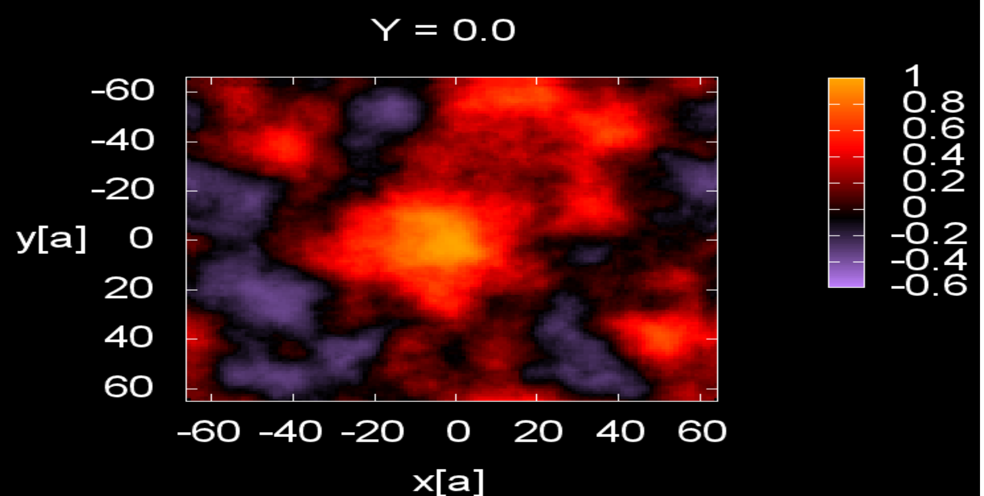
(Turbulent) many-body gluodynamics



``Boosted" gluon configurations inside a proton

Raju Venugopalan

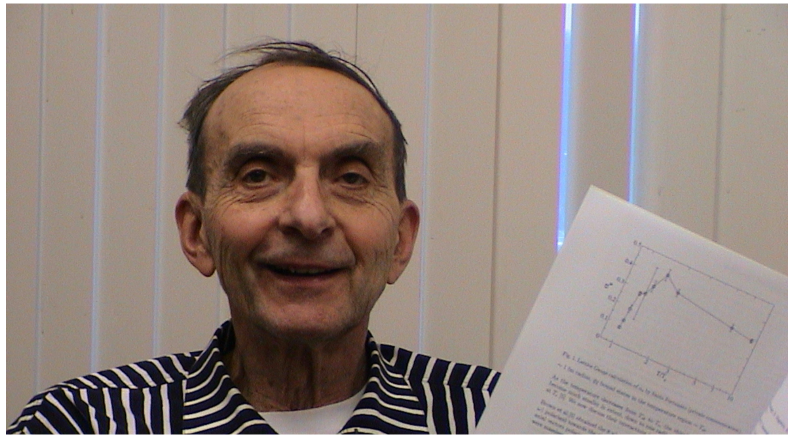
Brookhaven National Laboratory

Gerry Brown memorial meeting, November 24th-26th 2013, Stony Brook

Outline

- Gerry Brown
- The proton as a many-body system
- Colliding strongly correlated gluon systems
- Universal turbulent attractors in non-Abelian plasmas

Gerry Brown (1926-2013)

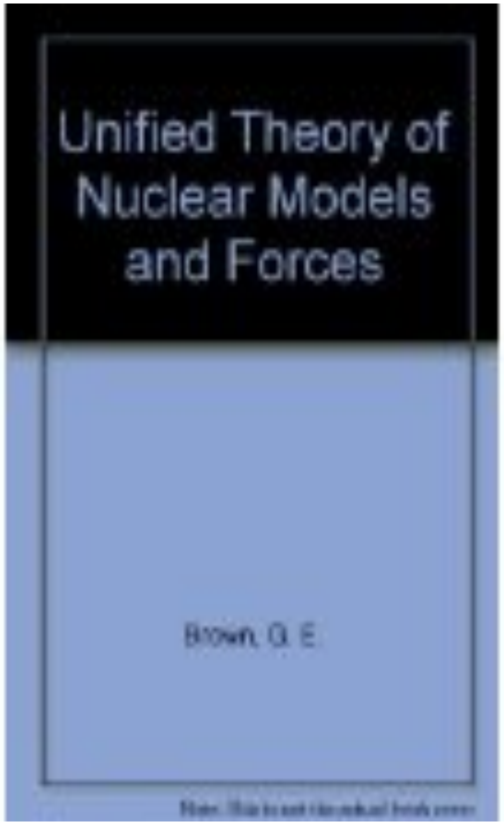


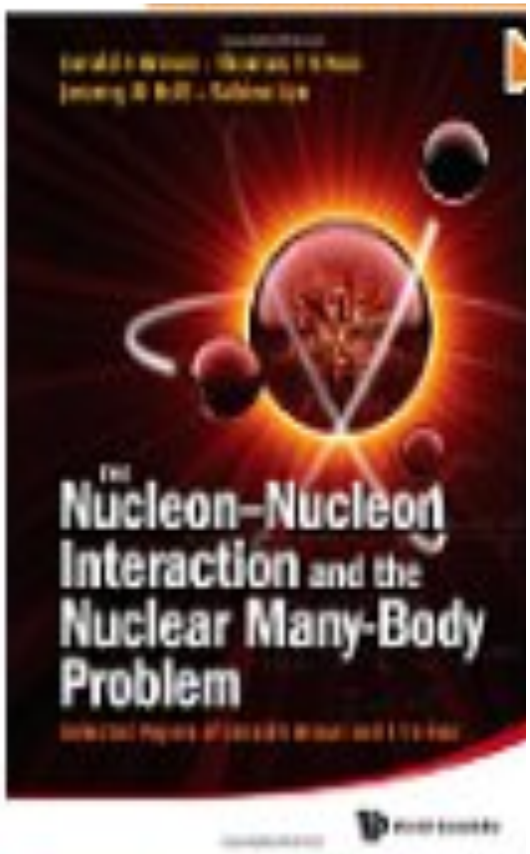
Gerry Brown recruited me while I was still an undergraduate
- He had an enormous impact on me and on my career

Though I worked only briefly with him, he was very kind to me
- he sent me to spend summers at Berkeley, Copenhagen, Jyvaskyla and Minneapolis, while still a graduate student

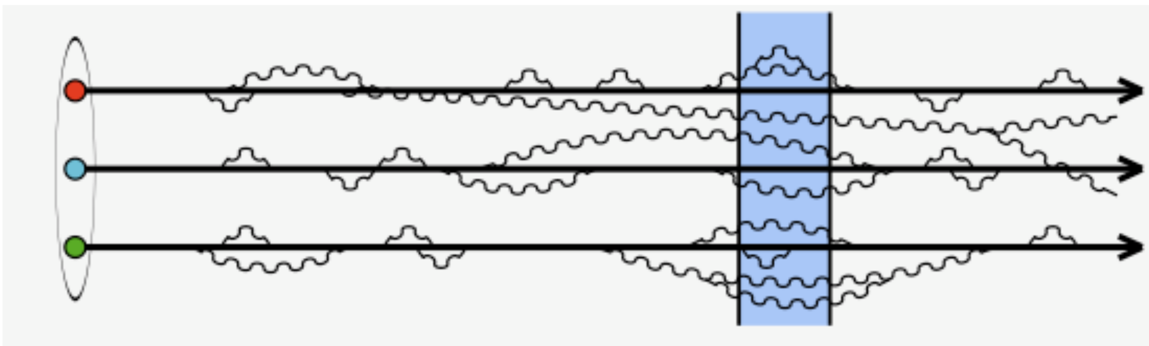
Gerry was *sui generis* – his style combined light hearted (very witty) banter with deep seriousness

