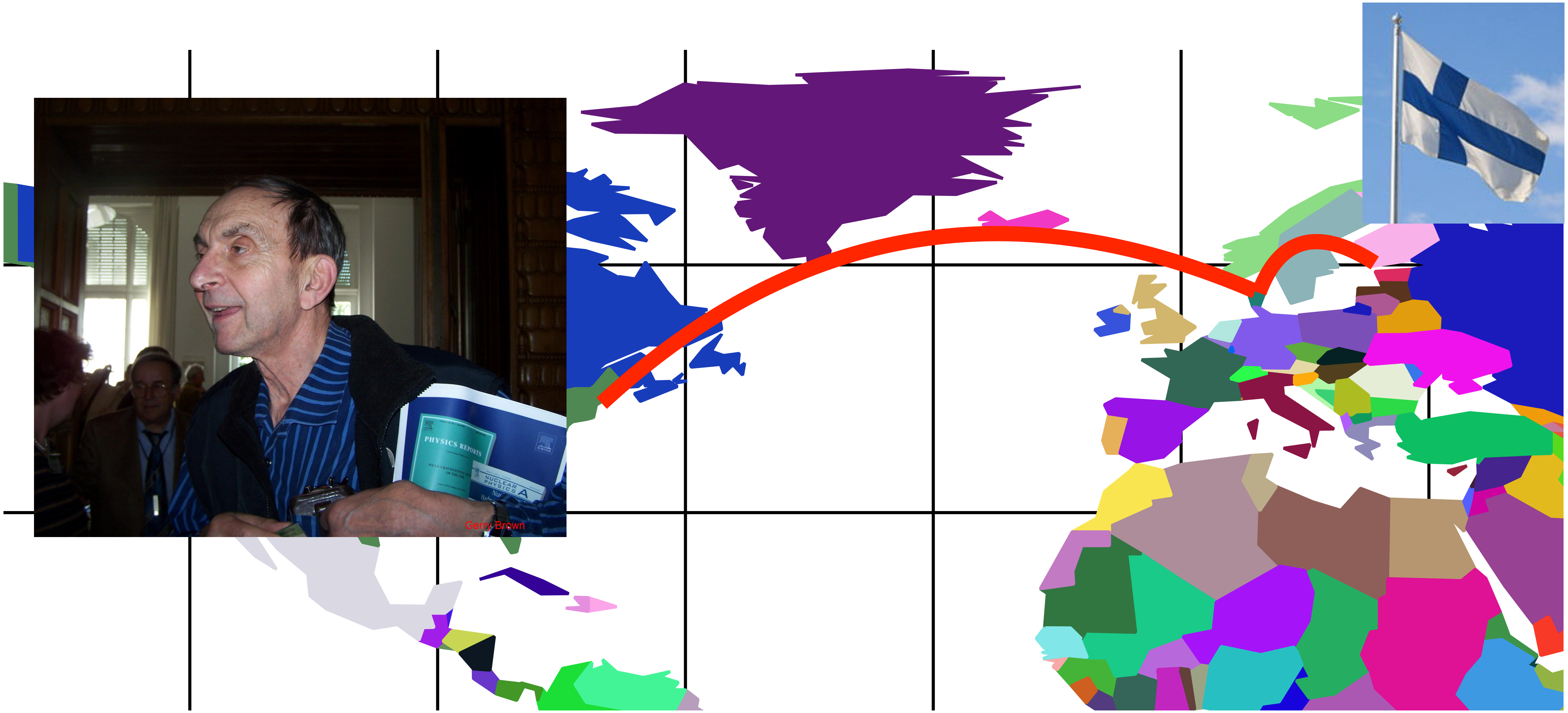
Probing the QCD Phase Diagram with Fluctuations

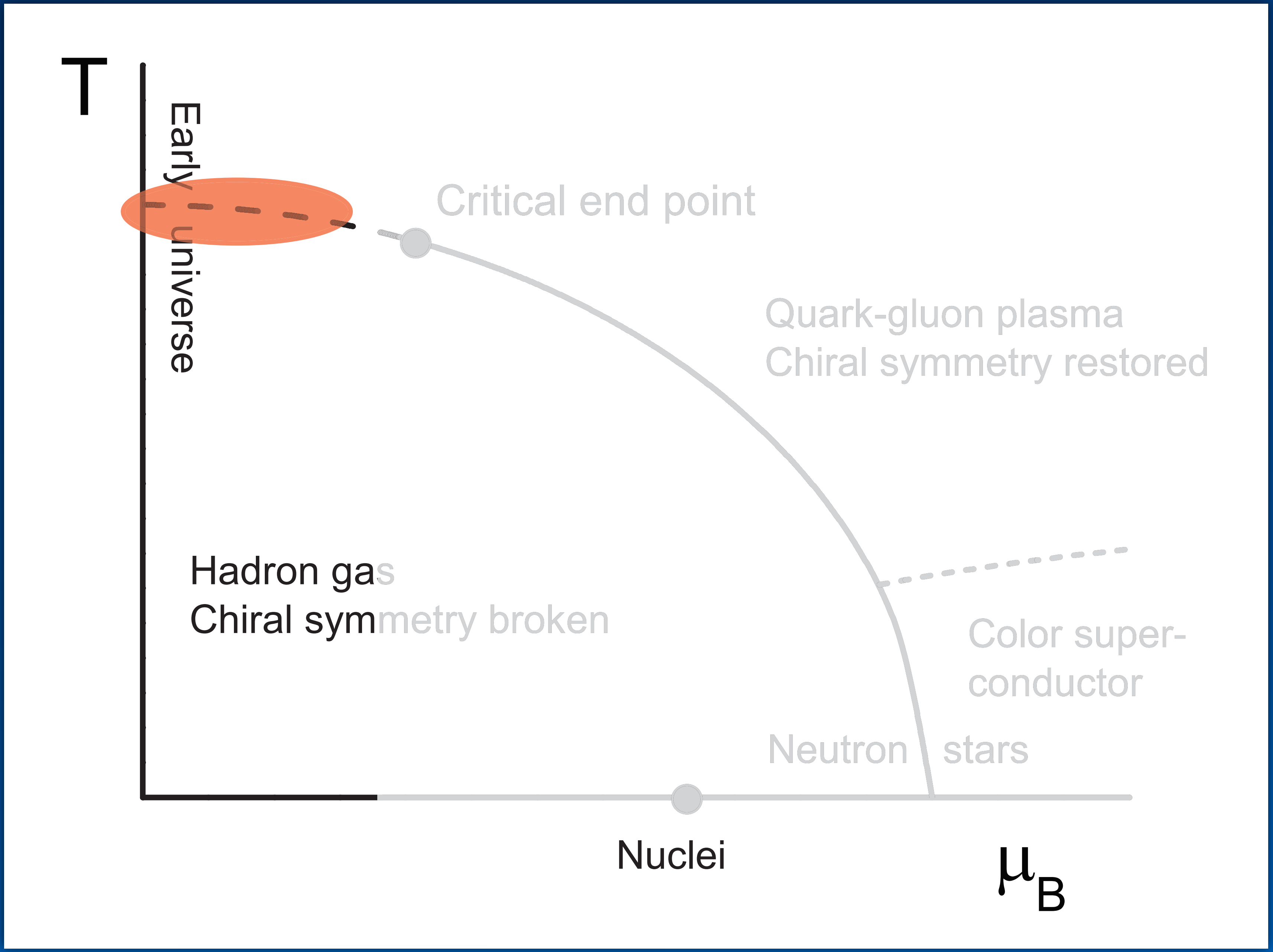
Bengt Friman
GSI Darmstadt

A Tribute to Gerald E. Brown
Stony Brook Nov. 24-26, 2013



- Gerry played a crucial role for the education of young nuclear theorists in Finland
- A large number moved with Gerry's help from NORDITA to research positions in the US and later got faculty jobs in Finland

