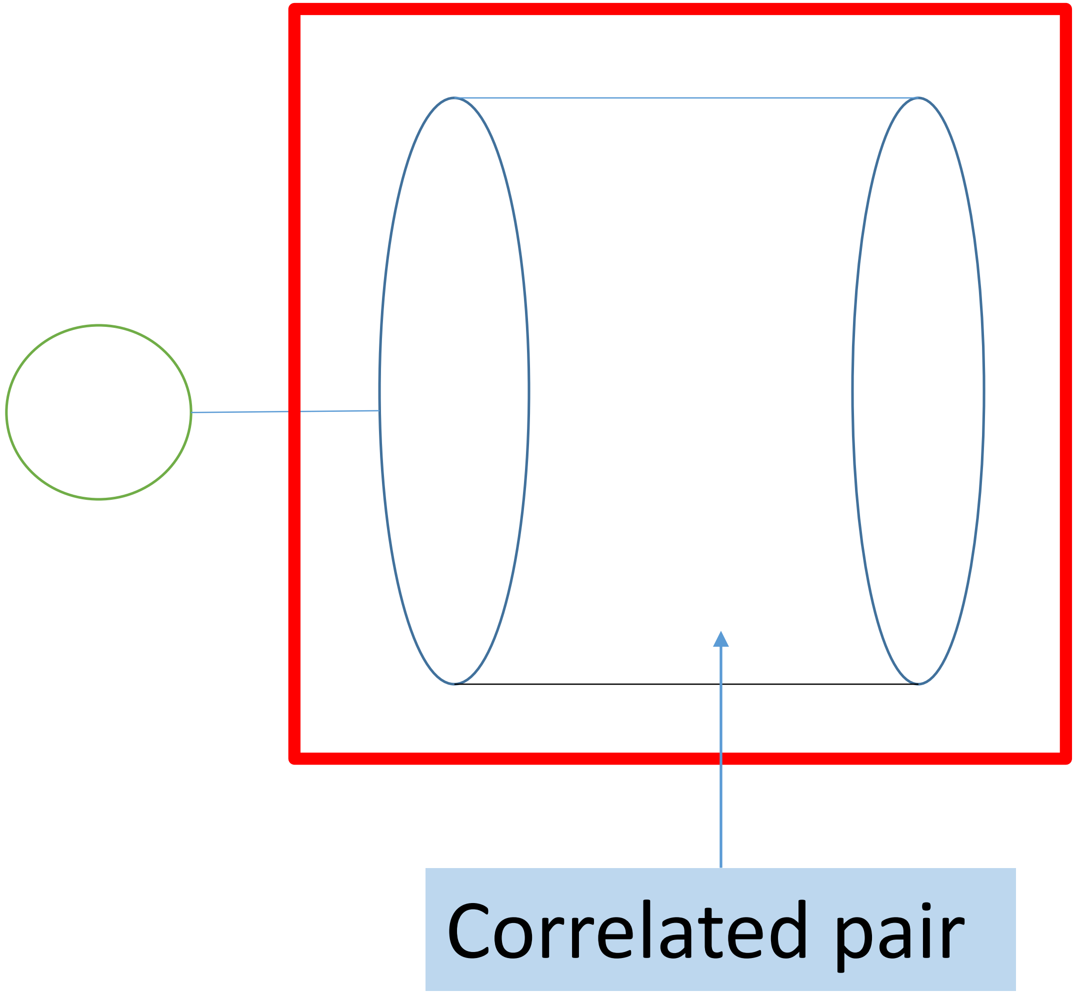
The K-matrix sums ladder and mean-field propagations to all orders. Infinite nuclear matter calculations show saturation and binding energy remarkable well. Important physics is included in this first order in K approximation. Improved results can (in principle) be obtained by higher orders.

It is a zero-width approximation. Spectral widths are included Green’s function calculations but show little difference in calculated values.

Calculations by Brueckner and coworkers for infinite nuclear matter as well as finite nuclei were very promising.
Binding energies and saturation properties were remarkably well reproduced suggesting that important physics was included.
Other calculations were made also including higher order terms.

Typical Energy-diagrams included in first order K-matrix calculation




Correlated pair

An important paper on nuclear matter was Brown Schappert and Wong in 1964.
Gerry was also much interested in nuclear matter and compressibility in his work with Hans Bethe on supernova explosions.