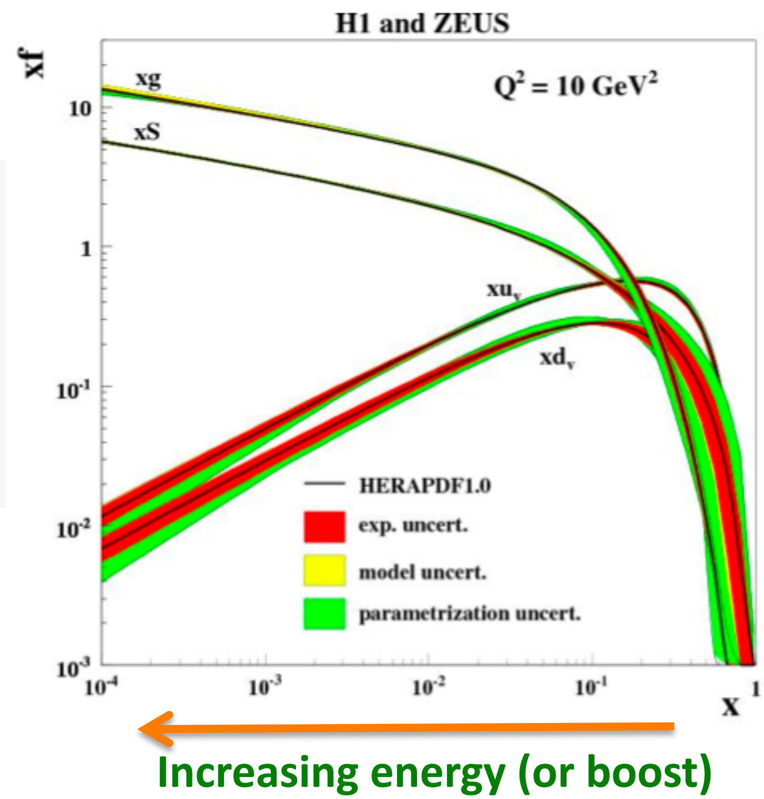
Gerry and many-body dynamics





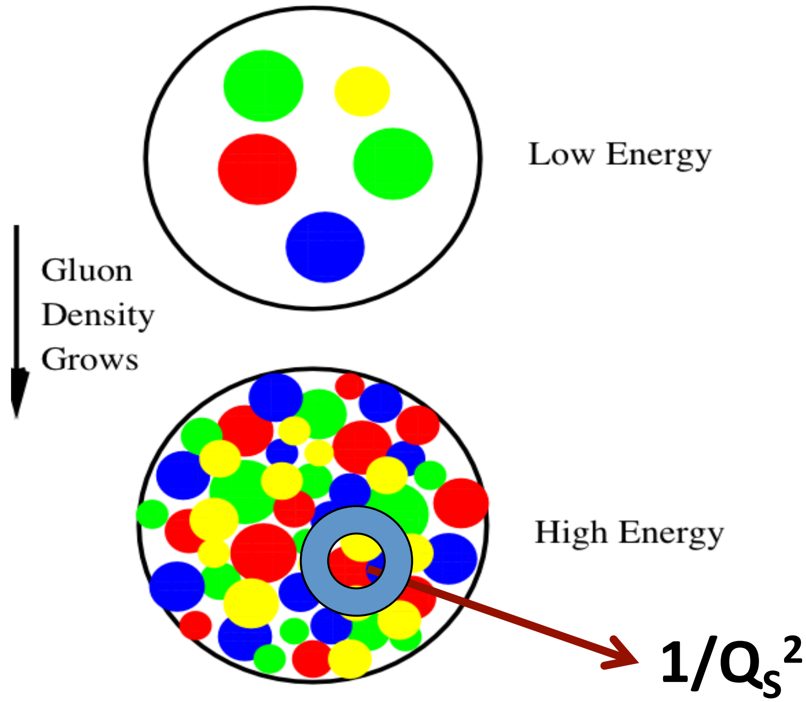
Lifting the veil: the many-body proton





``Wee” gluon (and sea quark) fluctuations are dilated on strong interaction time scales

Lifting the veil: the many-body proton



Gluons in QCD have maximal occupancy of ~ $1/\alpha_S$
-- their growth tapers with energy – gluon saturation

VIRTUAL PAIR CREATION IN A STRONG BREMSSTRAHLUNG FIELD: A QED model for parton saturation

A H MUELLER[1]

Physics Department, Columbia University, New York, NY 10027, USA and
Institute for Theoretical Physics, University of California, Santa Barbara, California 93106, USA

Received 10 March 1988

Virtual pair creation in a strong, virtual, bremsstrahlung field is considered in QED as a model for parton saturation In a weak field the virtual pair density increases quadratically in the external field, however, at large values of the field the number density becomes independent of the strength of that field A similar effect is found in scalar electrodynamics

1. Introduction

At small values of the Bjorken-x-variable parton (quark and gluon) number densities are expected to grow rapidly [1] However, when, say, the gluon distribution in a hadron, $xG(x, Q^2)$, reaches a value as large as $Q^2 r^2/\alpha$, with r the radius of the hadron, these gluons are so densely packed that one expects scattering and annihilation of partons to become important, thus limiting the ultimate number density to be of the size indicated above [1, 3]

This high density quark-and-gluon system is a most fascinating regime of QCD On the one hand, if $Q^2 \gtrsim 1$ GeV^2 the coupling, $\alpha(Q^2)$, is small and the usual non-perturbative condensates are unimportant while, on the other hand, the system is strongly interacting because of the high parton densities. That is, this regime of weak coupling but large numbers of partons is a new regime of QCD Such a high-density parton system occurs in a number of different high-energy processes (i) In deeply inelastic scattering one can directly measure such high-density systems at small x using the virtual photon as a probe [1, 3]. (ii) In the very early stages of a heavy ion collision such a system is produced over a large transverse area [4]. (iii) Two-jet correlations in high-energy reactions can trigger on local hot spots [5], high parton density regions which are smaller than the radius of a normal hadron

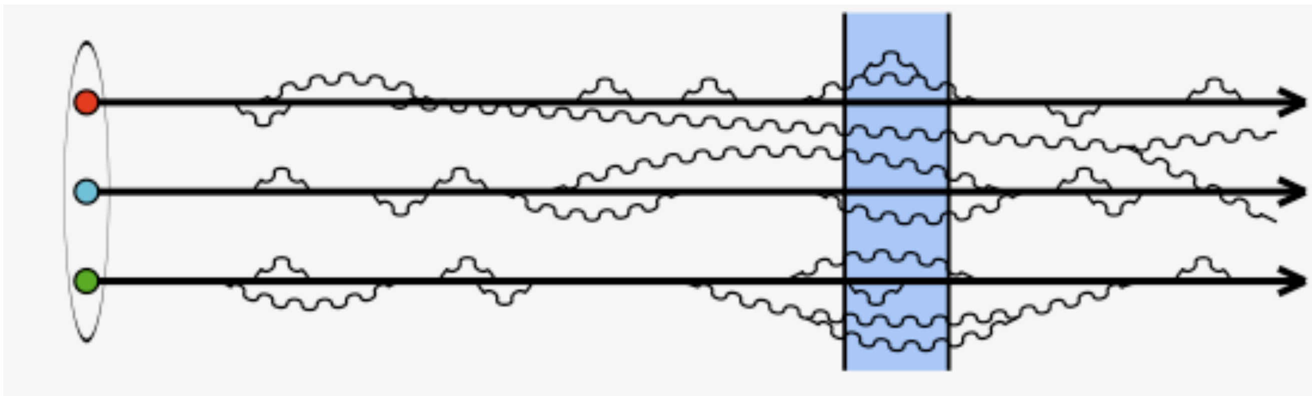
Many-body gluodynamics

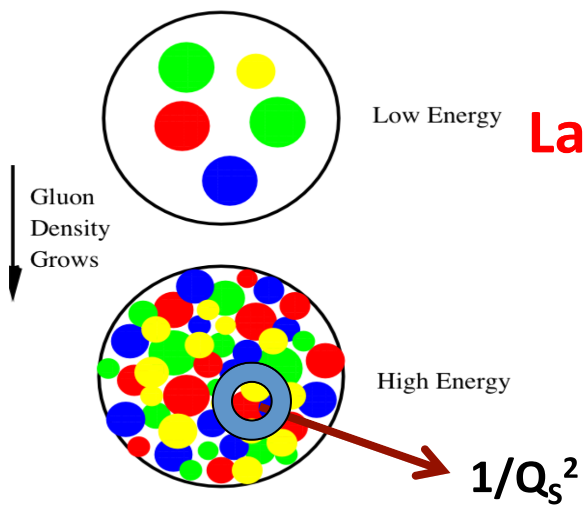
Gribov, Levin, Ryskin (1983)
Mueller, Qiu (1986)
Blaizot, Mueller (1987)

This high density quark-and-gluon system is a most fascinating regime of QCD On the one hand, if $Q^2 \gtrsim 1$ GeV^2 the coupling, $\alpha(Q^2)$, is small and the usual non-perturbative condensates are unimportant while, on the other hand, the system is strongly interacting because of the high parton densities. That is, this regime of weak coupling but large numbers of partons is a new regime of QCD Such a high-density parton system occurs in a number of different high-energy processes

QCD at high energies is a many-body theory
— weak coupling many body techniques may be applicable

Gluon Saturation in large nuclei:
classical coherence from quantum fluctuations





McLerran, RV (1994)