Exploring the QCD phase diagram



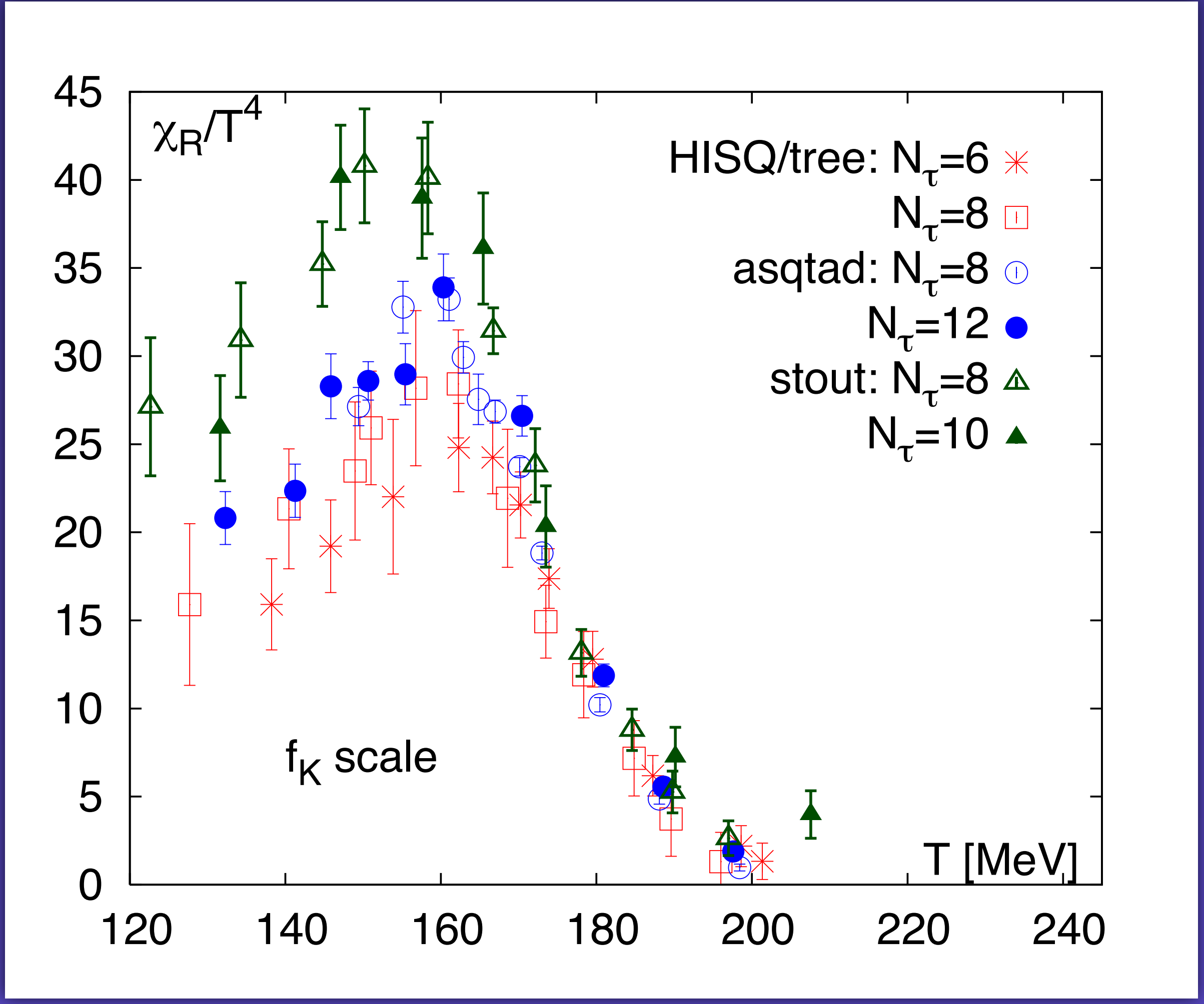
- Experiment
- Lattice QCD
- Models

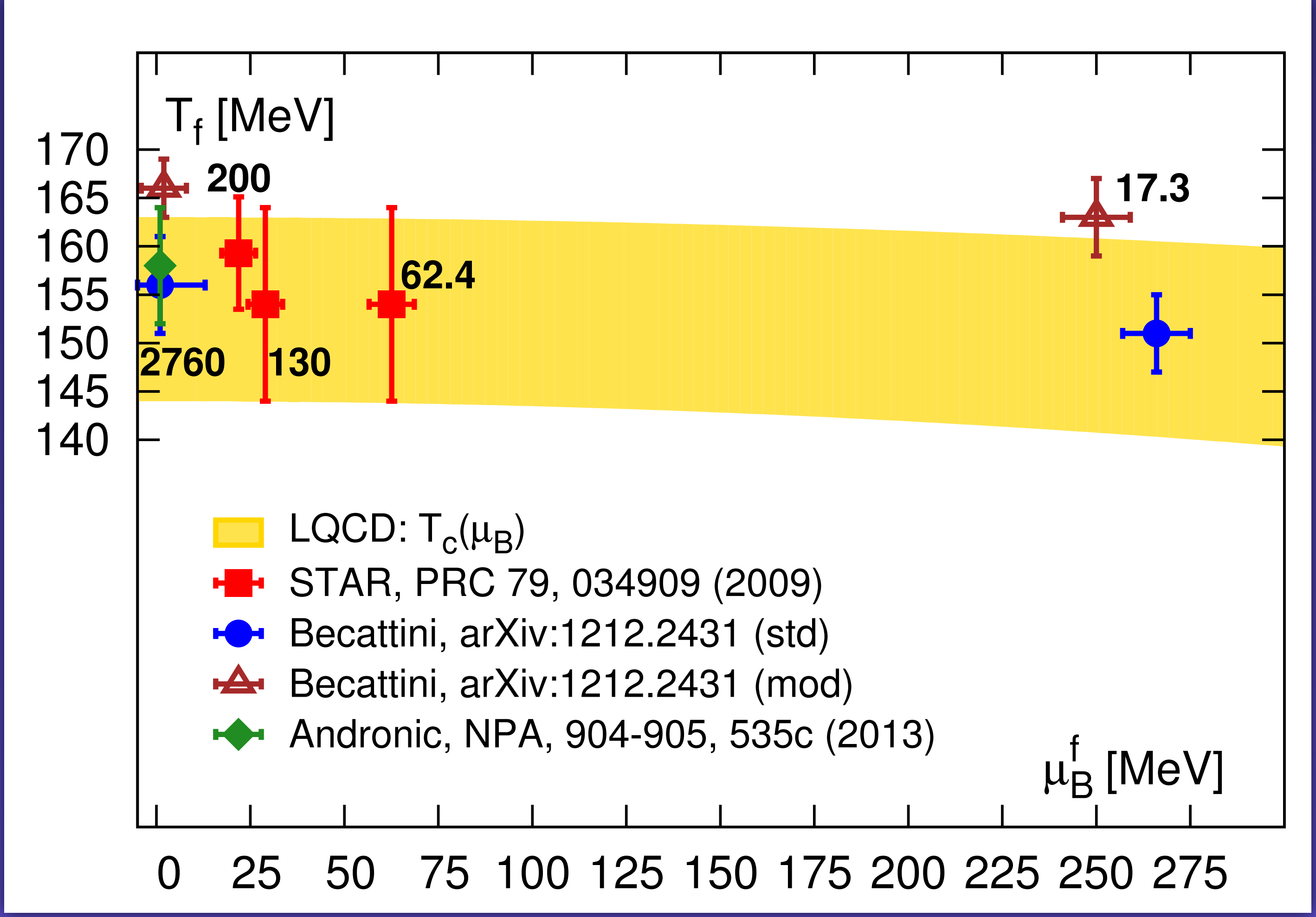
- Focus on $\mu \simeq 0$
- O(4) transition(?)
- Chiral vs. deconf

Fluctuations → phase boundary?

