From Nuclear Structure to Nucleon Structure

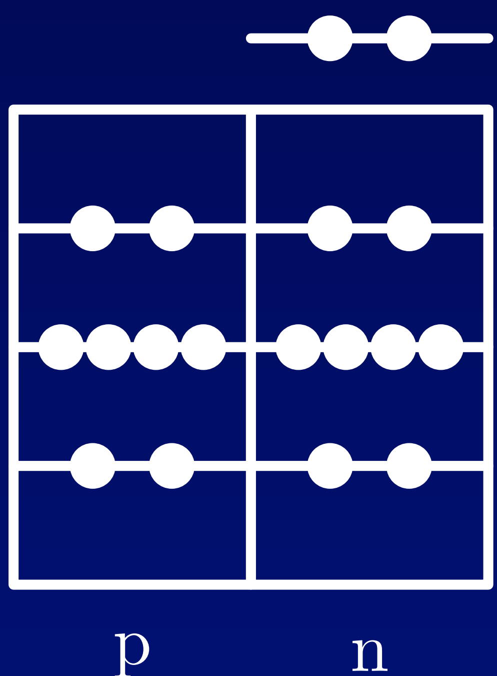
- Analogy between nuclear structure and nucleon structure
  - Collectivity
  - Z graphs and connected sea partons
  - Core polariation vs. vacuum polarization of the disconnected sea
- Proton spin components from lattice QCD

A Tribute to Gerald E. Brown
Stony Brook, Nov. 24, 2013

Nucleus

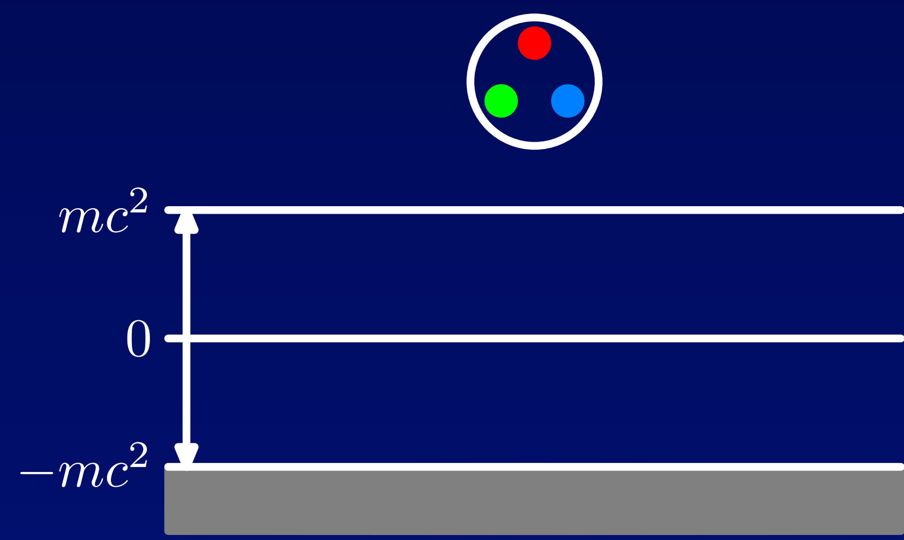
Many-Body Theory
Fermi surface at the last filled shell

^{18}O



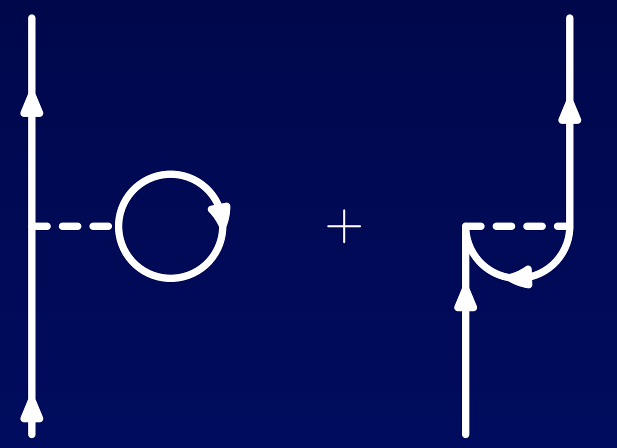
Nucleon

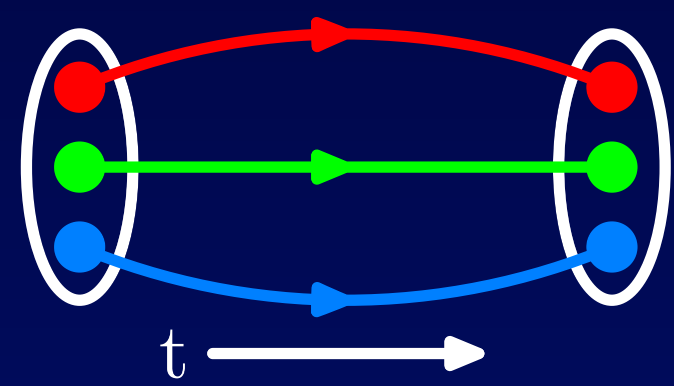
Quantum Chromodynanmics
Dirac sea is filled



Nucleus

Nucleon





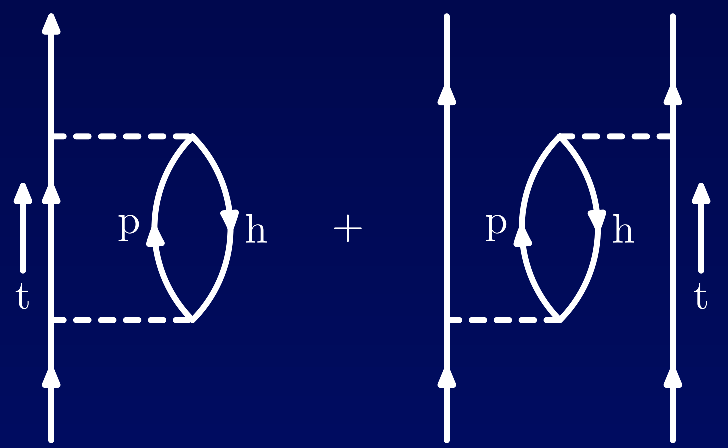
Quark Model
Syrmion (chiral soliton)
Instanton Liquid Model
Chiral Perturbation Theory