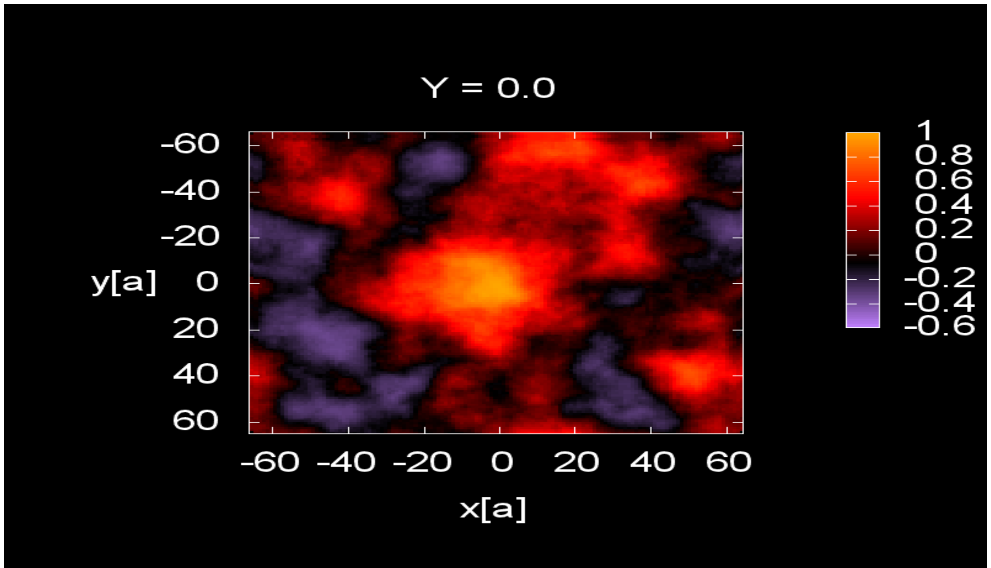
Large occupation # => classical color fields (and sources)

|P>_pert → |P>_classical

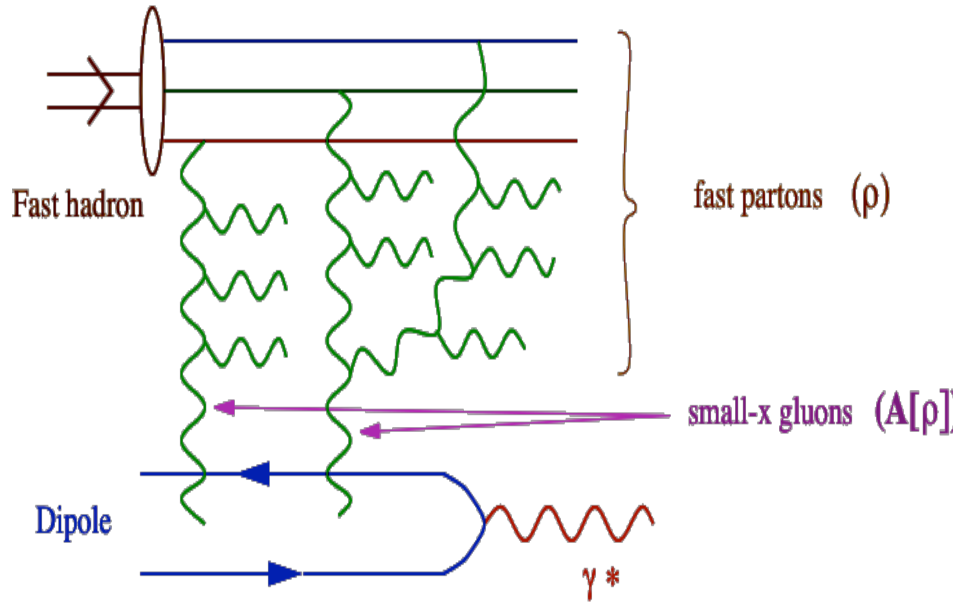
"Regge-Gribov" extreme QCD is a classical effective many-body theory

B-JIMWLK: the BBGKY hierarchy of gluodynamics

Balitsky (1996); Kovchegov (1999)
Jalilian-Marian,Kovner,McLerran,Weigert (1997)
Iancu, Leonidov, McLerran (2001)

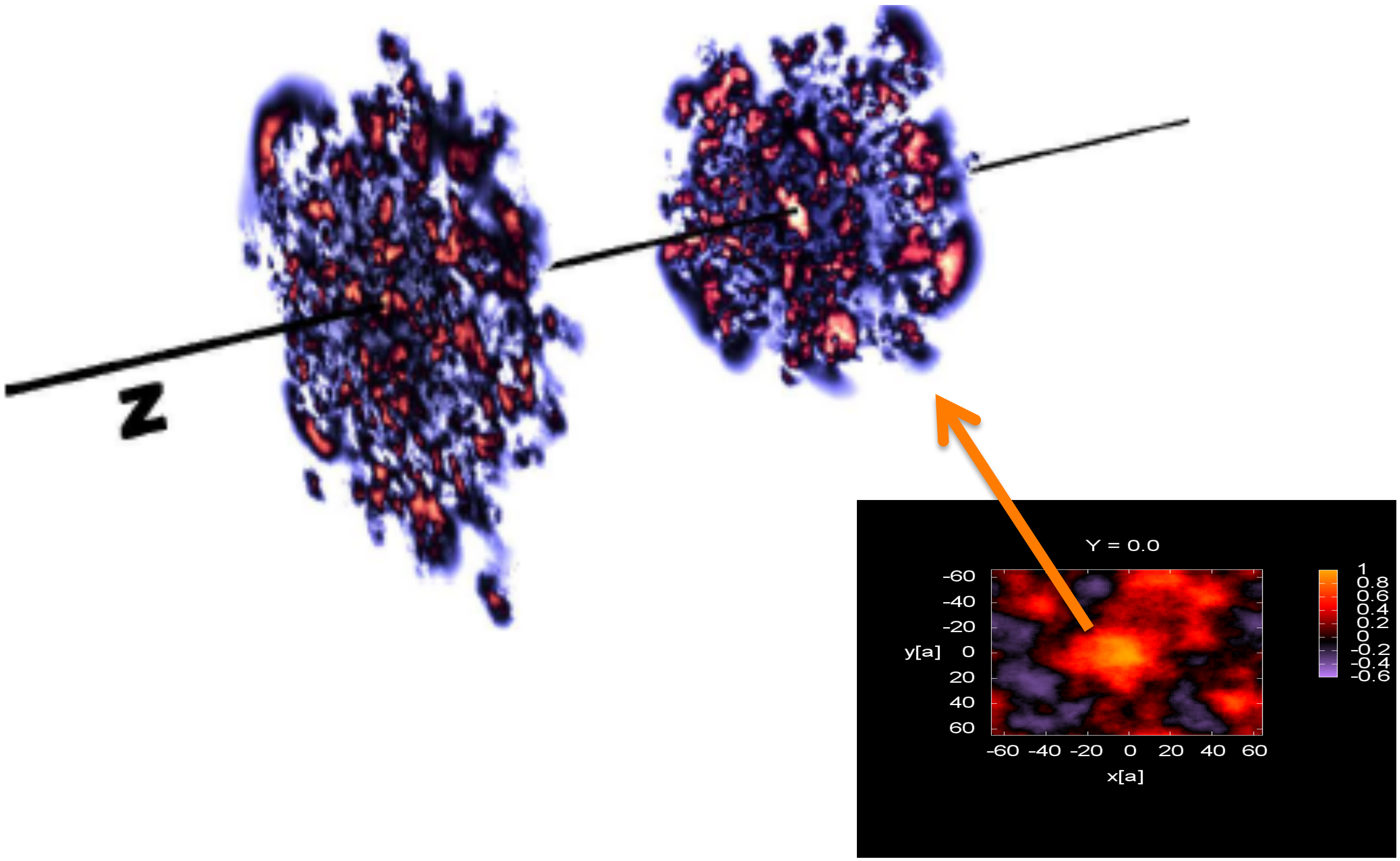


Dumitru,Jalilian-Marian,Lappi,Schenke,Venugopalan (2011)



This hierarchy of n-gluon correlators has been solved numerically --very important for phenomenology of multi-particle production

Ultrarelativistic nuclear collisions

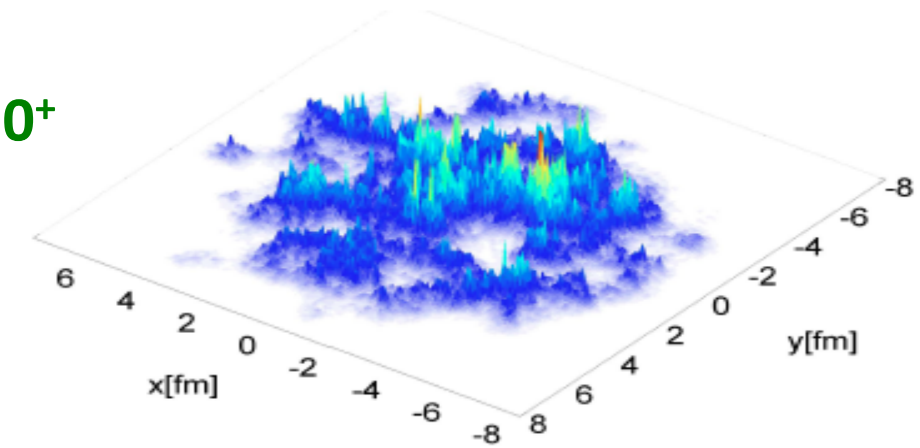


Incoming nuclei are Color Glass Condensates: Highly occupied gluon states with maximal occupancy in QCD