BINDING ENERGY OF NUCLEAR MATTER

G. E. BROWN † and G. T. SCHAPPERT

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and

C. W. WONG

Harvard University

and

University of Minnesota

Received 17 February 1964

Abstract: Various calculations of the binding energy of nuclear matter are discussed. The question of off-the-energy shell propagation is considered in detail. Numerical approximations made in the calculations are investigated and it is shown that they are such as to give too much binding energy, so that the present discrepancy between theoretical and experimental values is worsened.

A SIMPLIFIED EQUATION OF STATE NEAR NUCLEAR DENSITY

H.A. BETHE*

W.K. Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California 91125, USA

G.E. BROWN** and J. COOPERSTEIN**

Physics Department, State University of New York, Stony Brook, New York 11794, USA

and

JAMES R. WILSON***

Lawrence Livermore National Laboratory, Livermore, California 94550, USA

Received 14 December 1982

Abstract: The equation of state near nuclear density influences shock formation in stellar collapse supernovae. The drop in the adiabatic index below $\frac{4}{3}$ in this region, due to the negative nuclear pressure, disturbs the homology of the inner core and decreases its size. The initial shock energy and formation dynamics are particularly sensitive to matter in this regime.

Only matter at low entropies ($S \leqslant 1.5$) in the unshocked inner core approaches nuclear densities. We derive a simple equation of state for this material and find that nuclear properties are close to those at $S = 0$. The entropy associated with the nuclear surface can be absorbed into an "effective

1.E.2

Nuclear Physics **A99** (1967) 65—82; © *North-Holland Publishing Co., Amsterdam*

CALCULATION OF ^{16}O BINDING ENERGY

R. J. McCARTHY and H. S. KÖHLER

Rice University, Houston, Texas †

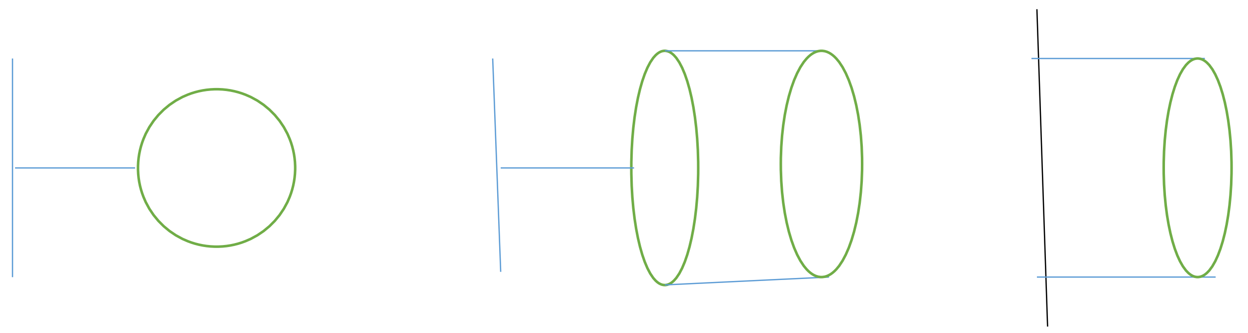
Received 13 February 1967

Abstract: The binding energy of ^{16}O has been calculated for the Hamada-Johnston and Brueckner-Gammel-Thaler potentials using an approximation scheme similar to the reference spectrum method. A first-order approximation to the *K*-matrix is obtained by neglecting both the Pauli principle and the potential energy in intermediate states. The only correction term calculated is that due to the Pauli principle but the method can be extended to include a more general energy spectrum. Harmonic oscillator wave functions are used to describe the single-particle orbitals and a self-consistent calculation of the *K*-matrix elements is made for several values of the oscil-

From the energy diagrams the mean field and effective interaction diagrams are obtained by first and second order functional derivatives

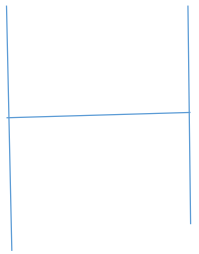
$$\boxed{U(i) = \frac{\partial E}{\partial i}}$$ and $$\boxed{V_{eff}(i,j) = \frac{\partial\ E}{\partial i \partial j}}$$