Shell Model
Mean-field Approach
Hatree-Fock, variation…

$$< T[O_1(A,\overline{\Psi},\Psi)O_2(A,\overline{\Psi},\Psi)...] >= \frac{1}{Z}\int DA_\mu\, e^{-S_G} \det M\; Tr\{M^{-1}M^{-1}...A...\}$$

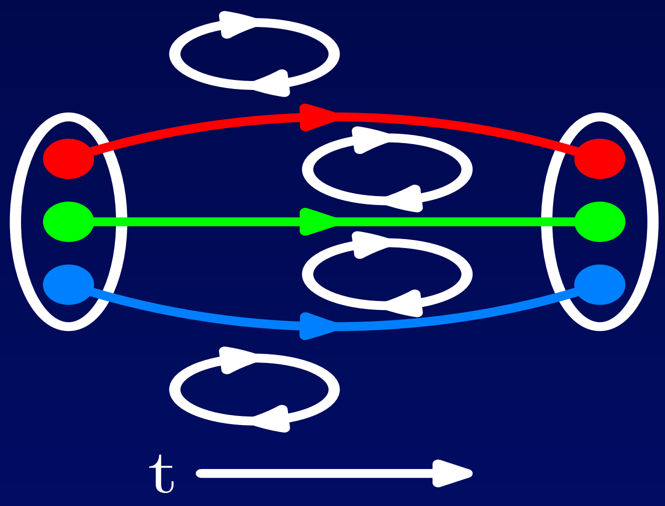
Path-integral with Quenched Approximation
(det M = constant)

Nucleus



Lee-Yang Mean Field
Folded Diagrams
Green's Function MC
Lattice Effective Theory

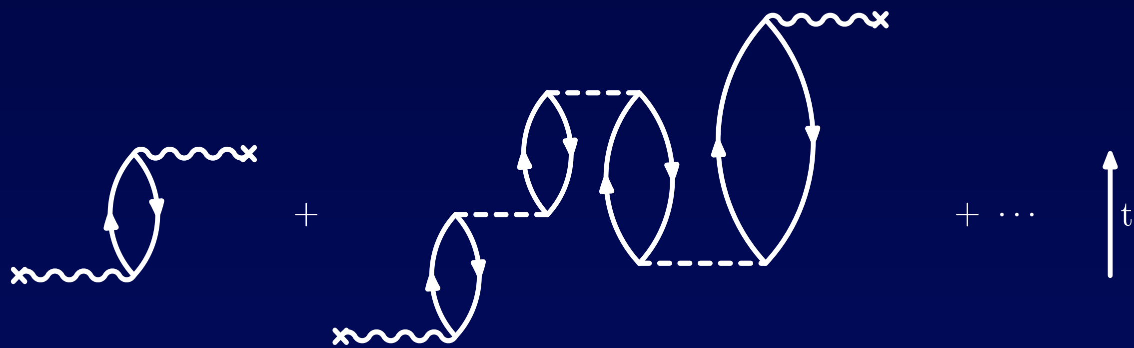
Nucleon



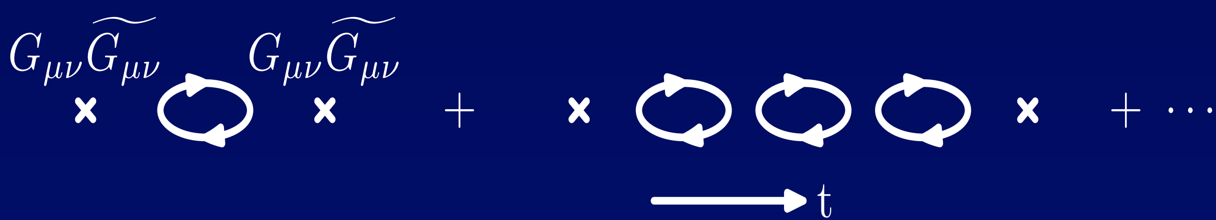
Lattice QCD with
dynamical fermions

$$< T[O_1(A,\overline{\Psi},\Psi)O_2(A,\overline{\Psi},\Psi)...] > = \frac{1}{Z}\int DA_\mu\, e^{-S_G} \det M\; Tr\{M^{-1}M^{-1}...A...\}$$

Collectivity



Schematic model – G. Brown
TDA
RPA --- Giant Resonances



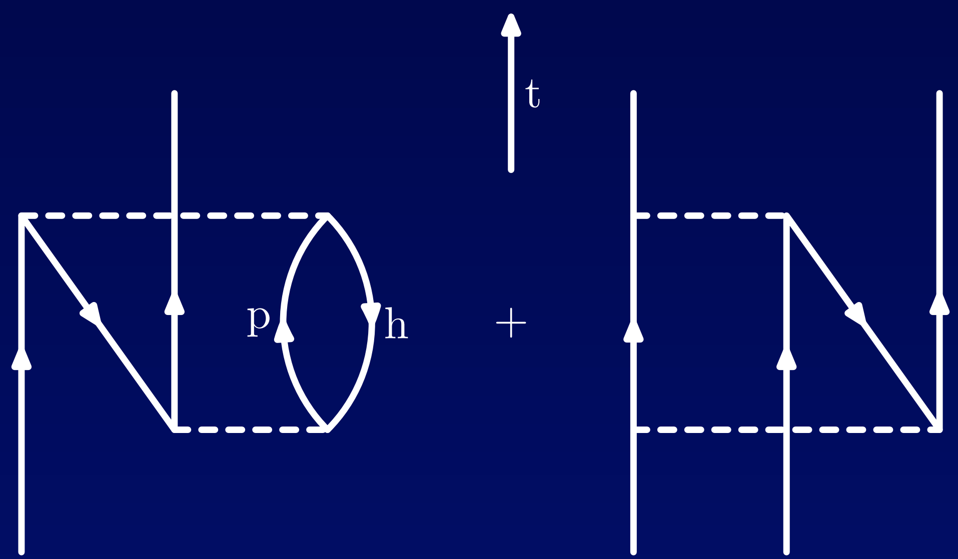
η' mass – U(1) anomaly
Witten – Veneziano formula

$$m_{\eta'}^2 = \frac{2N_F\chi}{f_\pi^2}, \quad \chi = \frac{\langle Q^2 \rangle}{V}$$

Veneziano – schematic model

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

Degree of Freedom in Fock Space



Z –graphs in Bethe-Goldstone diagram to account for particle-hole states in connected insertions