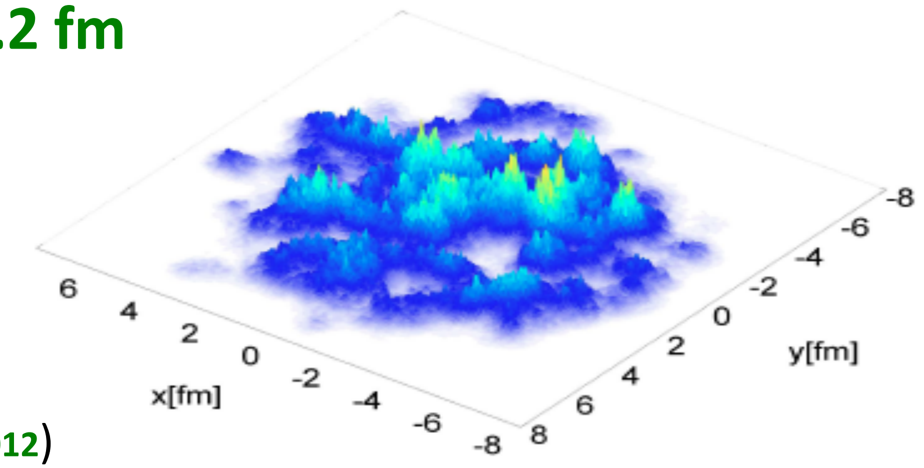
Non-equilibrium dynamics of non-Abelian plasmas

Energy density spatial profiles

T=0$^+$



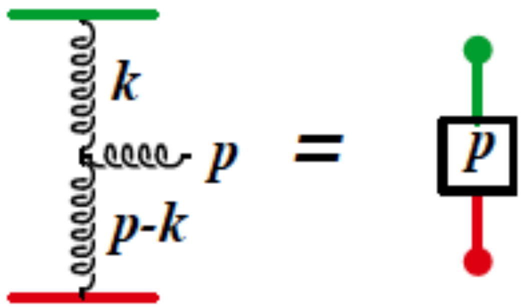
T=0.2 fm



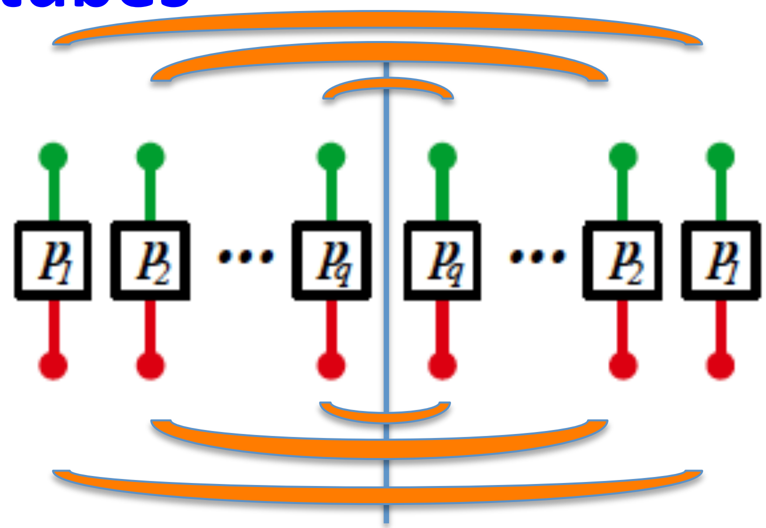
Schenke,Tribedy,RV (2012)

Early time non-equilibrium dynamic is classical-statistical dynamics

Lasing gluons: Stimulated emission from Glasma flux tubes

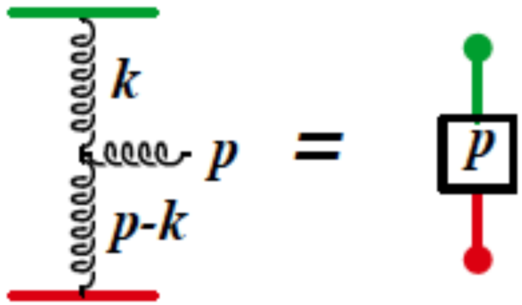


Dumitru,Gelis,McLerran,RV (2008)
Dusling,Fernandez-Fraile,RV (2009)
Gelis,Lappi,McLerran (2009)



Color combinatorics of cut graphs: a negative binomial distribution

Lasing gluons: Stimulated emission from Glasma flux tubes



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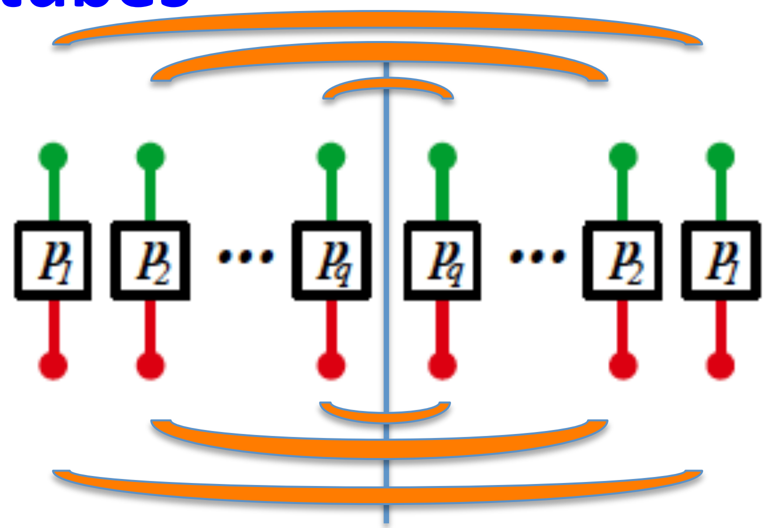
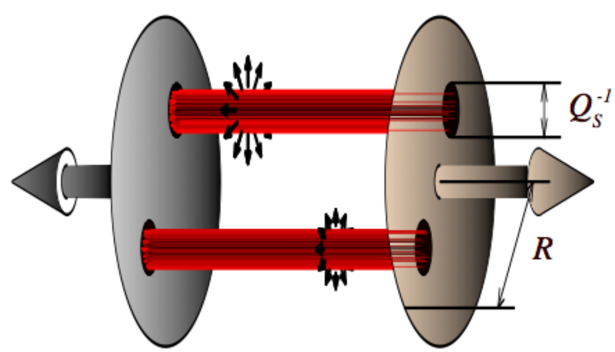


Color combinatorics of cut graphs: a negative binomial distribution

$$P_n^{\mathrm{NB}} = \frac{\Gamma(k+n)}{\Gamma(k)\Gamma(n+1)}\frac{\bar{n}^n k^k}{(\bar{n}+k)^{n+k}}$$

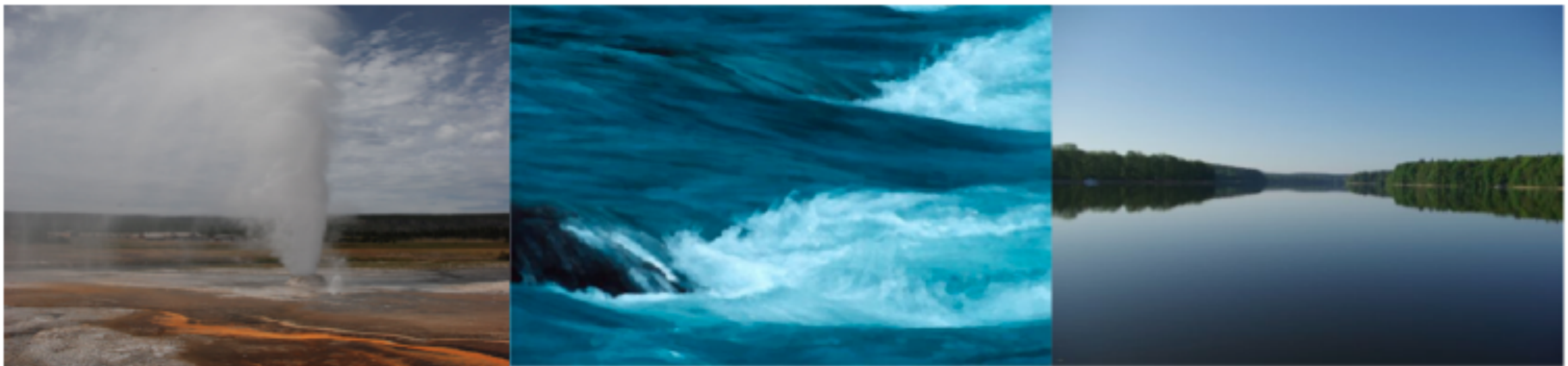
k=1: Bose-Einstein dist.
k=∞: Poisson distribution

$$k = \kappa \frac{(N_c{}^2-1)Q_s^2 S_\perp}{2\pi}$$



For $Q_S^2 \approx 1/S_T$ close to a Bose-dist!