Mean-field diagrams

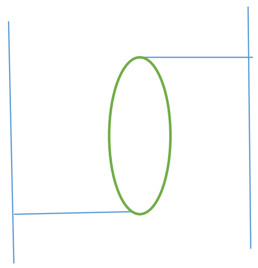


Effective Interaction

First order



Core polarisation



1.B

Nuclear Physics **74** (1965) 234—240; © *North-Holland Publishing Co., Amsterdam*

ROLE OF CORE POLARIZATION IN TWO-BODY INTERACTION

GEORGE F. BERTSCH †

Palmer Physical Laboratory, Princeton, New Jersey ††

Received 28 May 1965

Abstract: The correction to the interaction of two valence particles in ^{18}O and ^{42}Sc, due to perturbations of the closed shell wave functions, is calculated and found to be as large as 30 % of the first-order interaction. The qualitative behaviour of this interaction is: attractive for $T = 1$,

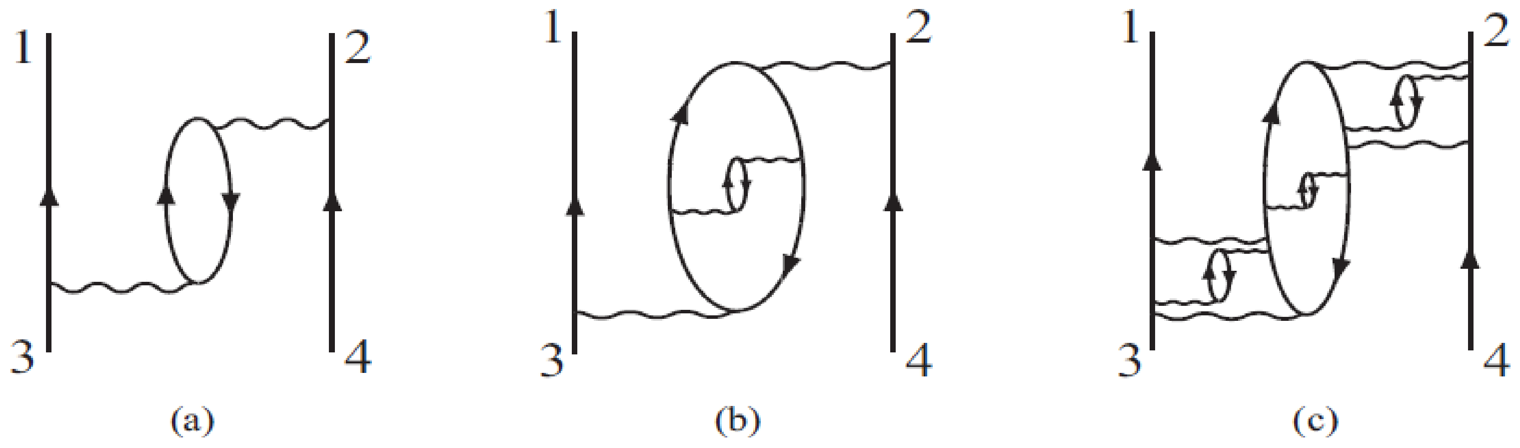


Figure 1. Second- and higher-order core polarization diagrams.

STRUCTURE OF FINITE NUCLEI AND THE FREE NUCLEON-NUCLEON INTERACTION

Kuo-Brown interaction

An Application to ^{18}O and ^{18}F

T. T. S. KUO and G. E. BROWN

Palmer Physical Laboratory, Princeton University, Princeton, New Jersey †

Received 4 March 1966

Abstract: The intention of this work is to investigate the applicability of the free nucleon-nucleon potential determined by the scattering data in the shell-model description of finite nuclei. The potential is chosen to be that of Hamada and Johnston. We have chosen ^{18}O and ^{18}F as our first numerical calculations. A major part of the work reported here concerns the evaluation of the shell-model reaction matrix elements. They are evaluated using the separation method for the singlet-even and triplet-even states and the reference spectrum method for the singlet-odd and triplet-odd states. The second-order Born term for the triplet-even tensor force is found to be very important. It can be calculated conveniently and with good accuracy using the closure

Barrett and Kirson (N.P. A148 (1970)145)
questioned the convergence.
Anastasio et al (N.P. A271(1976) 109
showed the influence of the shell-model potential.