Hadronic Tensor in Euclidean Path-Integral Formalism

- Deep inelastic scattering

  In Minkowski space

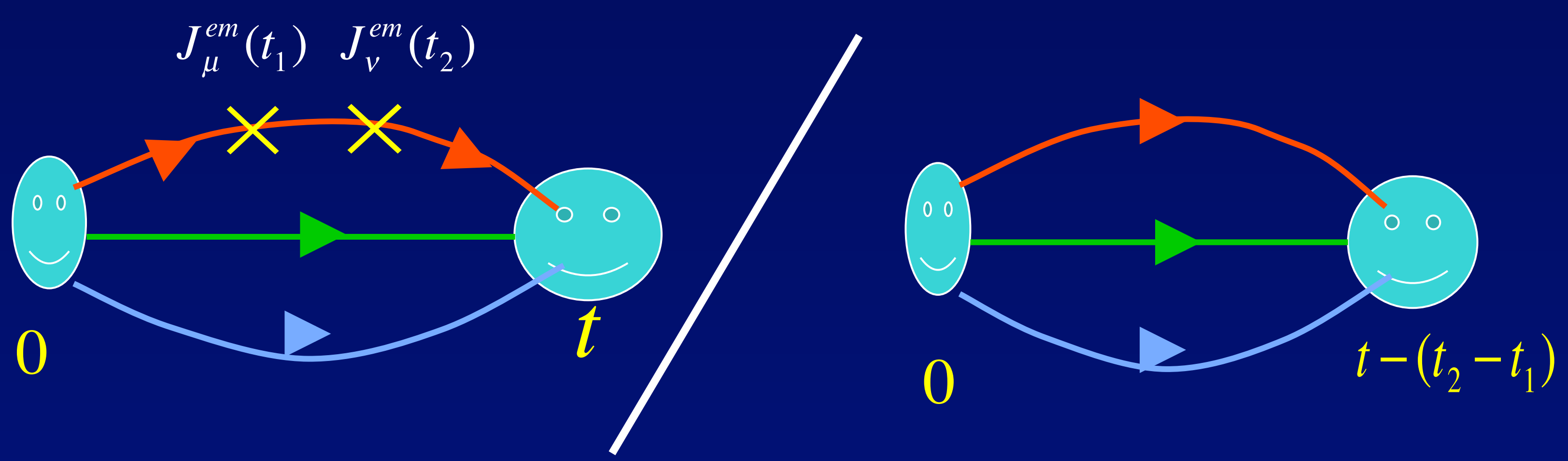
$$\frac{d^2\sigma}{dE'd\Omega} = \frac{\alpha^2}{q^4}\left(\frac{E'}{E}\right) l^{\mu\nu} W_{\mu\nu}$$

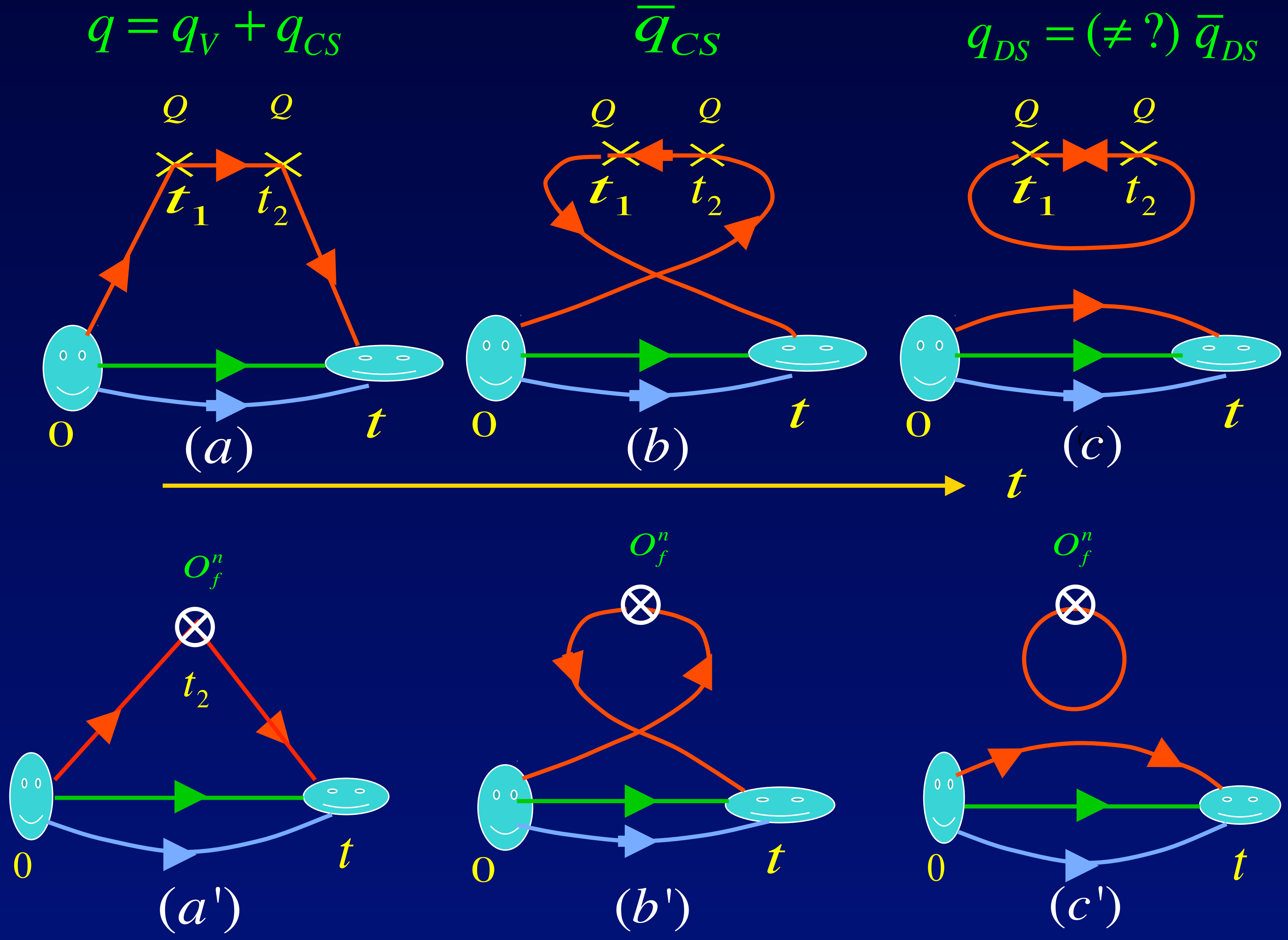
K.F. Liu
PRD 62, 074501 (2000)

$$T_{\mu\nu}(q^2,\nu) = \frac{1}{2M_N}\int \frac{d^4x}{(2\pi)^4}\, e^{iq\cdot x} < p\,|\,T[J_\mu^{em}(x)J_\nu^{em}(0)\,|\,P>,$$

$$W_{\mu\nu}(q^2,\nu) = \frac{1}{\pi}\mathrm{Im}\, T_{\mu\nu} = \frac{(2\pi)^3}{2M_N}\sum_n \delta^4(p_n - p + q) < P\,|\,J_\mu^{em}\,|\,n >< n\,|\,J_\nu^{em}\,|\,P>$$