Non-equilibrium dynamics of strongly correlated non-Abelian plasmas



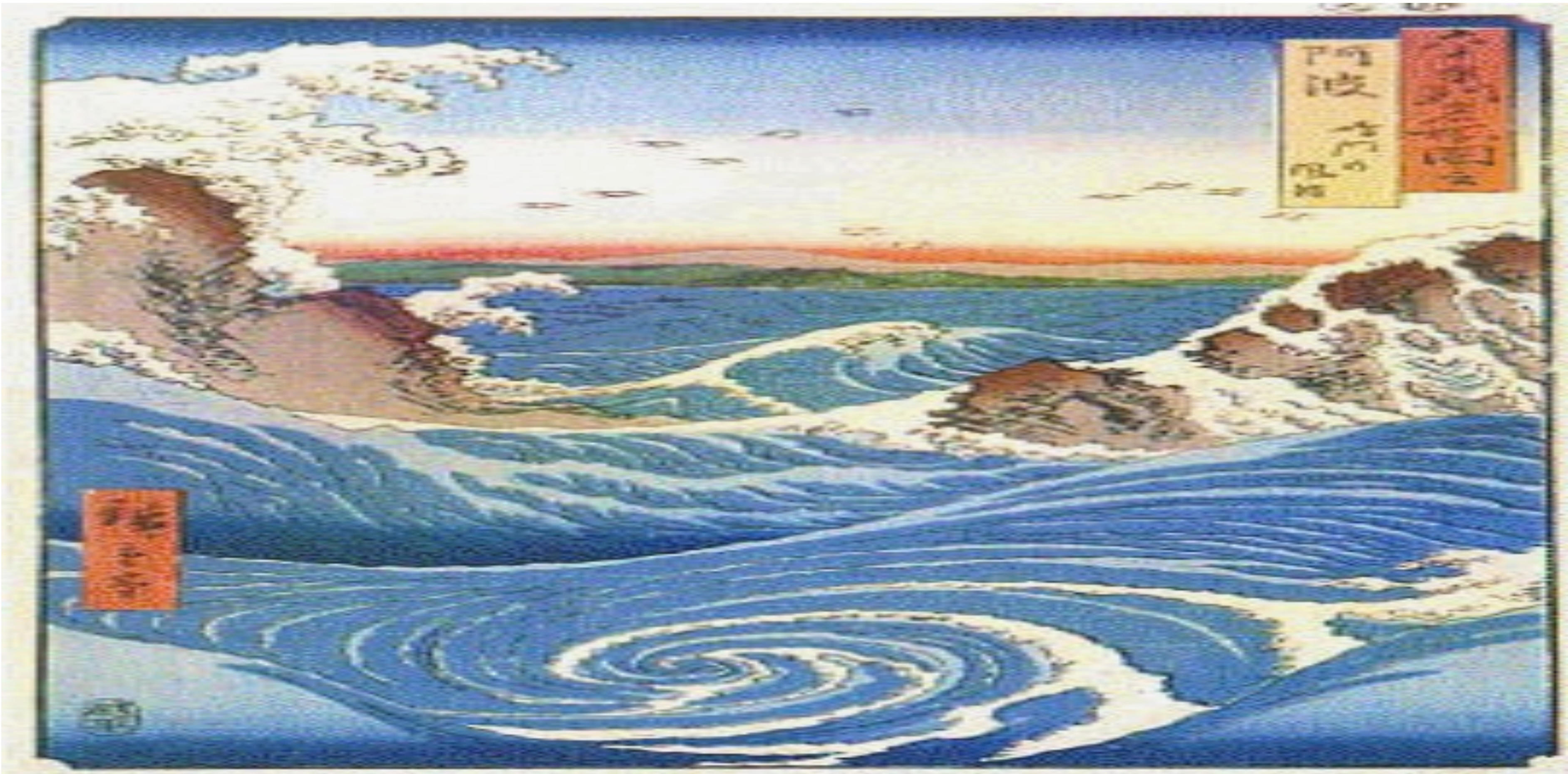
Initial state:
Far from equilibrium

Non-equilibrium dynamics

Final state:
Thermal equilibrium

How is thermal equilibrium achieved?

Universal non-thermal attractor in QCD



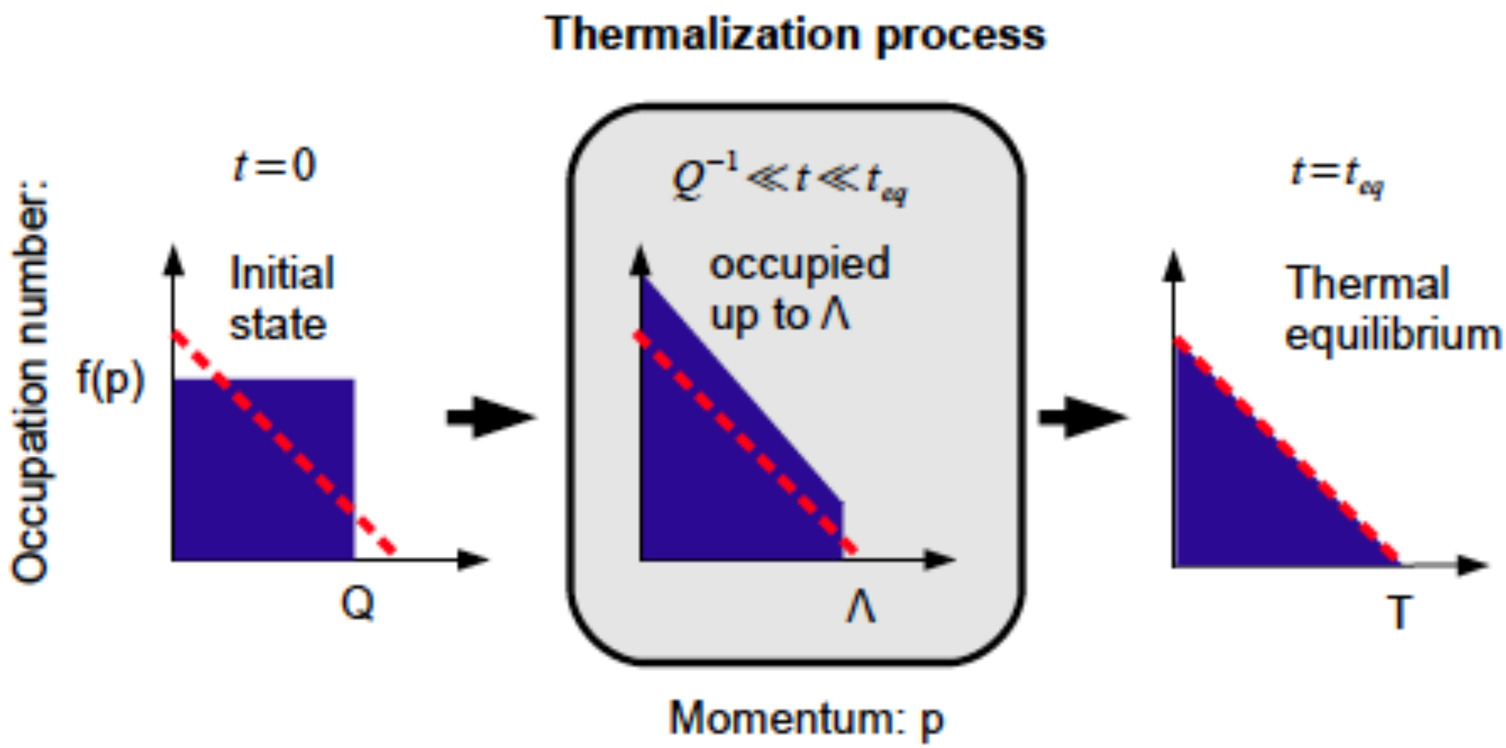
“Big whorls have little whorls, which feed on their velocity,
And little whorls have lesser whorls, and so on to viscosity.”