2.1. THE SEPARATION METHOD

The separation method was first introduced by Moszkowski and Scott for evaluating the reaction matrix elements in nuclear matter [25,26]. The idea is to divide the potential V into two parts, the short-range part V_s and the long-range part V_L. Namely

$$V = V_s\theta(d-r)+V_L\theta(r-d), \tag{2.16}$$

where $\theta(x)$ is the step function which equals to one if $x > 0$ and zero otherwise. Roughly speaking, the separation distance d is chosen so that the attractive part of V_s balances the repulsive core. Then what remains is essentially V_L. Let us choose the approximate reaction matrix and wave operator as

$$G_s = V_s - V_s \frac{1}{e_A} G_s\,, \tag{2.17}$$

Kuo-Brown paper

ANNALS OF PHYSICS: 11, 65–115 (1960)

Nuclear Forces and the Properties of Nuclear Matter*

S. A. MOSZKOWSKI AND B. L. SCOTT

University of California, Los Angeles, California

The application of the Brueckner theory to the nuclear many-body problem can be greatly simplified if one separates the two-nucleon interaction (for any given state and relative momentum) into a short range part v_s and a long range part v_l . The cut is made such that v_s alone gives no phase shift for free particle scattering. If the separation is made in this way, then the

which gives

Dispersion correction, 3-body term

$$\Delta t_{00}(D) = 2\Delta U I_D\,, \tag{II-62}$$

where

Wound-integral

$$I_D = \int (\Psi^s - 1)^2\, d\mathbf{r} \tag{II-63}$$

for a pair with zero relative momentum.

We sum over the Fermi sea, making the same assumptions as in the calculation of the Pauli correction. Then:

$$\frac{\Delta E(D)}{A} = \frac{k_f^{\,3}}{4\pi^2}\,\Delta t_{00}(D), \tag{II-64}$$

ANNALS OF PHYSICS: **16:** 375–386 (1961)

On the Separation Method for Calculating the Nuclear Reaction Matrix*

H. S. Köhler†

University of California, Los Angeles, California

The separation method for calculating the nuclear reaction matrix that was presented by Moszkowski and Scott (*1*) has been investigated. Estimates are made which suggest that their dispersion, interference, and Pauli corrections are underestimated. A modification in the treatment of these terms is presented that gives considerable improvement. Our treatment also permits good insight into the nature of the approximations. The errors in the new method are dis-

The estimates that we have made in this paper are only rough but it seems justified to make more exact calculations following the suggestions in this paper.[7]

We summarize the approximation for G^N that we have derived

$$\boxed{\begin{aligned} G^N = {} & G_s^F + v_l + (\Omega_s^F - 1)e(Q-1)(\Omega_s^F - 1) \\ & + (\Omega_s^F - 1)(e_0 - e)(\Omega_s^F - 1) + 2v_l Q(\Omega_s^F - 1) + v_l(Q/e)v_l\,. \end{aligned}} \tag{23}$$

The separation distance should be chosen so that G_s^F is small or zero. Still better results would be obtained by following the somewhat more elaborate approximation scheme in the preceding chapter.

Finally we wish to point out that the value of an approximation of the kind presented in this paper is not only in its accuracy but also in the better understanding of the problem that it makes possible. A treatment of desired accuracy can nowadays be achieved with enough effort of an electronic computer.

“Exact results by Monte Carlo.

ACKNOWLEDGEMENTS

A many body problem is always a two-part problem:
1. Interactions between particles e.g. 2-,3- etc interaction potentials.
2. A many body theory.

The theory of nuclear forces has been a long-standing problem. (Machleidt).It is easy to construct potentials that fit NN phase-shifts e.g. by inverse scattering and separable potentials. But that is in general not enough. Off-shell scattering information is needed in the many-body system. This was emphasized already in the 1964 paper by G E Brown,Schappert and Wong.

BINDING ENERGY OF NUCLEAR MATTER

G. E. BROWN † and G. T. SCHAPPERT
Department of Physics, Massachusetts Institute of Technology
and
C. W. WONG
Harvard University
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Received 17 February 1964

Abstract: Various calculations of the binding energy of nuclear matter are discussed. The question of off-the-energy shell propagation is considered in detail. Numerical approximations made in the calculations are investigated and it is shown that they are such as to give too much binding energy, so that the present discrepancy between theoretical and experimental values is worsened.