- Euclidean path-integral



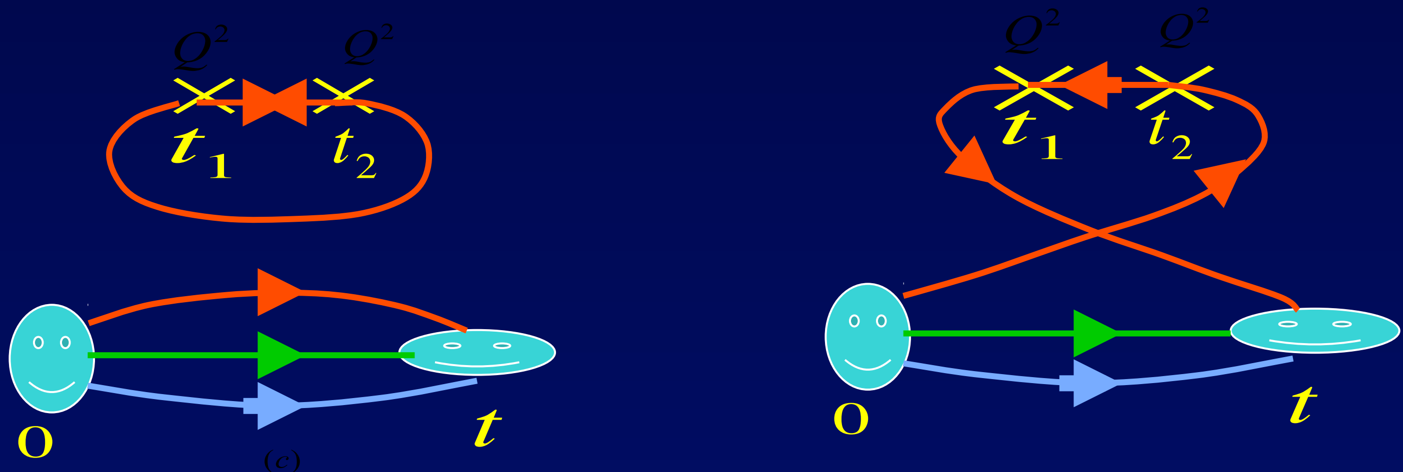


$q = q_V + q_{CS}$
$\overline{q}_{CS}$
$q_{DS} = (\neq ?)\ \overline{q}_{DS}$
Q
Q
t_1
t_2
0
t
(a)
(b)
(c)
O_f^n
(a')
(b')
(c')

2) Gottfried Sum Rule Violation

$$S_G(0,1;Q^2) = \frac{1}{3} + \frac{2}{3}\int_0^1 dx\ (\bar{u}_P(x) - \bar{d}_P(x)); \quad S_G(0,1;Q^2) = \frac{1}{3}\text{(Gottfried Sum Rule)}$$

NMC: $S_G(0,1;4\ \text{GeV}^2) = 0.240 \pm 0.016\ (5\sigma\ \text{from GSR})$



two flavor traces ($\bar{u}_{DS} = \bar{d}_{DS}$) one flavor trace ($\bar{u}_{CS} \neq \bar{d}_{CS}$)

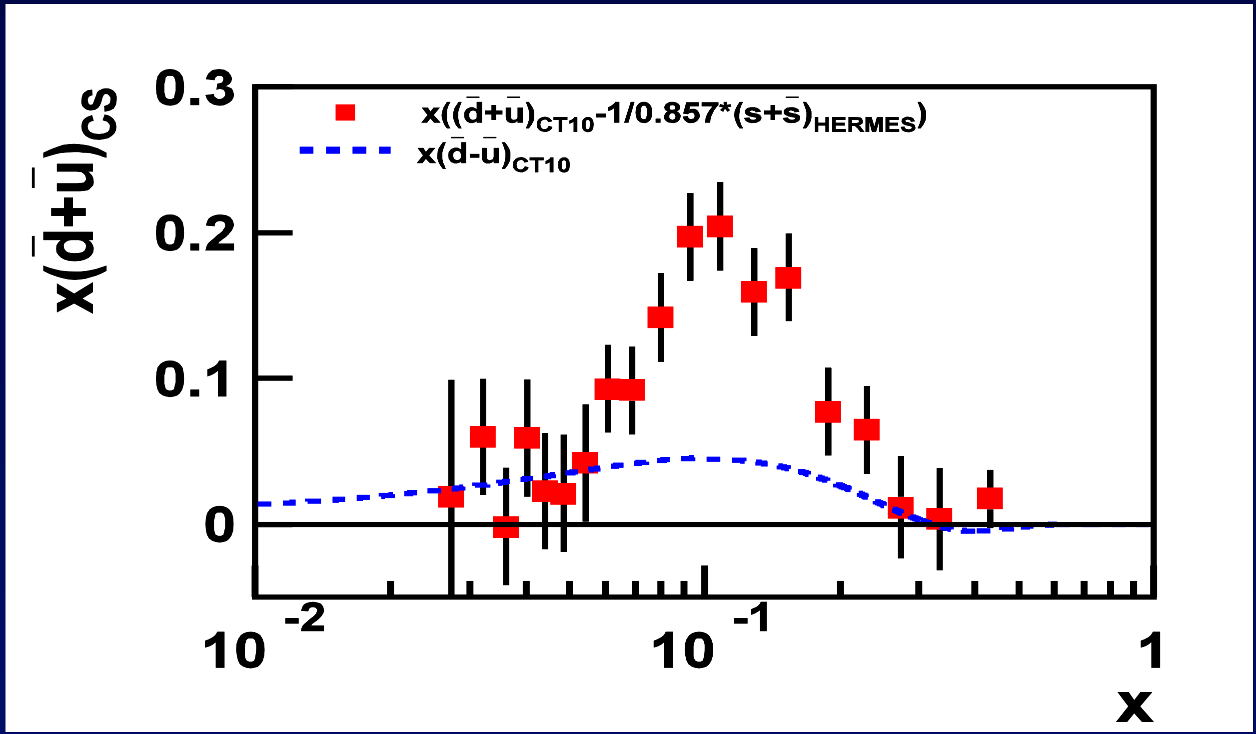
K.F. Liu and S.J. Dong, PRL 72, 1790 (1994)