Turbulence: a preamble

`I am an old man now, and when I die and go to heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is the turbulent motion of fluids. And about the former I am rather optimistic.'
- Horace Lamb

“Simple” problem: overoccupied plasma in a box

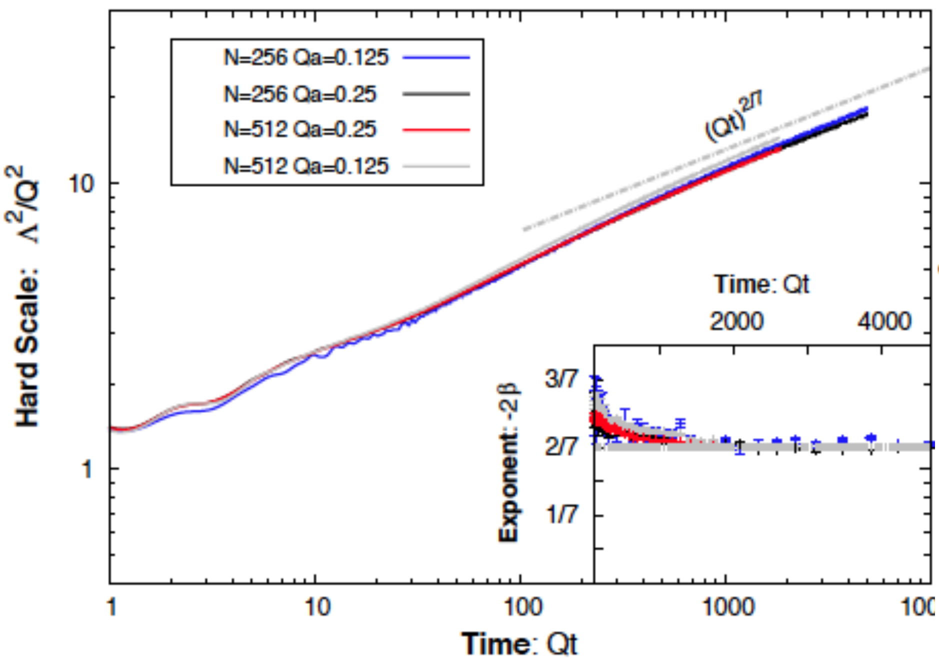
Schlichting/Kurkela,Moore (2012)
Berges,Boguslavski,Schlichting,RV:1311.3005

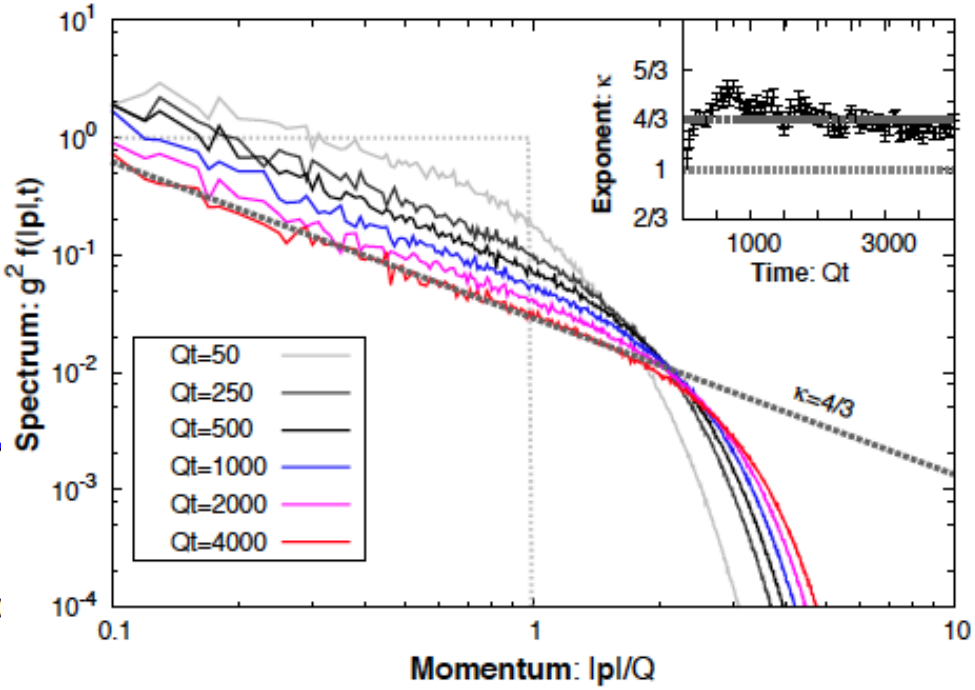


$$\boxed{\begin{aligned} f(p,t) &= (Qt)^{\alpha} f_S(p/\Lambda) \\ \Lambda &= Q(Qt)^{-\beta} \end{aligned}}$$

Overoccupied non-Abelian plasma in box:

Weak coupling results from classical-statistical real-time simulations

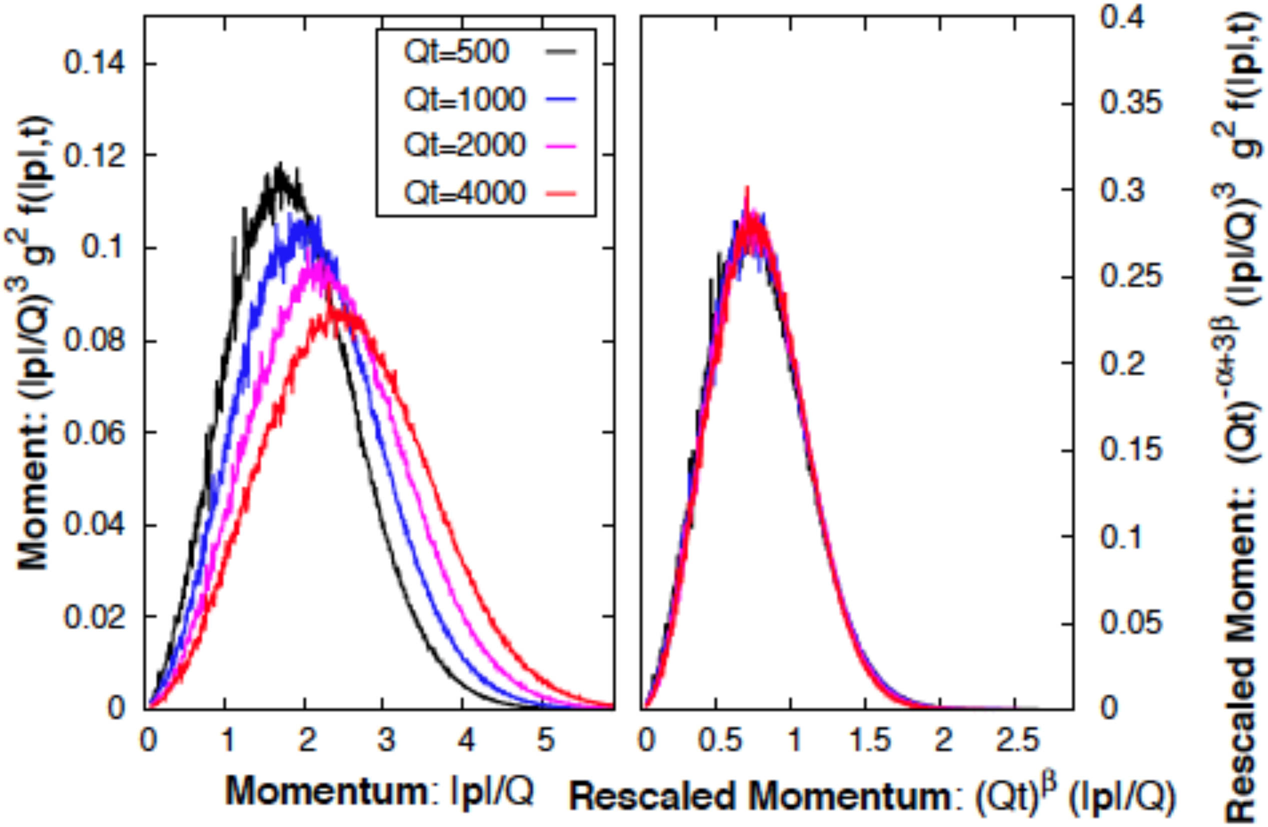




Universal exponents: $\alpha = 4/7 \;\; ; \;\; \beta = 1/7 \;\; ; \;\; \kappa = 4/3$

- ✓ **α and β understood from: energy conservation+self-similar scaling of kinetic eqn.**
- ✓ **κ is the Kolmogorov-Zakharov spectral index for (weak turbulent) particle cascades**

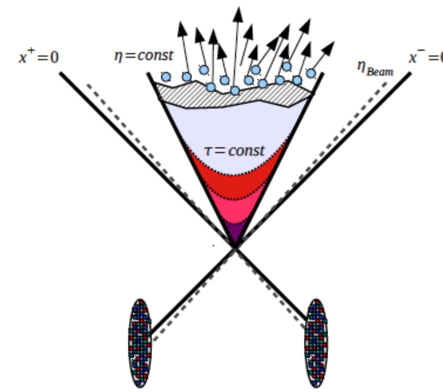
Self-similarity



``Thermalization time" in static box:

$$t_{\text{eq.}} \sim Q^{-1} \alpha_S^{-7/4}$$

Longitudinal expanding non-Abelian plasma: aka Heavy Ion collisions



- **Huge $1/\tau$ redshift at early times generates anisotropy in longitudinal and transverse pressures**

- **Will report here on real time classical-statistical lattice simulations of a weakly coupled (strongly correlated) overoccupied plasma.**

- **These are by far the largest real-time Yang-Mills simulations to date -- fully capture important infrared dynamics.**

Berges,Boguslavski,Schlichting,RV:1303.5650
Berges,Boguslavski,Schlichting,RV:1311.3005

Self-similar quasi-stationary solutions

$$f(p_T, p_z, \tau) = (Q\tau)^\alpha \; f_S\left(\frac{p_T}{\Lambda_T}, \frac{p_z}{\Lambda_L}\right)$$