- Chiral susceptibility: $\chi_m = \frac{\partial^2 p}{\partial m_q^2}$

Freeze-out close to crossover





BNL-Bielefeld, PRD 85,054503

F. Karsch, 2013

Fluctuations of conserved charges

- Consider a subvolume V
- Susceptibility (2nd cumulant)

$$\chi = VT\frac{\partial^2 p}{\partial \mu^2} = \langle N^2 \rangle - \langle N \rangle^2 \qquad N = N_B - N_{\bar{B}}$$

- Less singular than order parameter fluctuations, but easier to access.
- Criticality shows up in higher cumulants:

$$\chi_n = c_n = VT\frac{\partial^n p}{\partial \mu^n}$$

Critical scaling

- Free energy density: backgound + singular part

$$f = f_r + f_s \quad \text{where}$$

$$f_s(t,h) = h^{1+1/\delta} f_f(z) \qquad M = h^{1/\delta} f_G(z) + bg.$$

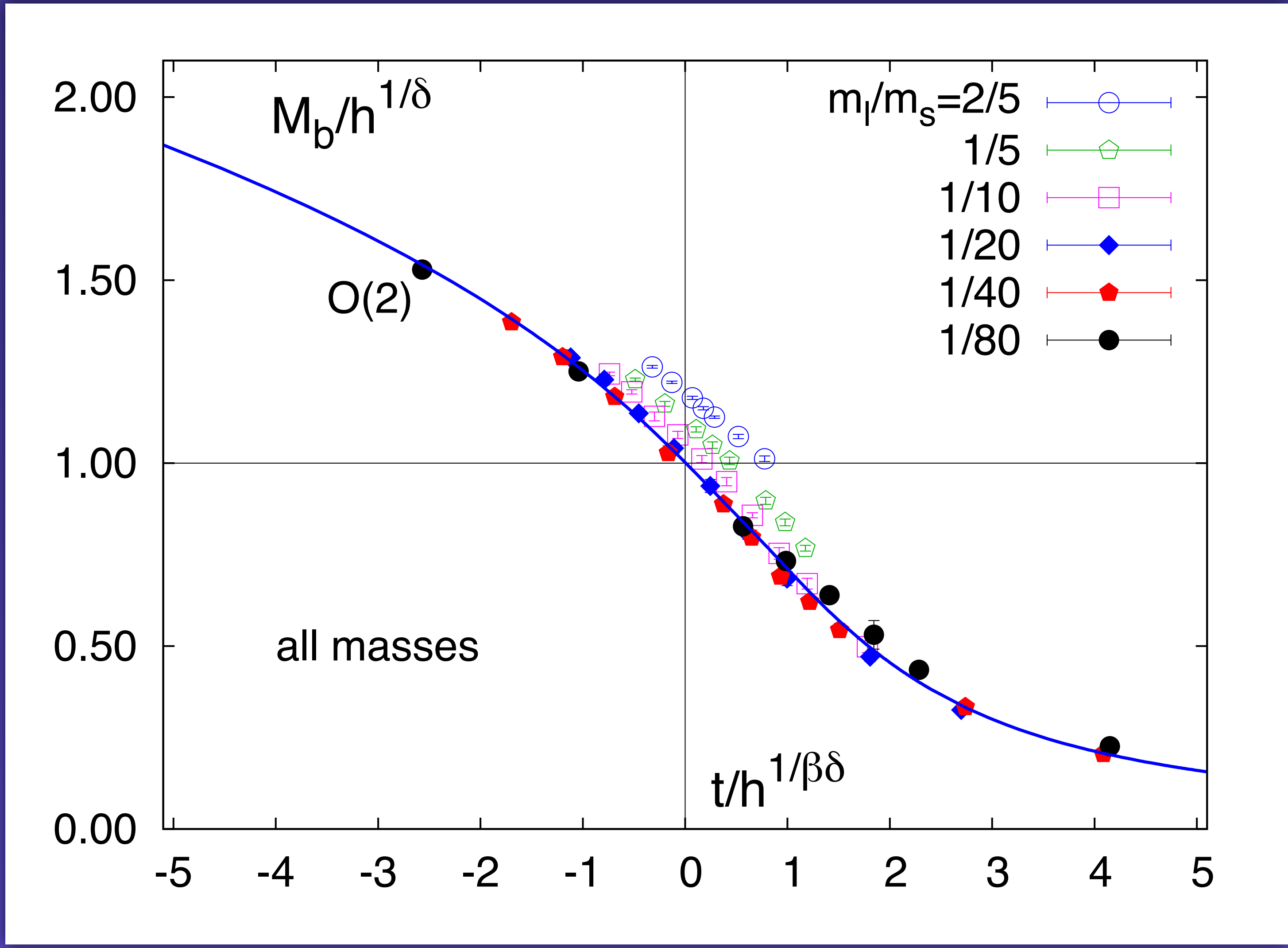
- Scaling variable

$$z = t/h^{1/\beta\delta}$$

$$t = (T - T_c)/T_c$$

$$h = m_q/T_c$$

- Physical m_π in O(N) scaling regime



Ejiri et al., PRD 80, 094505

Baryon number cumulants $(\mu = 0)$

- Generalized scaling parameter

$$t = (T - T_c)/T_c + \kappa(\mu/T_c)^2, \quad z = t/h^{1/\beta\delta}$$

- Only even cumulants:

$$\chi_B^{2n} = -T\,\frac{\partial^{2n} F}{\partial \mu^{2n}} \sim -h^{(2-\alpha-n)/\beta\delta} f_f^{(n)}(z) + \ldots$$

$\chi_B^4 \sim -h^{-\alpha/\beta\delta} f_f^{(2)}(z) \quad \to 0$ in chiral limit $(\alpha \simeq -0.2)$

$\chi_B^6 \sim -h^{-(1+\alpha)/\beta\delta} f_f^{(3)}(z) / \sim \xi^{1.1}$

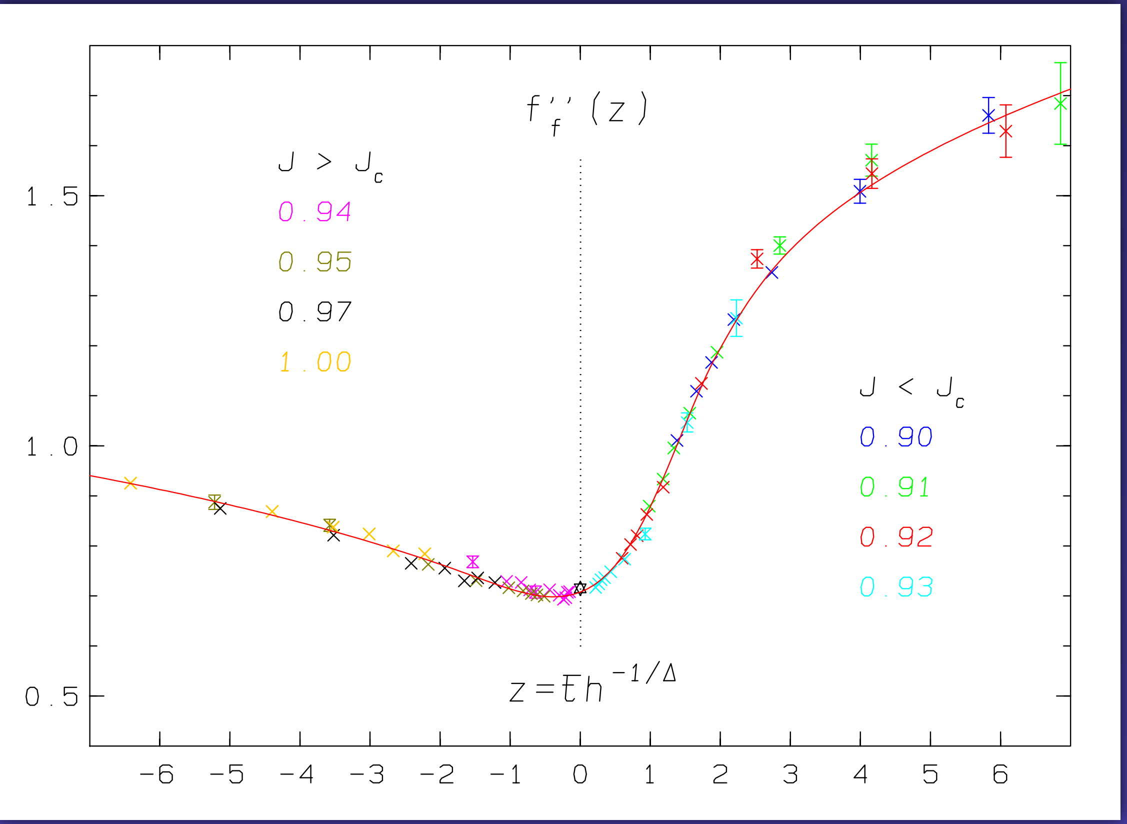
$\chi_B^8 \sim -h^{-(2+\alpha)/\beta\delta} f_f^{(4)}(z) / \sim \xi^{2.4}$