$$Sum = \frac{1}{3} + \frac{2}{3}\int_0^1 dx\ (\bar{u}_{CS}(x) - \bar{d}_{CS}(x)),$$

$$= \frac{1}{3} + \frac{2}{3}\left[n_{\bar{u}_{CS}} - n_{\bar{d}_{CS}}\right]\ (1 + O(\alpha_s))$$

Connected Sea Partons

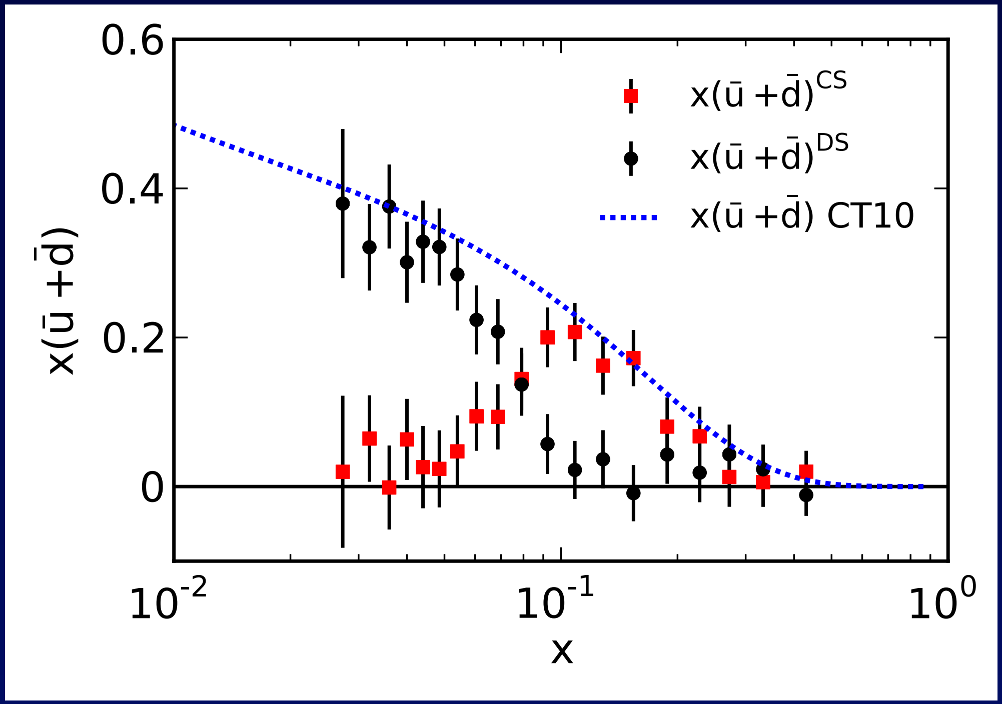
K.F. Liu, W.C. Chang, H.Y. Cheng, J.C. Peng, PRL 109, 252002 (2012)



$$x(\bar{d}+\bar{u})_{CS}(x) = x(\bar{d}+\bar{u})(x) - \frac{1}{R}x(s+\bar{s})(x);$$

CT10 lattice expt

$$R = \frac{\langle x\rangle_s}{\langle x\rangle_u (DI)} \text{(lattice)} : \; 0.857$$

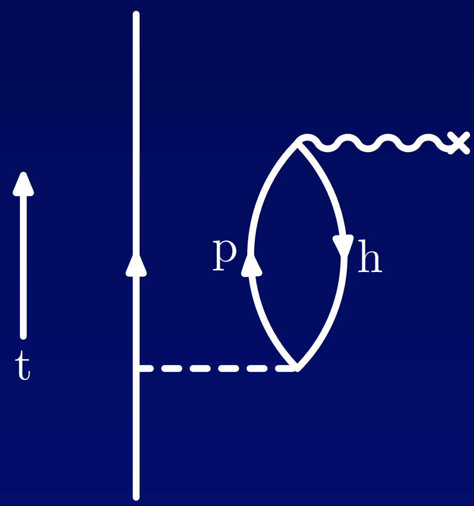


$$q_V\,,\ q_{CS}\ ,\ \bar{q}_{CS} \sim_{x\to 0}\ \ x^{-\alpha_R}\,(x^{-1/2})$$

$$q_{DS}\ ,\ \bar{q}_{DS} \sim_{x\to 0}\ \ x^{-1}$$

Polarization

Nucleus



Core polarization
Arima-Horie Effect

Nucleon

QCD with confinement, asymptotic freedon

Vacuum polarization – sea quarks
Gluon contributions
Axial anomaly
Trace anomaly

Where does the spin of the proton come from?

Status of Proton Spin

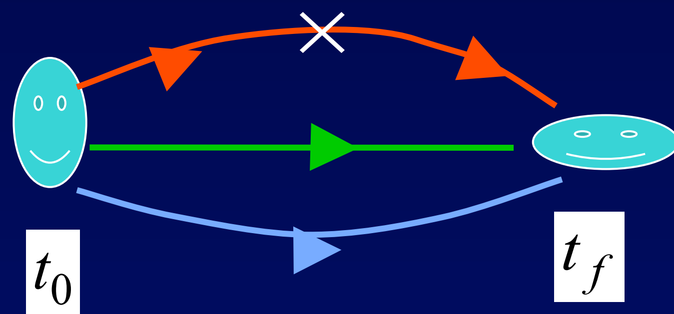
- Quark spin $\Delta\Sigma \sim 20 - 30\%$ of proton spin (DIS, Lattice)
- Quark orbital angular momentum? (lattice calculation (LHPC,QCDSF) → ~ 0)
- Glue spin $\Delta G/G$ small (COMPASS, STAR) ?
- Glue orbital angular momentum is zero (Brodsky and Gardner) ?