(Gauge invariant) hard scales: $\Lambda_L = Q\,(Q\tau)^{-\gamma}$

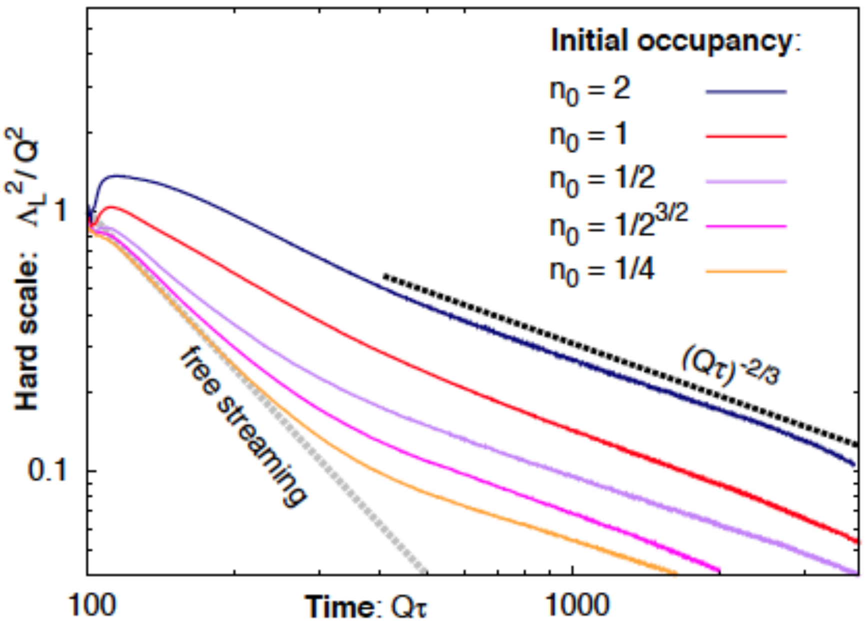
$\Lambda_T = Q\,(Q\tau)^{-\beta}$

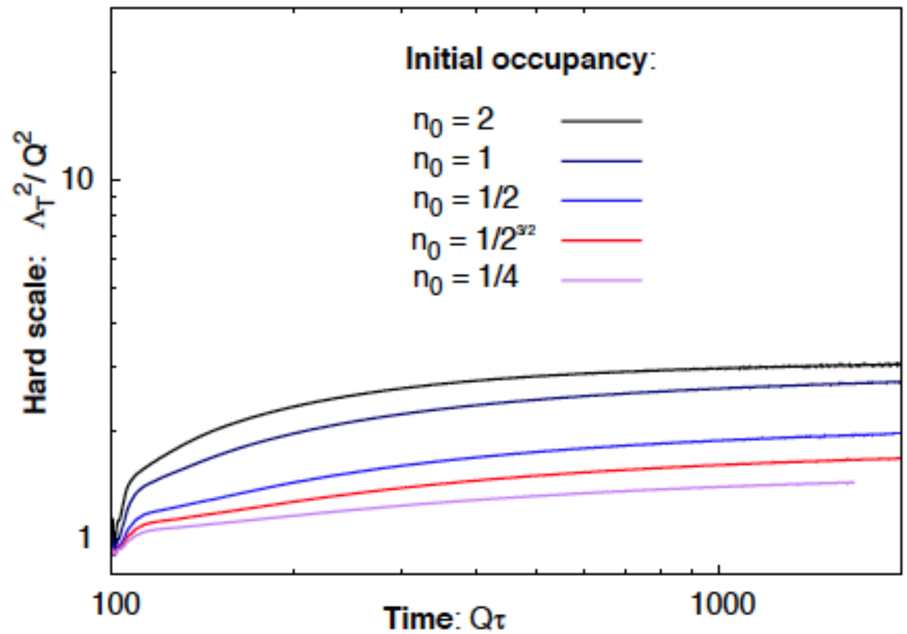
Model for overoccupied initial distribution

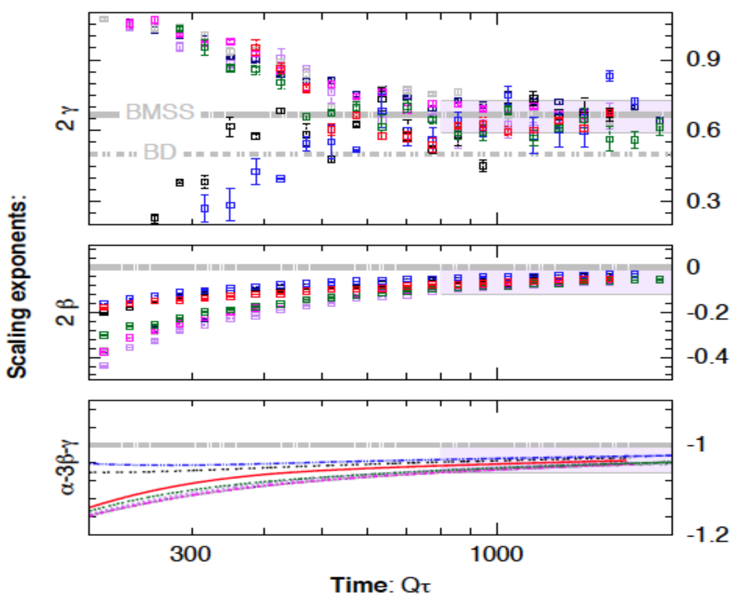
$$f(p_T, p_z, \tau_0) = \frac{n_0}{2g^2}\Theta\left(Q - \sqrt{p_T^2 + (\xi_0 p_z)^2}\right)$$

Vary n_0, τ_0, ξ_0 ; construct stochastic gauge fields that satisfy this initial condition; solve 3+1-D Yang-Mills; avg. over initializations