1. Introduction

Many calculations of the binding energy of nuclear matter have now been carried out. Since each is done with different methods, they cannot be cross-checked in detail, It can however be argued that for low-energy nuclear problems the high energy component of the interaction should be irrelevant. The low and high energy components are separated in The Moszkowski-Scott separation method shown earlier. The effect of high energy (short-ranged) correlations was contained as a correction: The ‘dispersion term’, that is proportional to the product of ‘correlation volume’ and the mean field.

A comparison with $\boxed{V_{low\ k}}$ is of interest.

This is $V_{low\ k}$

Low momentum nucleon-nucleon potential and shell model effective interactions

Scott Bogner[1], T. T. S. Kuo[1], L. Coraggio[2], A. Covello[2] and N. Itaco[2]

[1] *Department of Physics, SUNY, Stony Brook, New York 11794, USA*

[2] *Dipartimento di Scienze Fisiche, Università di Napoli Federico II and Istituto Nazionale di Fisica Nucleare, I-80126 Napoli, Italy*

(February 9, 2008)

A low momentum nucleon-nucleon (NN) potential V_{low-k} is derived from meson exchange potentials by integrating out the model dependent high momentum modes of V_{NN}. The smooth and approximately unique V_{low-k} is used as input for shell model calculations instead of the usual Brueckner G matrix. Such an approach eliminates the nuclear mass dependence of the input interaction one finds in the G matrix approach, allowing the same input interaction to be used in different nuclear regions. Shell model calculations of ^{18}O, ^{134}Te and ^{135}I using the same input V_{low-k} have been performed. For cut-off momentum Λ in the vicinity of 2 fm^{-1}, our calculated low-lying spectra for these nuclei are in good agreement with experiments, and are weakly dependent on Λ.

21.60.Cs; 21.30.Fe; 27.80.+j

Towards a model-independent low momentum nucleon-nucleon interaction

$V_{low\ k}$

S.K. Bogner[a,1], T.T.S. Kuo[a,2], A. Schwenk[a,3],
D.R. Entem[b] and R. Machleidt[b]

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Abstract

We provide evidence for a high precision model-independent low momentum nucleon-nucleon interaction. Performing a momentum-space renormalization group decimation, we find that the effective interactions constructed from various high precision

It is to be expected that high-momentum cut-offs would not affect nuclear structure results.
Compare the cut-off in coordinate space in the separation method.
The near equivalence of V_{low-k} and the MS separation method was shown by J W Holt and Gerry Brown.

A more fundamental approach is EFT originated by Weinberg. (Phys. Lett. 251B (1990) 288.) Not surprisingly, Gerry Brown was consulted.

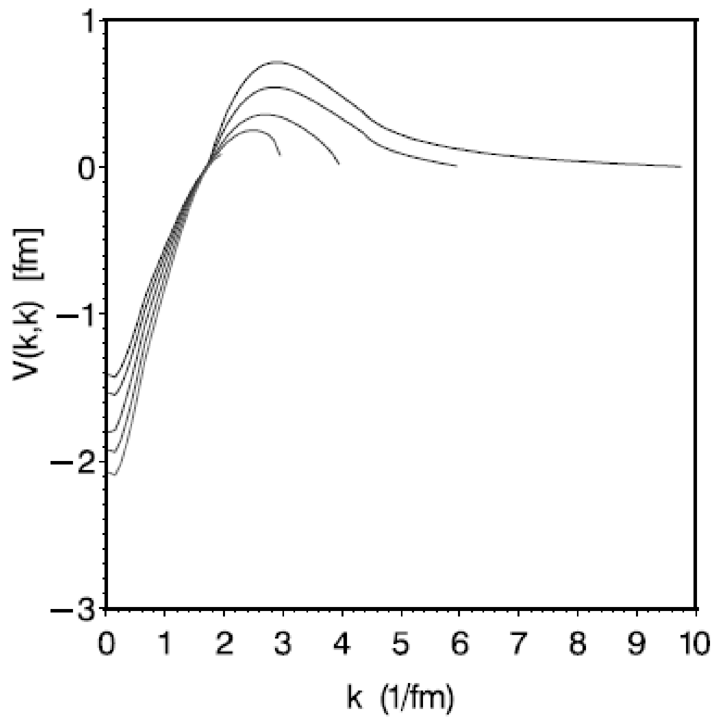
Effect of momentum cut-off

Binding energies in singlet and triplet states will be shown below.
Separable potentials are calculated by inverse scattering using the experimental phase-shifts and the Deuteron parameters as the only input.