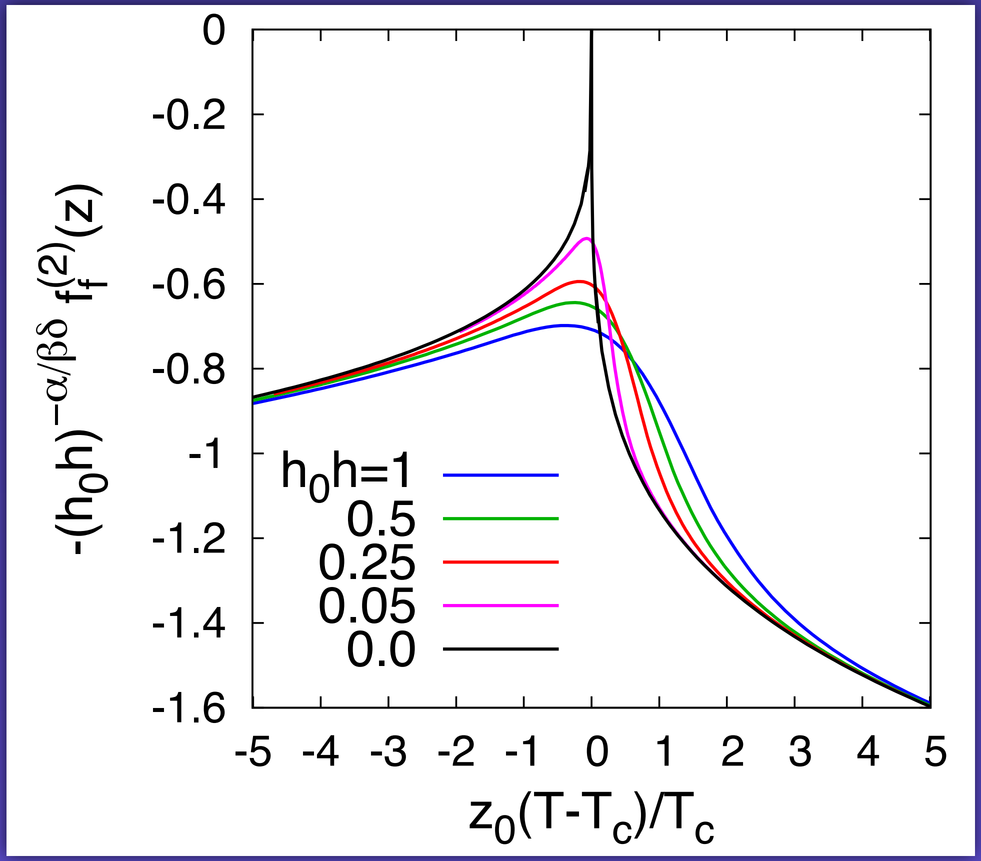
Universal O(4) scaling function

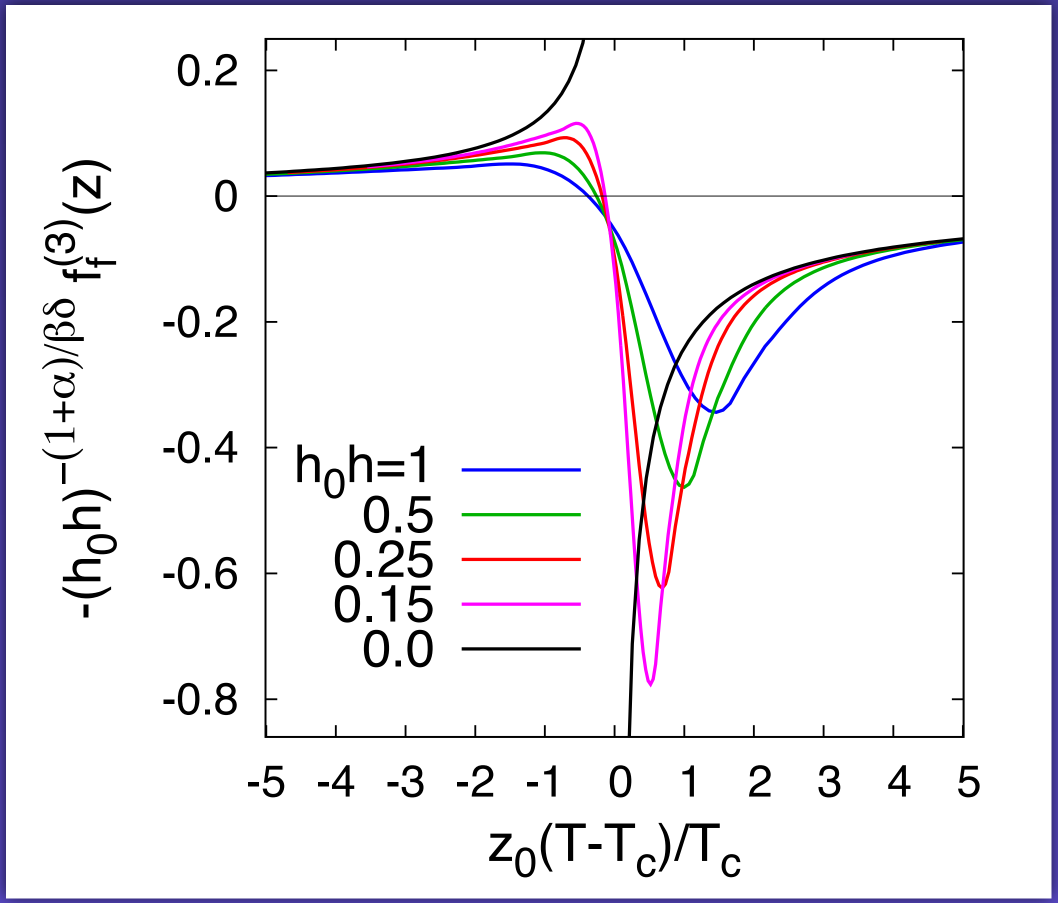
- Engels & Karsch
  PRD 85, 094506

$$\chi_B^4 \sim -h^{-\alpha/\beta\delta} f_f^{(2)}(z)$$

$$n \geq 6 \; : \; \chi_B^n < 0 \text{ for } T \simeq T_{pc}$$

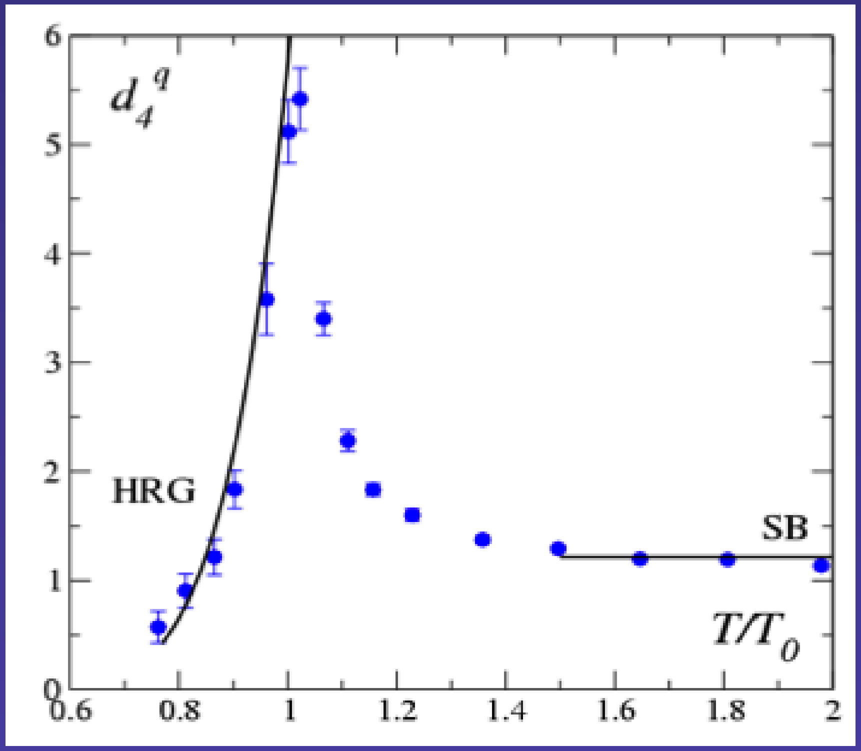




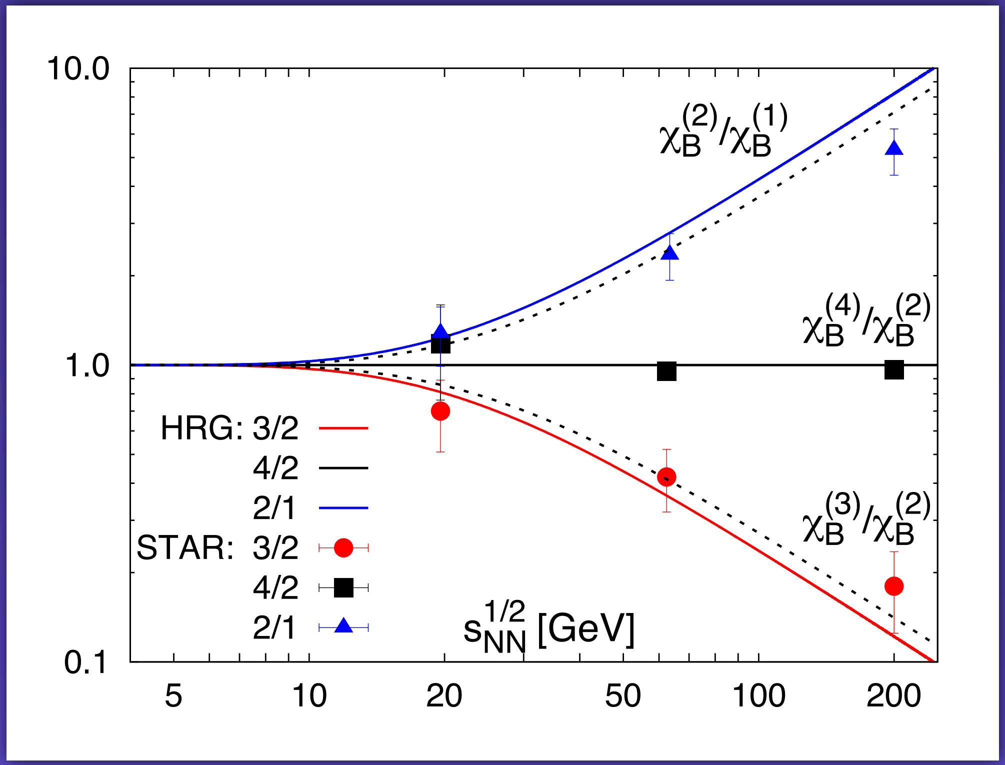


Baseline: HRG (no criticality)

- HRG reproduces lattice results, e.g. $\chi_4\ (T < T_{pc})$ (Ejiri et al. 2006)



- and experiment
  $$\chi_4/\chi_2 = 1$$
  $$\chi_3/\chi_2 = \tanh(\mu/T)$$
  $$\chi_2/\chi_1 = \coth(\mu/T)$$
  (Karsch, Redlich, 2011)

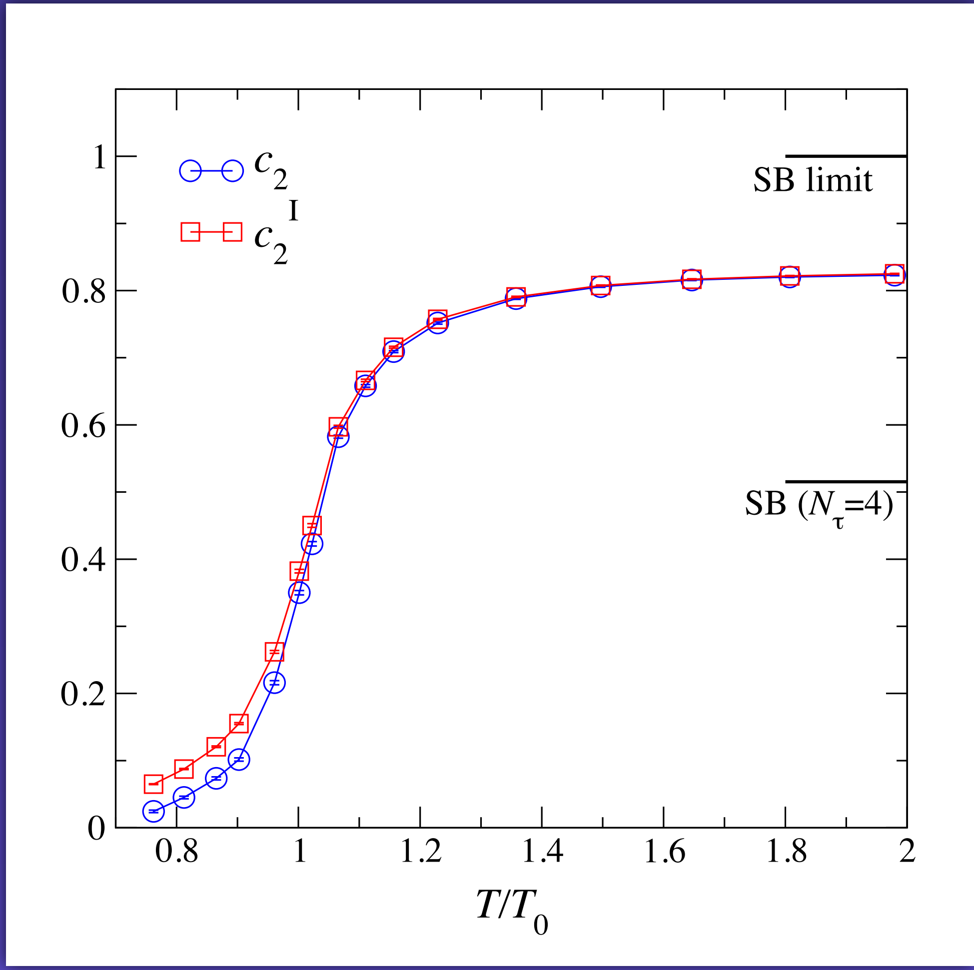


- Criticality in higher susc.