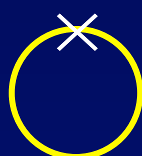
→ Dark Spin ?

Hadron Structure with Quarks and Glue

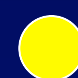
- Quark and Glue Momentum and Angular Momentum in the Nucleon

$$(\bar{u}\gamma_\mu D_\nu u + \bar{d}\gamma_\mu D_\nu d)(t)$$

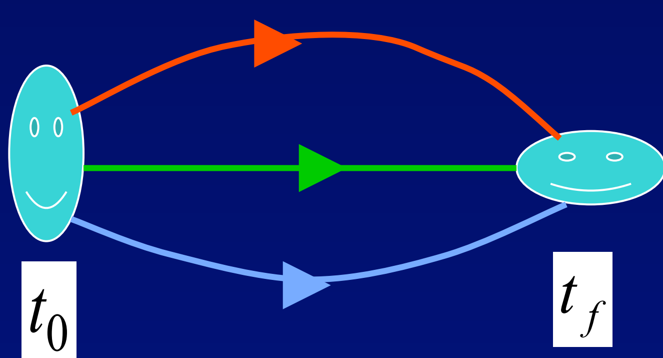


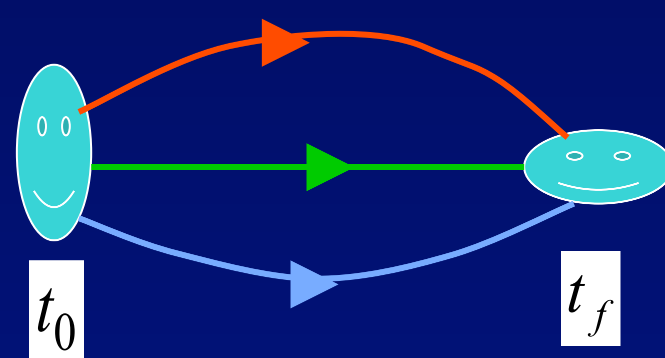


$$\bar{\Psi}\gamma_\mu D_\nu \Psi(t)(u,d,s)$$



$$F_{\mu\alpha}F_{\nu\alpha} - \frac{1}{4}\delta_{\mu\nu}F^2$$





Momenta and Angular Momenta of Quarks and Glue

- Energy momentum tensor operators decomposed in quark and glue parts gauge invariantly --- Xiangdong Ji (1997)

$$T^q_{\mu\nu} = \frac{i}{4}\left[\bar{\psi}\gamma_\mu \overleftrightarrow{D}_\nu \psi + (\mu \leftrightarrow \nu)\right] \rightarrow \vec{\mathrm{J}}_q = \int d^3x \left[\frac{1}{2}\bar{\psi}\vec{\gamma}\gamma_5\psi + \vec{x}\times\bar{\psi}\gamma_4(-i\vec{D})\psi\right]$$

$$T^g_{\mu\nu} = F_{\mu\lambda}F_{\lambda\nu} - \frac{1}{4}\delta_{\mu\nu}F^2 \qquad \rightarrow \vec{\mathrm{J}}_g = \int d^3x\left[\vec{x}\times(\vec{E}\times\vec{B})\right]$$

- Nucleon form factors

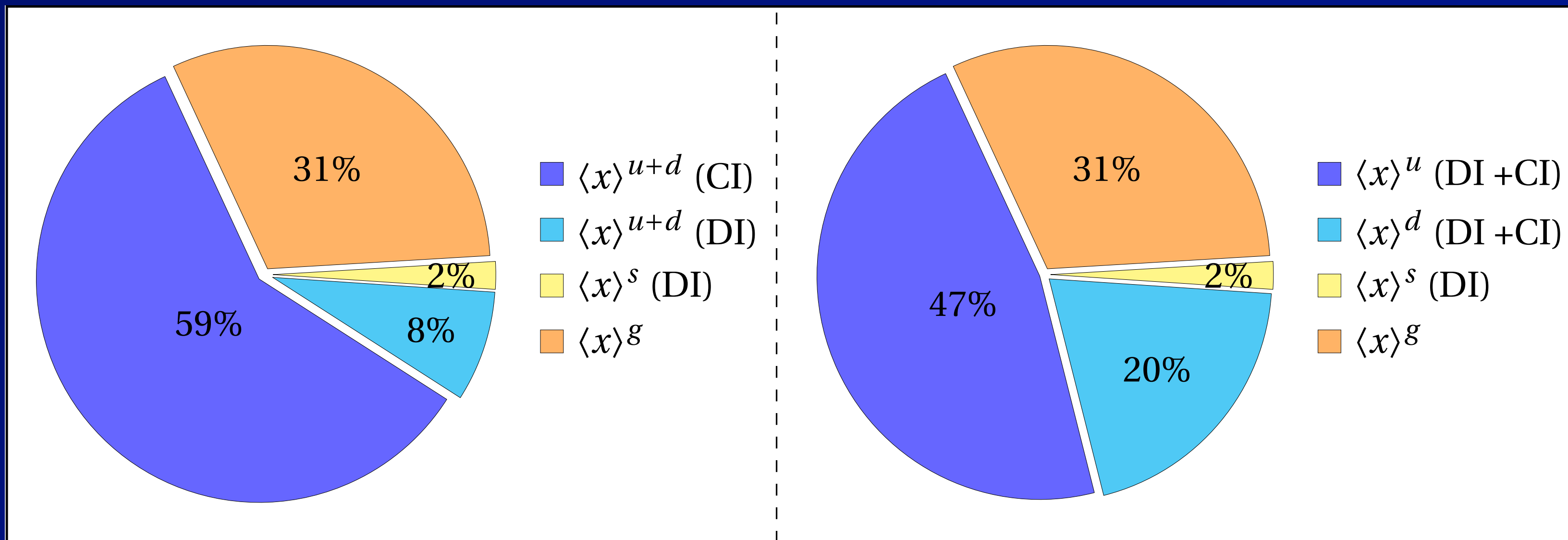
$$\langle p,s\,|\,T_{\mu\nu}\,|\,p's'\rangle = \bar{u}(p,s)[T_1(q^2)\gamma_\mu\bar{p}_\nu - T_2(q^2)\bar{p}_\mu\sigma_{\nu\alpha}q_\alpha/2m$$

$$\text{-i}\mathrm{T}_3(q^2)(q_\mu q_\nu - \delta_{\mu\nu}q^2)/m + T_4(q^2)\delta_{\mu\nu}m/2]u(p's')$$