Expanding plasma: weak coupling results-I



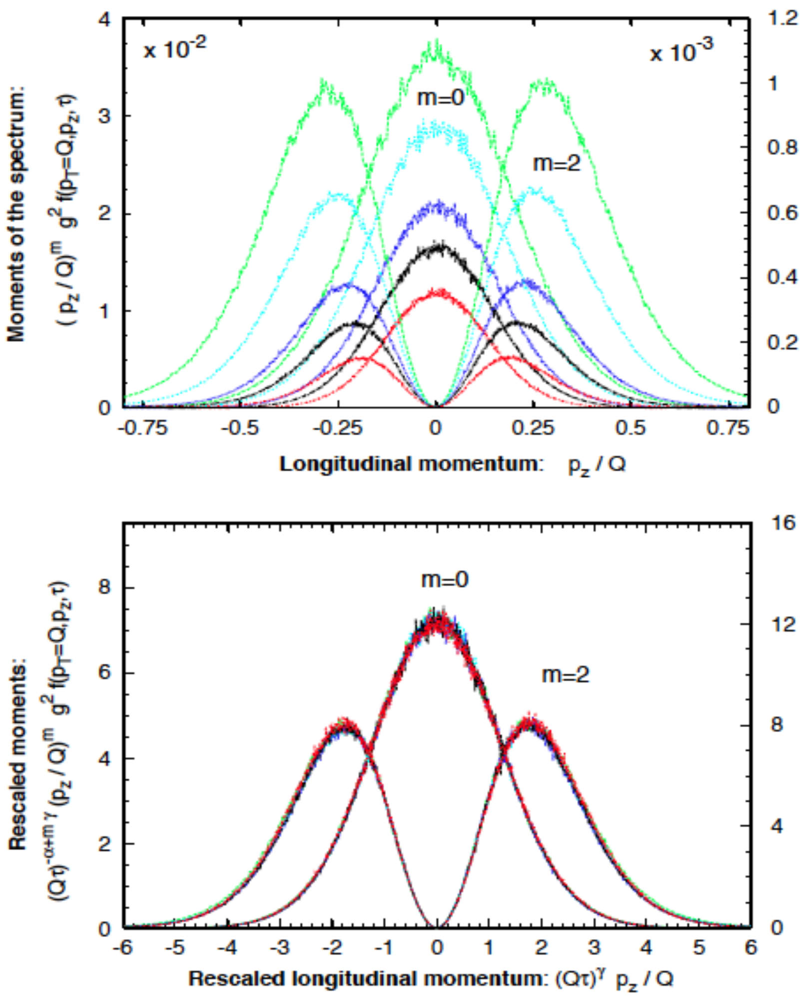


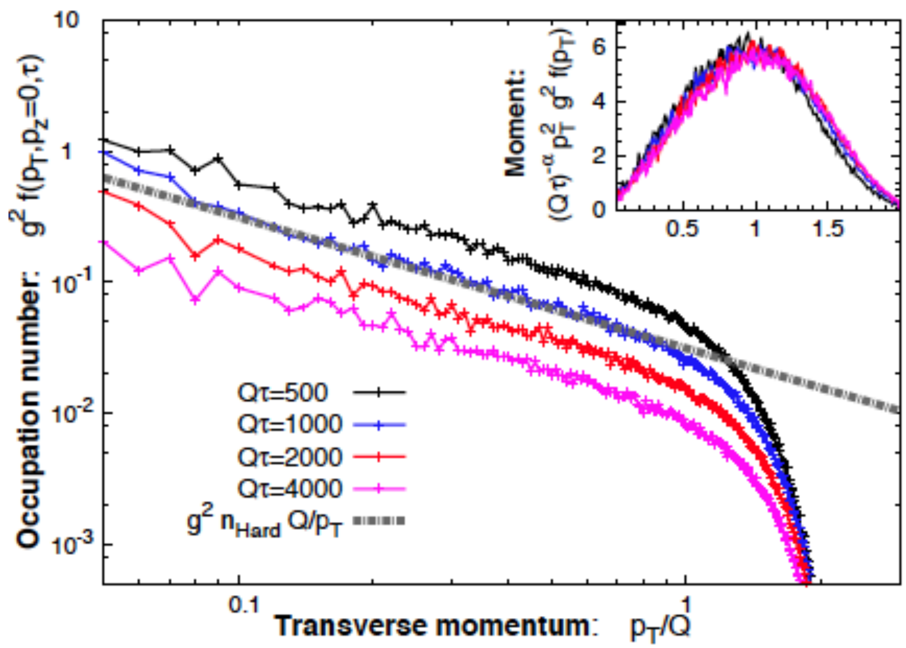


$$\alpha \sim -2/3$$

$$|\beta| < 0.06 \; ; \; 2\gamma = 0.67 \pm 0.07$$

Expanding plasma: weak coupling results-II





Transverse spectra has Bose T/p self-similar shape even as longitudinal spectra are being squeezed…

Kinetic theory in the overoccupied regime

For $1 < f < 1/\alpha_S$ a dual description is feasible either in terms of kinetic theory or classical-statistical dynamics …

Mueller,Son (2002)
Jeon (2005)

Different scenarios:

- ❑ Elastic multiple scattering dominates in the Glasma

BMSS: Baier,Mueller,Schiff,Son

- ❑ Rescattering due to plasma (Weibel) instabilities

KM: Kurkela, Moore

- ❑ Transient Bose condensation+multiple scattering

BGLMV: Blaizot,Gelis,Liao,McLerran,RV

Kinetic theory in the overoccupied regime

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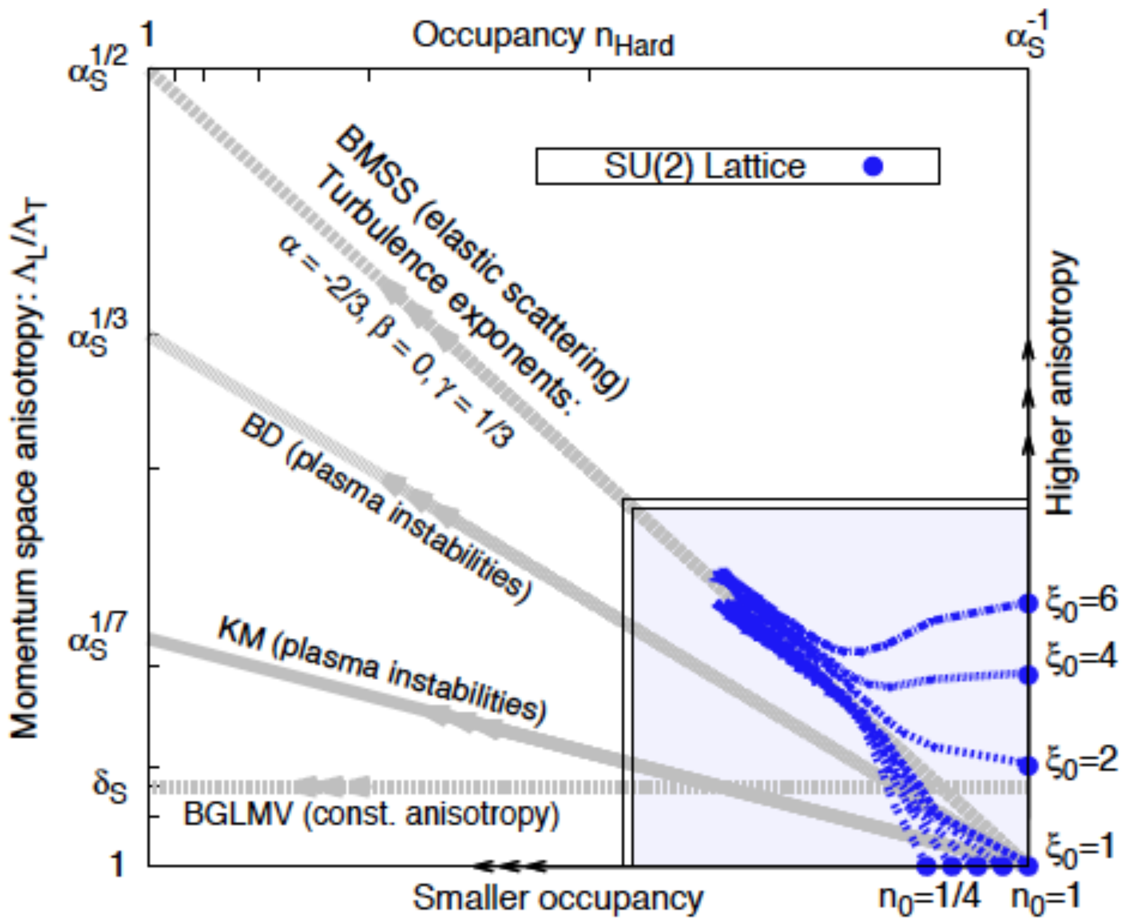


Quo vadis, thermalization?

- If multiple scattering a la BMSS is the right classical scenario, scattering starts early…

  but full thermalization only occurs in the quantum regime

  due to inelastic 2 <-> 3 processes

- **BMSS result:** $\tau_{\mathrm{iso.}} \sim Q^{-1}\alpha_S^{-5/2} \;\; ; \;\; T_{\mathrm{therm}} \sim \alpha^{2/5} Q$

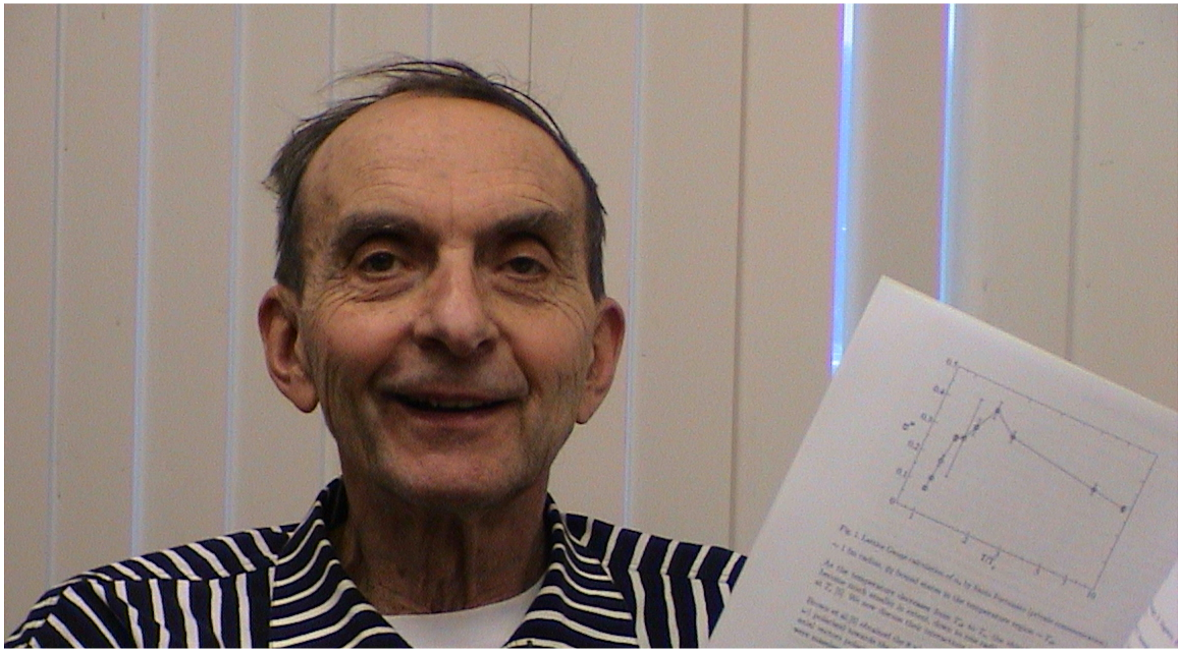
Notes:

1) for realistic couplings, prefactors may be very important.

Gelis,Epelbaum:1307.2214

2) inelastic mini-jet 2 <-> 3 processes also display turbulent dynamics
   -- may speed up thermalization

Blaizot,Iancu,Mehtar-tani:1301.6102



Gerry will live on in our memories:
many thanks to Dima, Edward, Ismail and Tom for giving us a chance to relive them