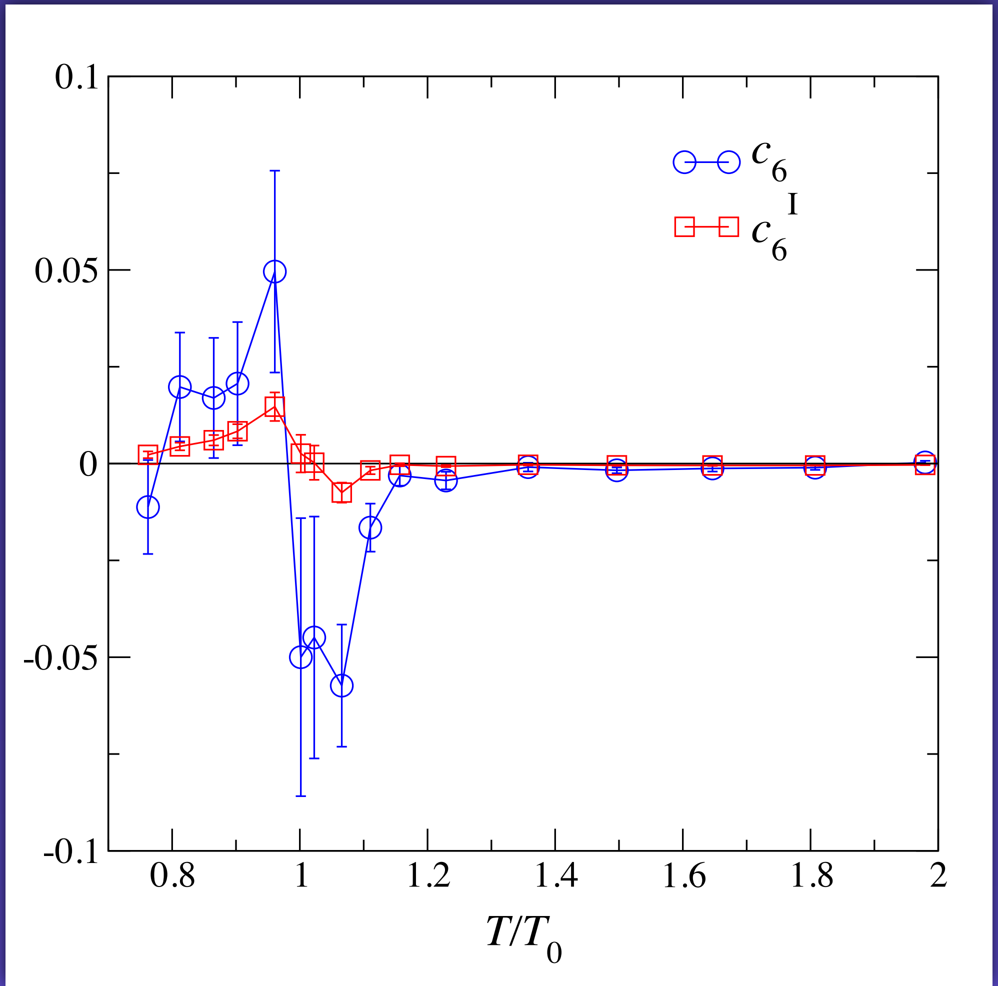
Tracking the phase boundary

- If $T_{fo} \simeq T_{pc}$, expect deviation from HRG $(\chi_6/\chi_2 \neq 1)$
- Allton et al. (Bielefeld-Swansea), PRD 71, 054508
- Look for $\chi_6/\chi_2 < 0$