Results of Brueckner calculations are presented as a function of the cut-off in momentum-space.

One finds that the diagonal elements in momentum-space of these potentials (Singlet-S will be shown) are functions of the cut-off although fitted to the same input.

The potentials are of course in themselves meaning-less in the sense of physics as they are not observables.



Separable Potential as a function of cutoff

Potential energies as a function of cut-off

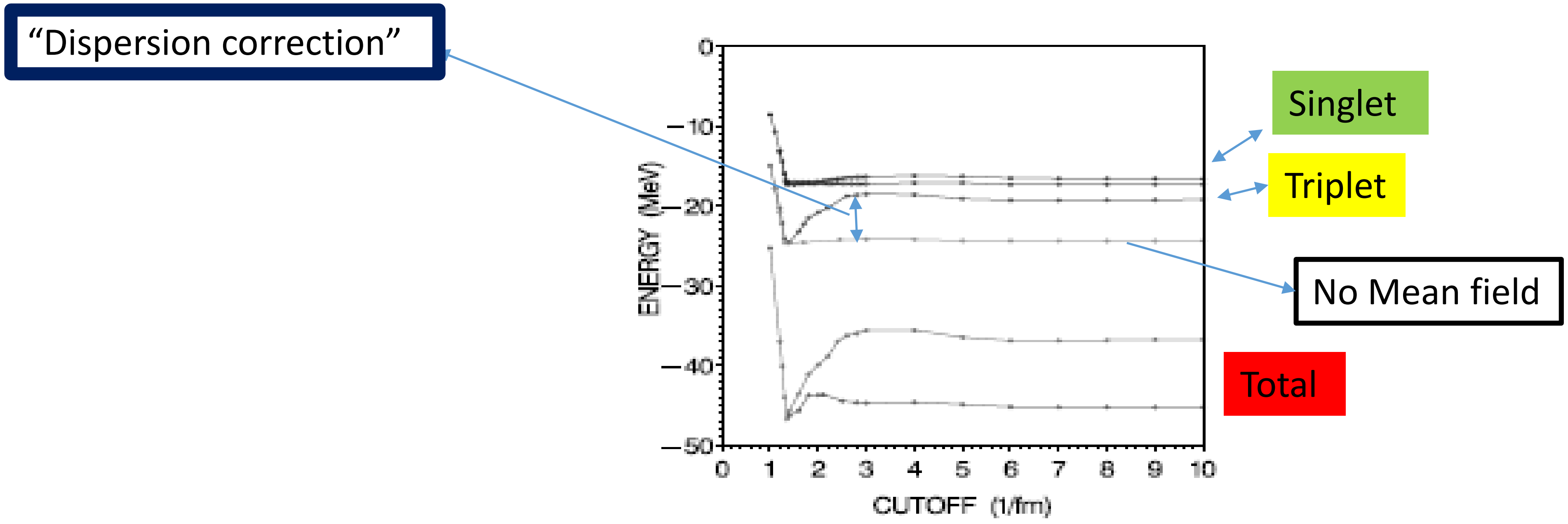


FIG. 1. Effects of the selfconsistent mean field (dispersion-correction). There are three sets of curves. The uppermost set shows the contribution to the potential energy per particle from the $^1S_0$ state, the middle from the $^3S_1$ and the bottom includes all (21) states. In each set of curves the lower curve is without the mean field $U(k)$ while the upper is with $U(k)$ included in the calculation. The difference between these two curves is the dispersion correction, which is seen to decrease as the cutoff Λ decreases below $\Lambda \sim 3.0\,fm^{-1}$ and approaches zero as $\Lambda \to k_F = 1.35\,fm^{-1}$.

Fig. 3 shows the importance of the dispersion correction in providing saturation in a Brueckner calculation of the binding energy. The separable interaction without any cut-off is used here. The upper curve is the full Brueckner calculation, while in the lower the selfenergy $U(k)$ is neglected so that the only many-body effect comes from the Q-operator..