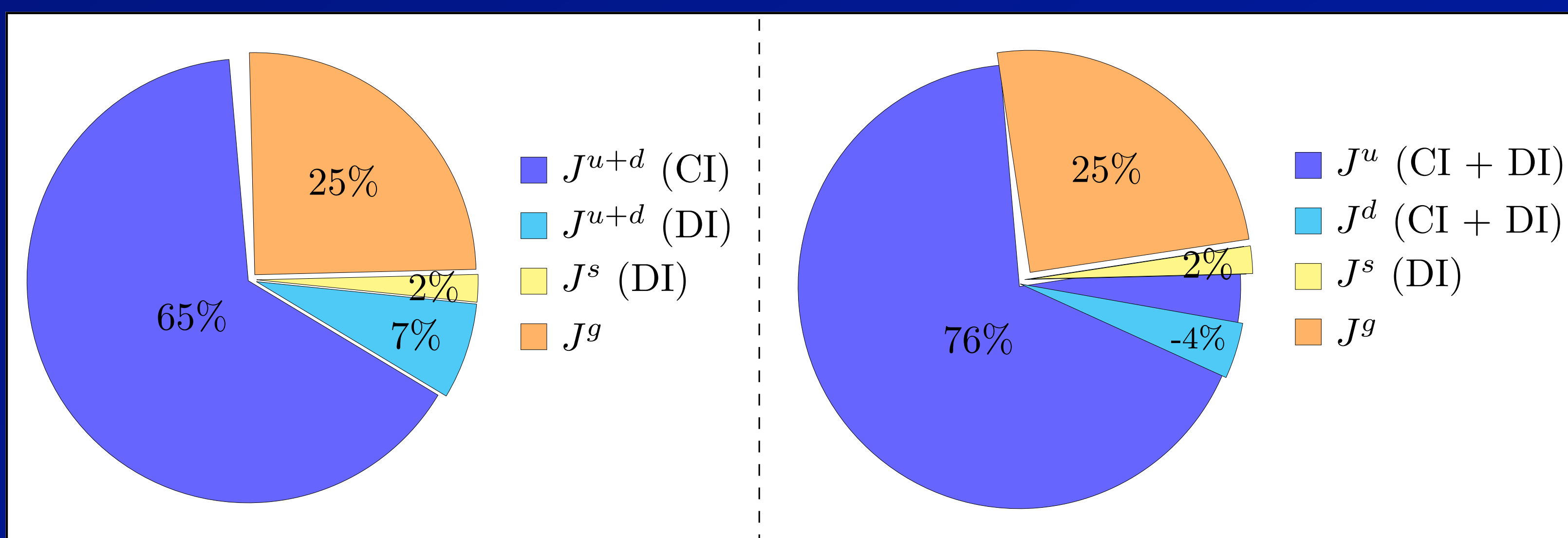
- Momentum and Angular Momentum

$$Z_{q,g}T_1(0)_{q,g} \ \left[\mathrm{OPE}\right] \rightarrow \langle x\rangle_{q/g}(\mu,\overline{\mathrm{MS}}), \quad Z_{q,g}\left[\frac{T_1(0)+T_2(0)}{2}\right]_{q,g} \rightarrow J_{q/g}(\mu,\overline{\mathrm{MS}})$$

Momentum fractions <x>q, <x>g



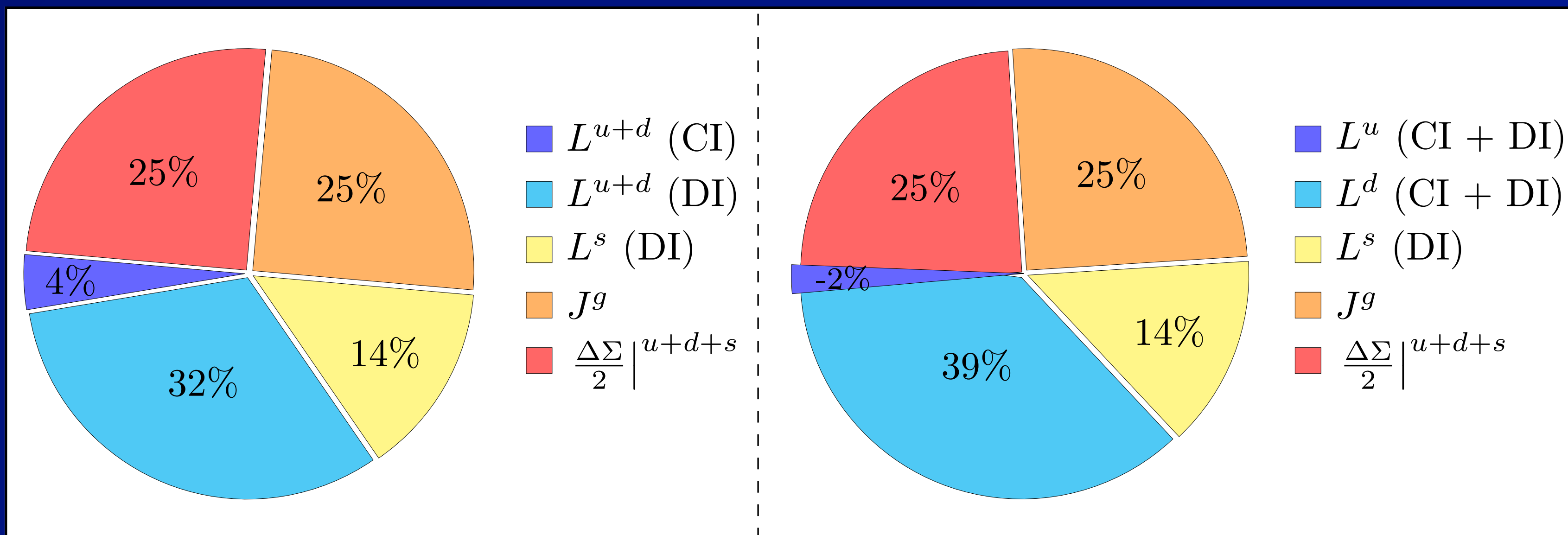
Angular Momentum fractions Jq, Jg



Renormalized results:

	CI(u)	CI(d)	CI(u+d)	DI(u/d)	DI(s)	Glue
2J	0.726 (128)	-.072 (82)	0.651 (51)	0.036 (7)	0.023 (7)	0.254 (76)
g_A	0.95 (11)	-0.32 (12)	0.65 (8)	-0.12 (1)	-0.12 (1)	
2 L	-0.25 (18)	0.26 (14)	0.00 (10)	0.17 (2)	0.15 (2)	

Quark Spin, Orbital Angular Momentum, and Gule Angular Momentum



$\Delta q \approx 0.25;$

$2\, L_q \approx 0.49\ (0.0(\mathrm{CI}){+}0.49(\mathrm{DI}));$

$2\ \mathrm{J}_g \approx 0.25$

2+1 flavor DWF configurations (RBC-UKQCD)

$La \sim 4.5$ fm
$m_\pi \sim 170$ MeV

32^3 x 64, a =0.12 fm

($O(a^2)$ extrapolation)

$La \sim 2.8$ fm
$m_\pi \sim 330$ MeV

24^3 x 64, a =0.115 fm

$La \sim 5.5$ fm
$m_\pi \sim 140$ MeV

48^3 x 96, a =0.115 fm

$La \sim 2.7$ fm
$m_\pi \sim 295$ MeV

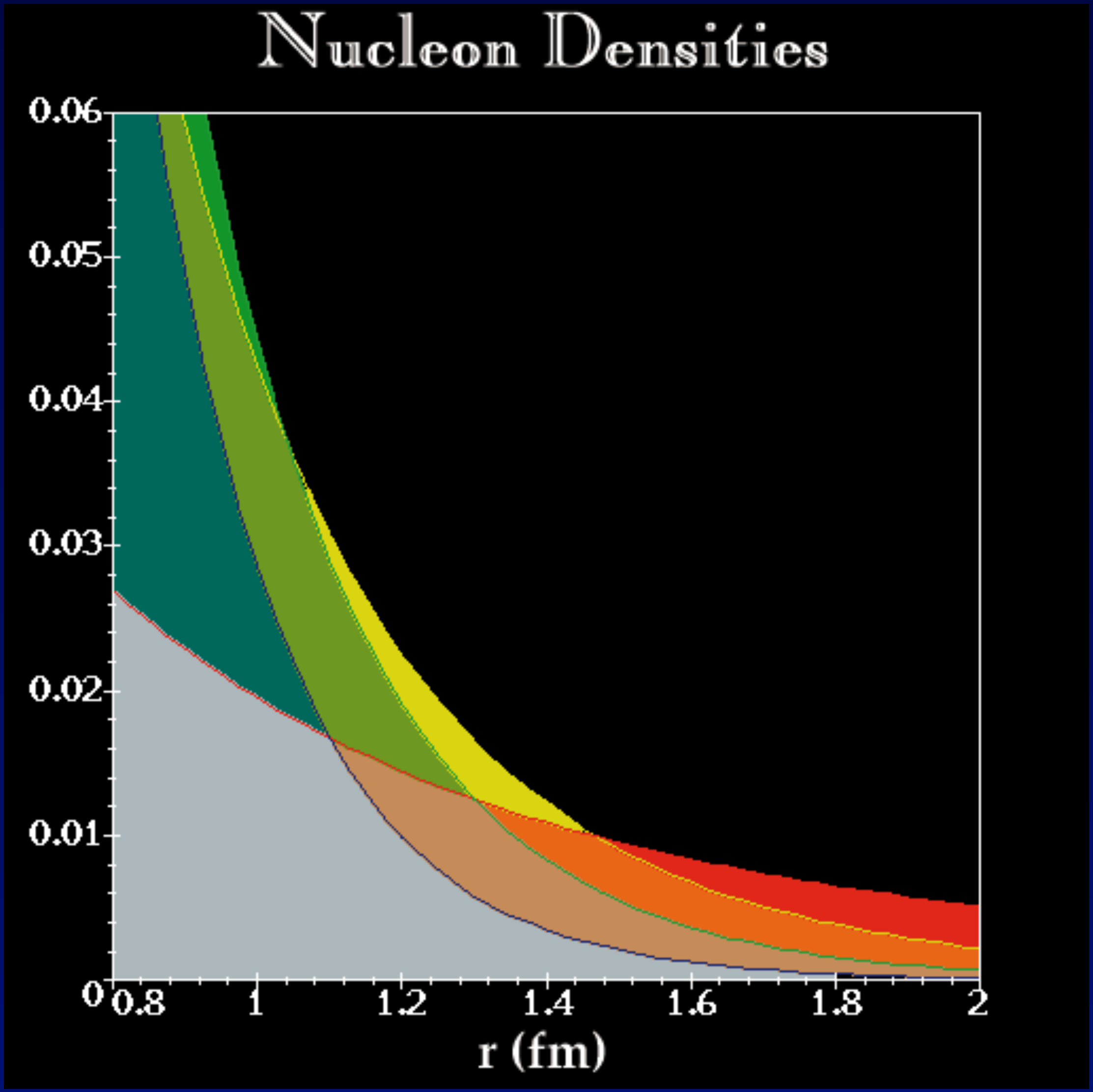
32^3 x 64, a =0.085 fm

$La \sim 5.5$ fm
$m_\pi \sim 140$ MeV

64^3 x 128, a =0.085 fm

Quark Spin from Anomalous Ward Identify

- Calculation of the axial-vector in the DI is very noisy
- Instead, try AWI $\partial_\mu A_\mu^0 = 2mP + \frac{N_f}{8\pi^2} G_{\mu\nu}\tilde{G}_{\mu\nu}$
  - Overlap fermion --> mP is RGI.
  - Overlap operator for $q(x) = -1/2\, \mathrm{Tr}\gamma_5 D_{ov}(x,x)$ is RGI.
  - P is totally dominated by small eigenmodes.
  - q(x) from overlap is exponentially local and is dominated by high eigenmodes.
  - Direct check the origin of `proton spin crisis'.


Nucleon Densities
0.06
0.05
0.04
0.03
0.02
0.01
0
0.8
1
1.2
1.4
1.6
1.8
2
r (fm)

My lifelong appreciation to Gerry for his encouragement and inspiring me to take the path from nuclear structure to nucleon structure.