χ_2

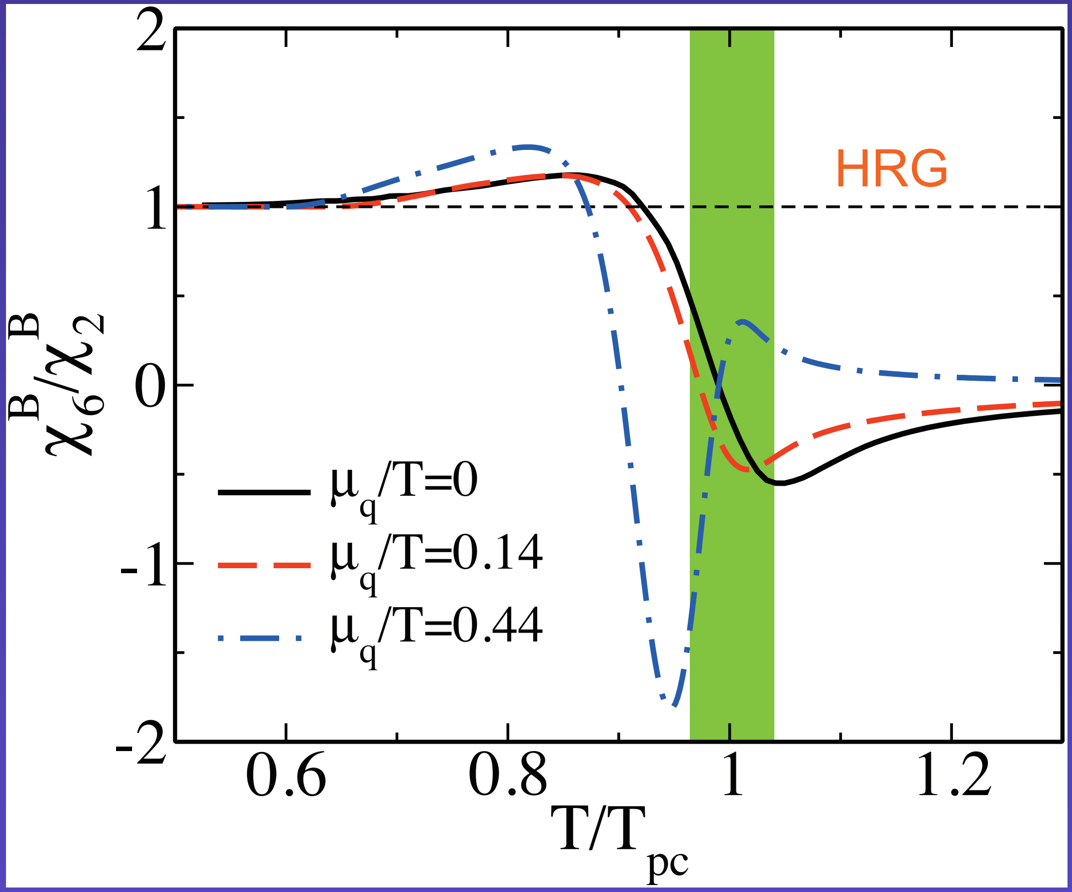


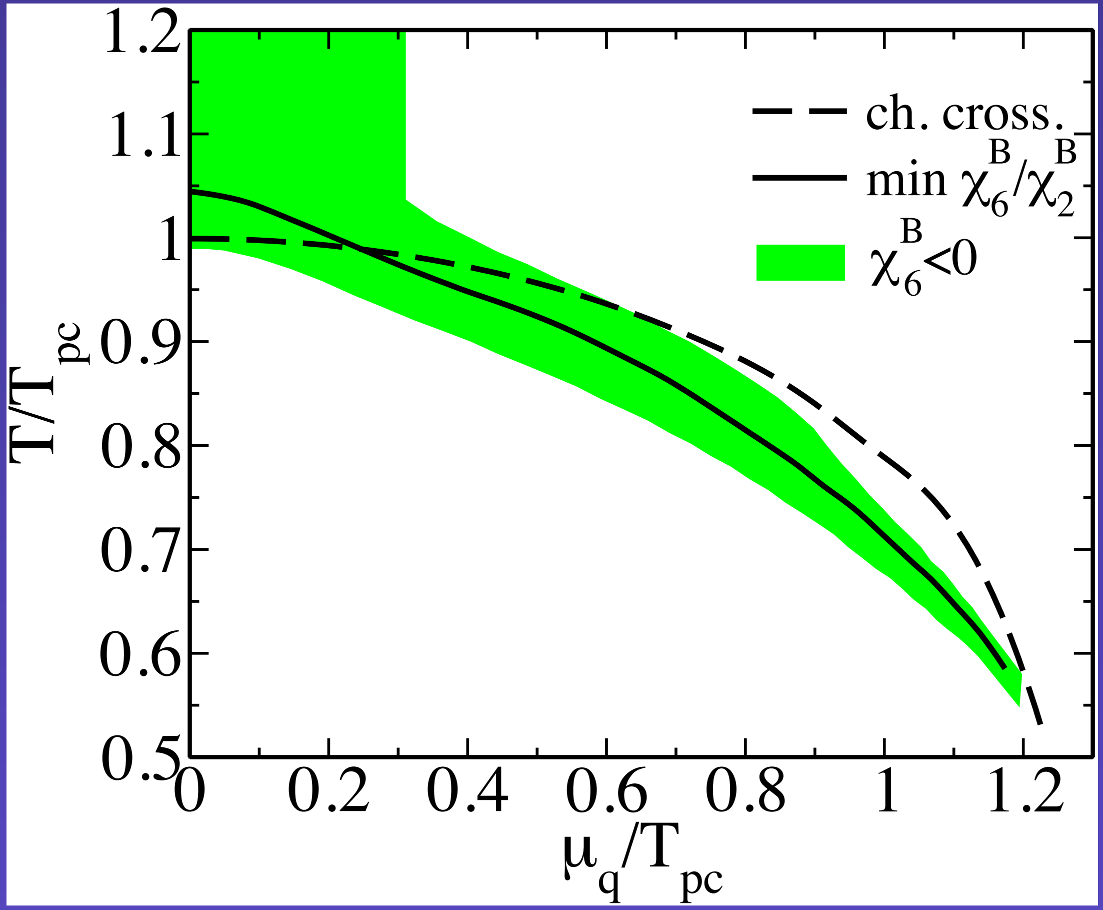
χ_6



Tracking the phase boundary

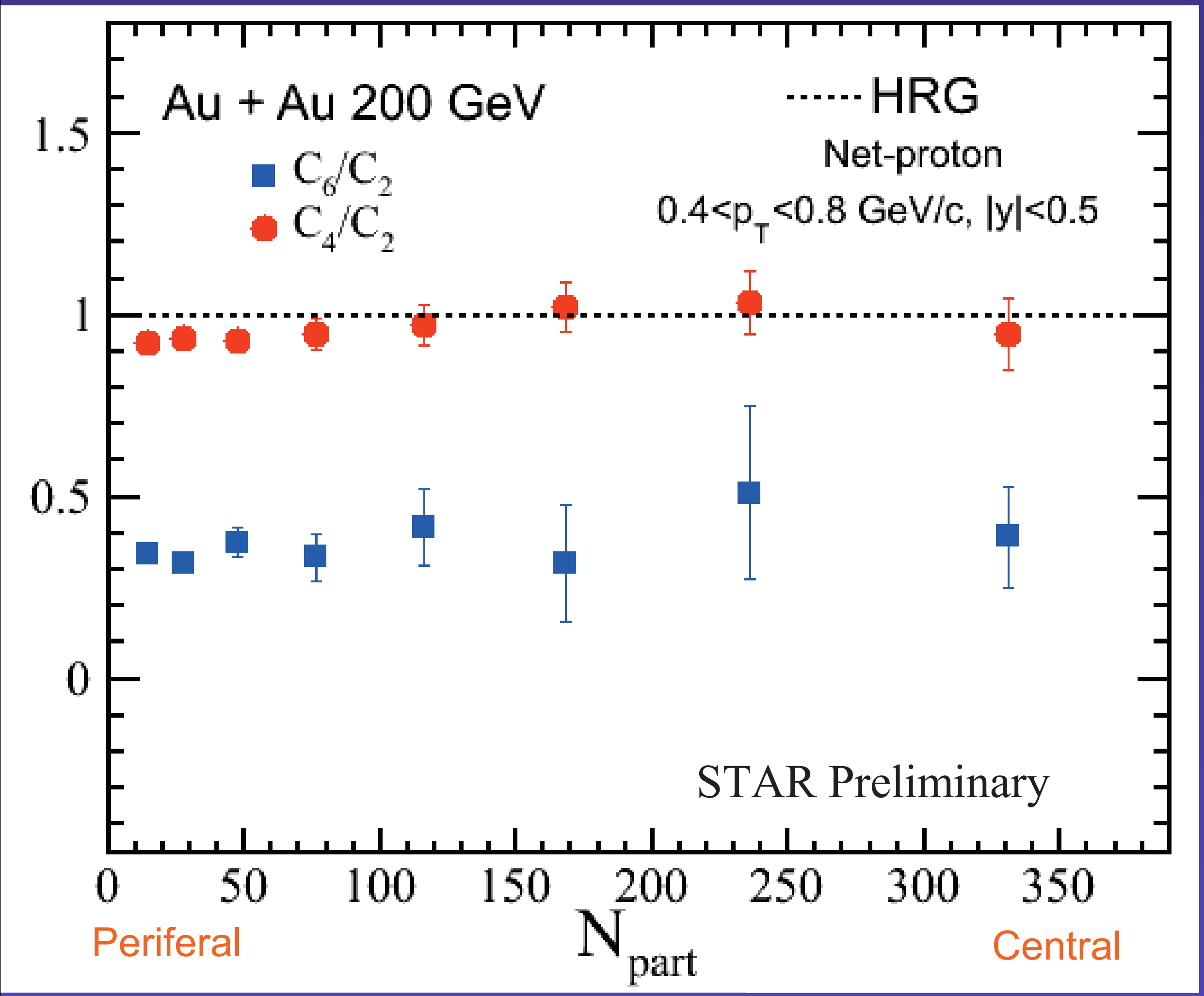
- If $T_{fo} \simeq T_{pc}$, expect deviation from HRG $(\chi_6/\chi_2 \neq 1)$