The effect of the high momentum cut-off is further illustrated by Fig. 4. With $\Lambda = 9.8\,fm^{-1}$ the phase-shifts for all

$\Lambda = 9.8$

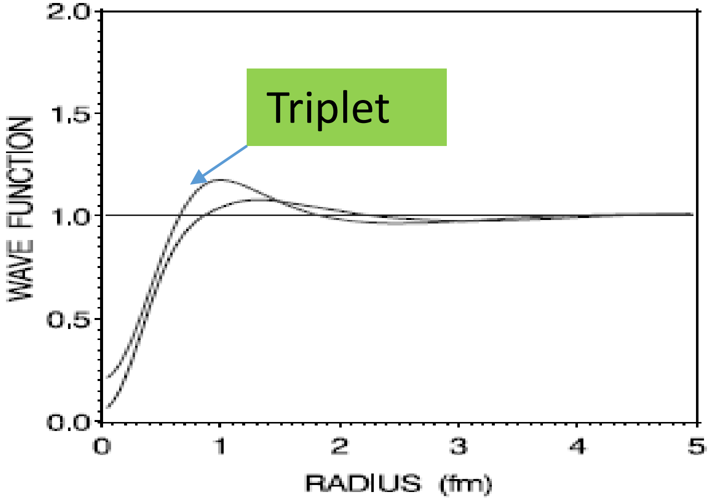


Figure 5: The straight line is the uncorrelated wavefunction Φ at $k = 0$. The lower curve shows the correlated $^1S_0$ while the upper is the correlated $^3S_1$ wavefunction Ψ for a relative momentum $k = 0$, a center of mass momentum $P = 0$ and cut-off $\Lambda = 9.8 fm^{-1}$. Note the 'healing'. For the singlet case this gives a $\kappa = .021$ and for the triplet one gets $\kappa = .029$ For small radius $\Psi \rightarrow 0$ and this is evidence of a short-ranged repulsion.

2.2 Depletion factor κ

Singlet and triplet NN-correlations

$\Lambda = 2$

No correlation
No wound
No repulsion

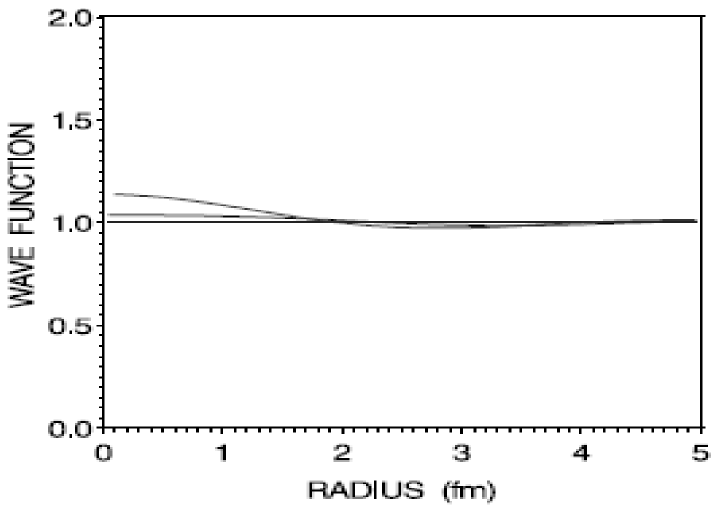


Figure 8: The straight line is the uncorrelated wavefunction Φ at $k = 0$. The lower curve shows the correlated $^1S_0$ and the upper the correlated $^3S_1$ wavefunction Ψ for a relative momentum $k = 0$, center of mass momentum $P = 0$ and $\Lambda = 2.0$. Compare with the singlet case in Fig. 5 for $\Lambda = 9.8$. Here $\kappa = .015$ for the $^1S_0$ less than the value for $\Lambda = 9.8$ consistent with the independently calculated average value of κ for shown in the Table below. Compare also with the triplet case in Fig. 5 for $\Lambda = 9.8$. In this case $\kappa_{ss} = .013$ There is no evidence of a short-ranged repulsion for this value of $\Lambda = 2.0$.

Singlet and triplet correlations

In V_{low-k} one seeks a minimal momentum cut-off Λ.

From the results above one would conclude that $\Lambda > 3\ fm^{-1}$ is necessary, otherwise the correlations, the dispersion term is lost.

END HISTORY

BEGIN FUTURE

Predictive power! Not only reproduce known experimental data.

EFT
DFT?
No Core
Skyrme $\rightarrow$ non-local interaction

Microscopic models

Computers

Mass Formula

Macroscopic model

High Density

Tensor component

Quantum Transport