- PQM model (O(4) & Z(3)) in Functional RG
  B.F., Karsch, Redlich, Skokov, EPJ C71, 1694

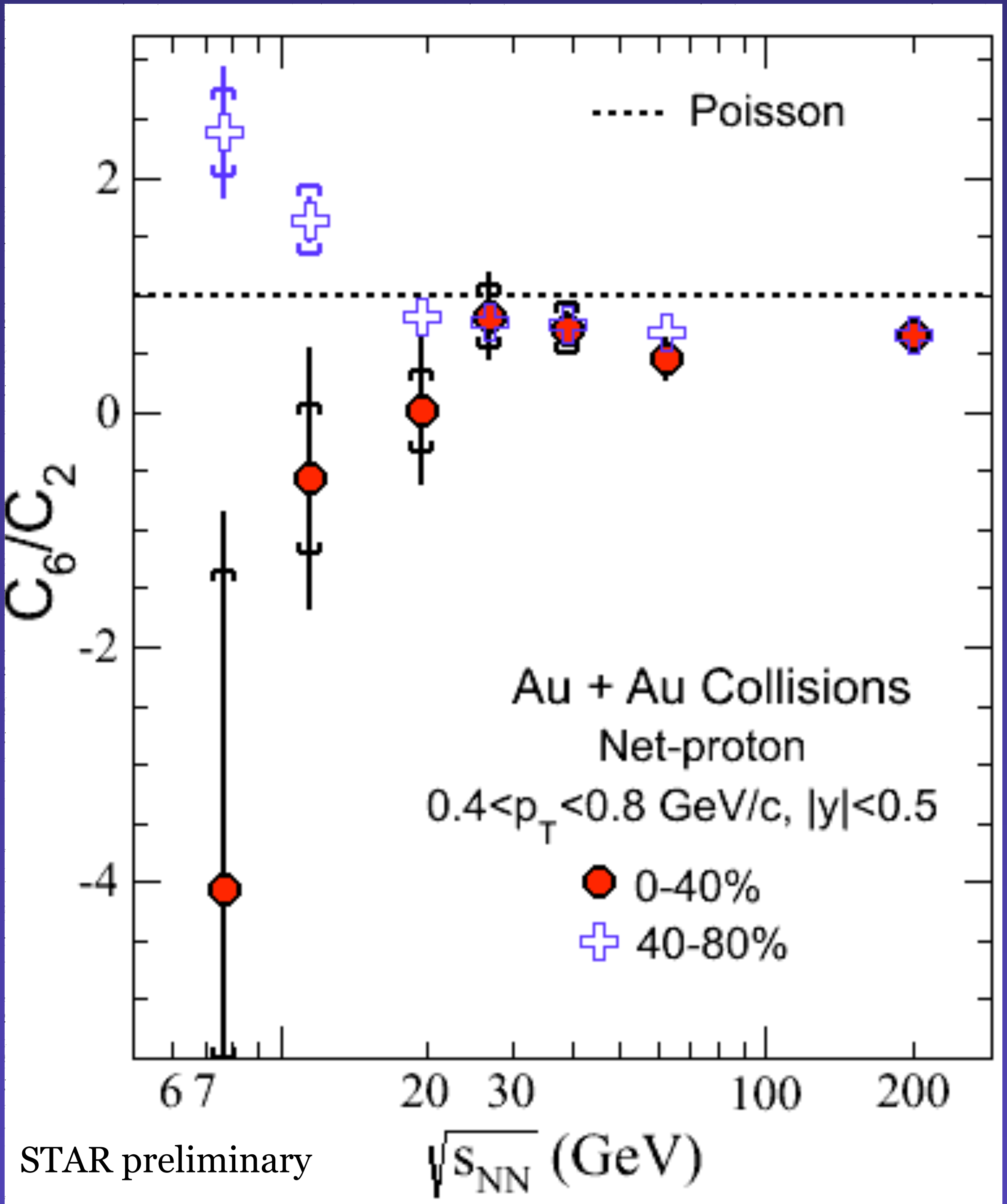




STAR data (net proton fluctuations)

- Au-Au collisions show clear deviation from HRG expect.





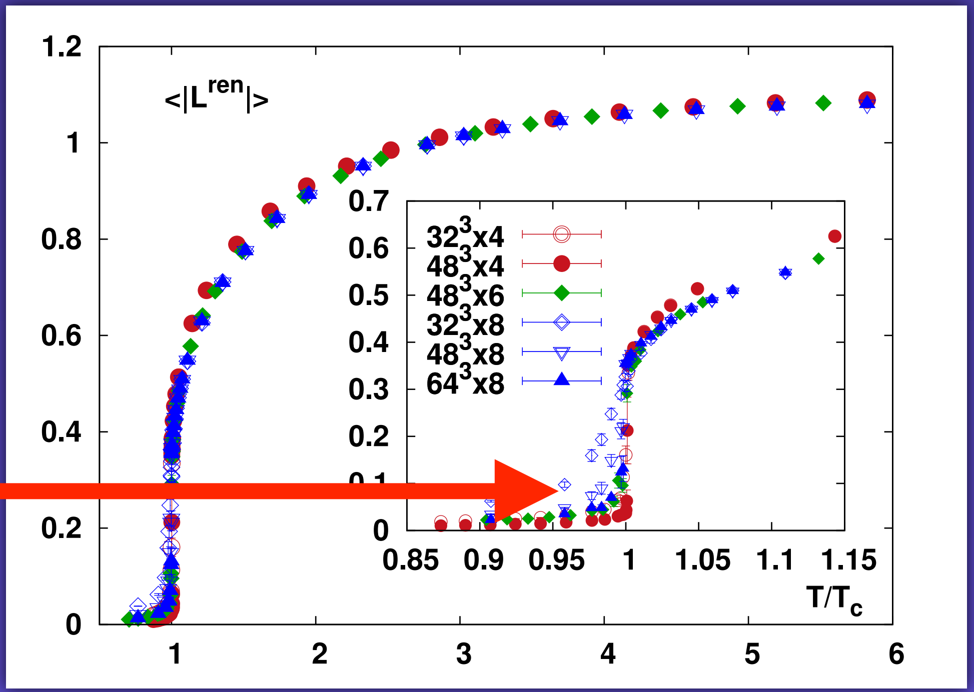
Uncertainties: finite volume, non-critical fluctuations, acceptance corrections

Deconfinement transition

- Polyakov loop: order parameter of deconfinement in pure gauge theory

$$L = \frac{1}{N_c} tr_c \mathcal{P} \exp\left[i \int_0^\beta A_4 \, d\tau \right] \qquad \langle L \rangle = e^{-\beta F_q}$$

- Z(N) symmetry $L \to e^{2\pi i n/N_c} L$ broken for $T > T_c$
- In SU(3) Polyakov loop complex $L = L_R + i\, L_I$
- Often use $\langle |L| \rangle$ as order parameter
- Finite size effects



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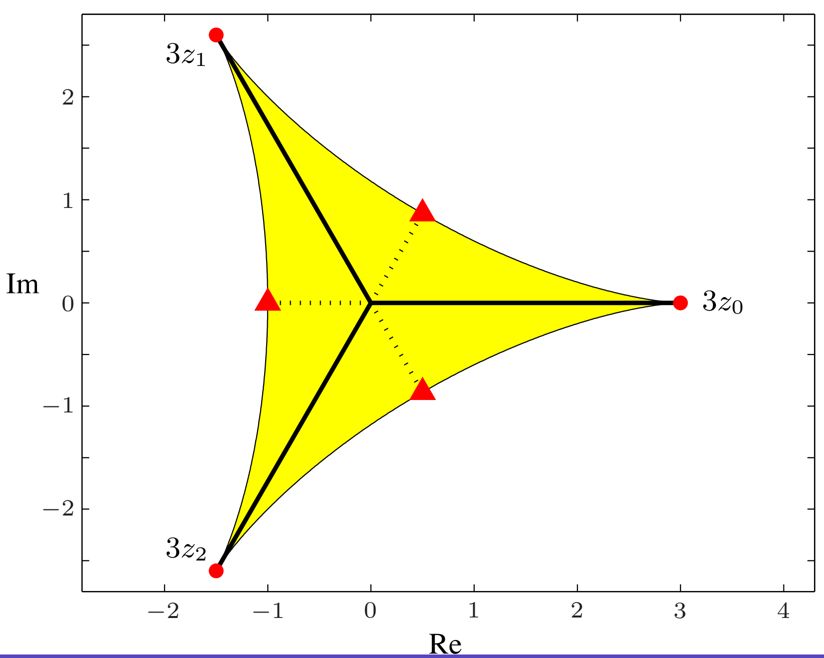
- Constraints from group struct. target space (SU(3)):



Polyakov loop fluctuations

- Polyakov loop susceptibilities in pure gauge th.

$$T^3\,\chi_A \;=\; \frac{N_\sigma^3}{N_\tau^3}\left(\langle |L|^2\rangle - \langle |L|\rangle^2\right)$$

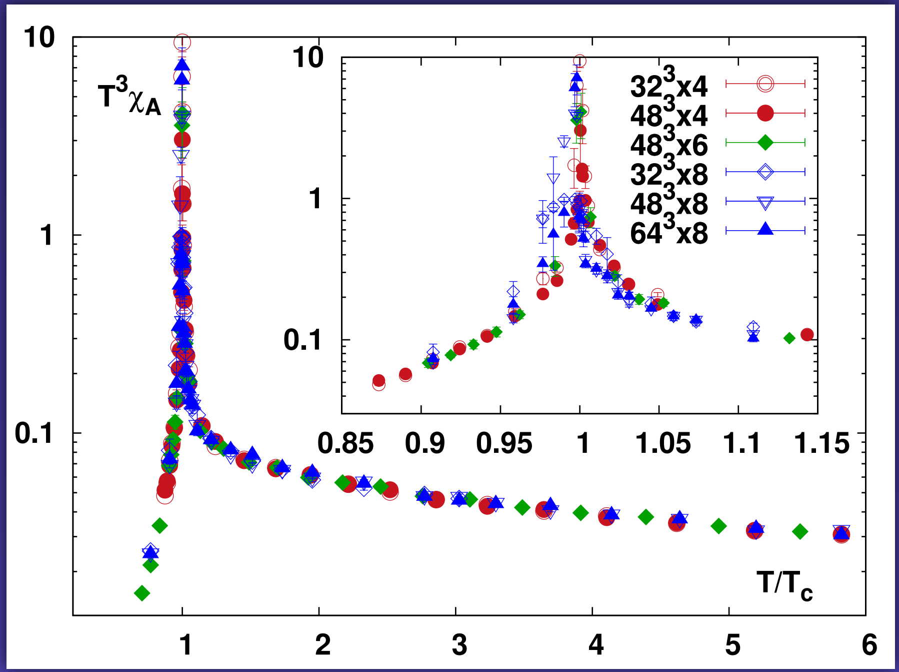
- Lo, B.F., Kaczmarek, Redlich, Sasaki

  PRD 88, 014506, 074502

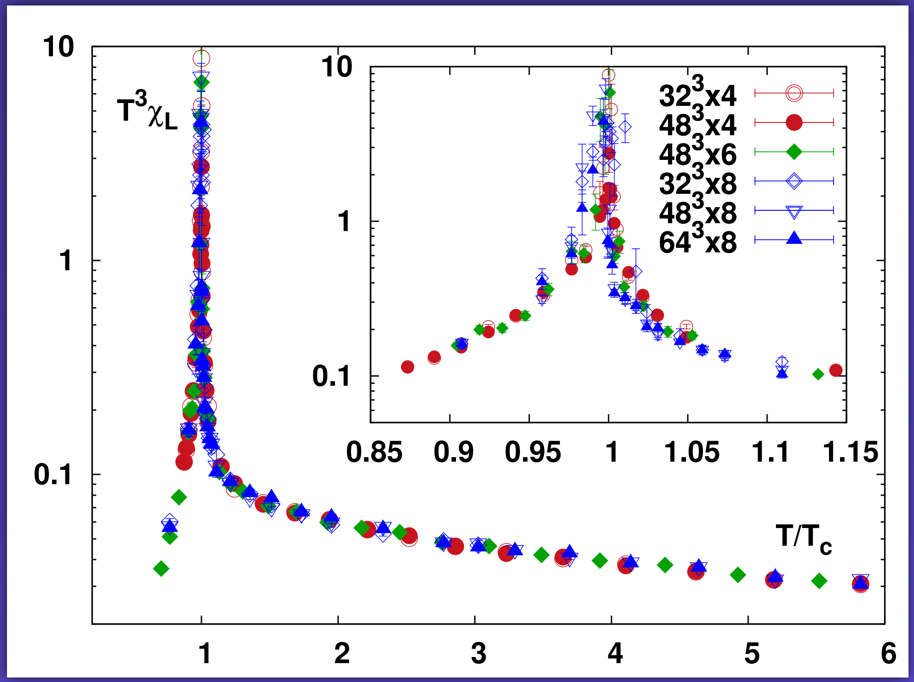
- Narrow transition region!

Renormalized

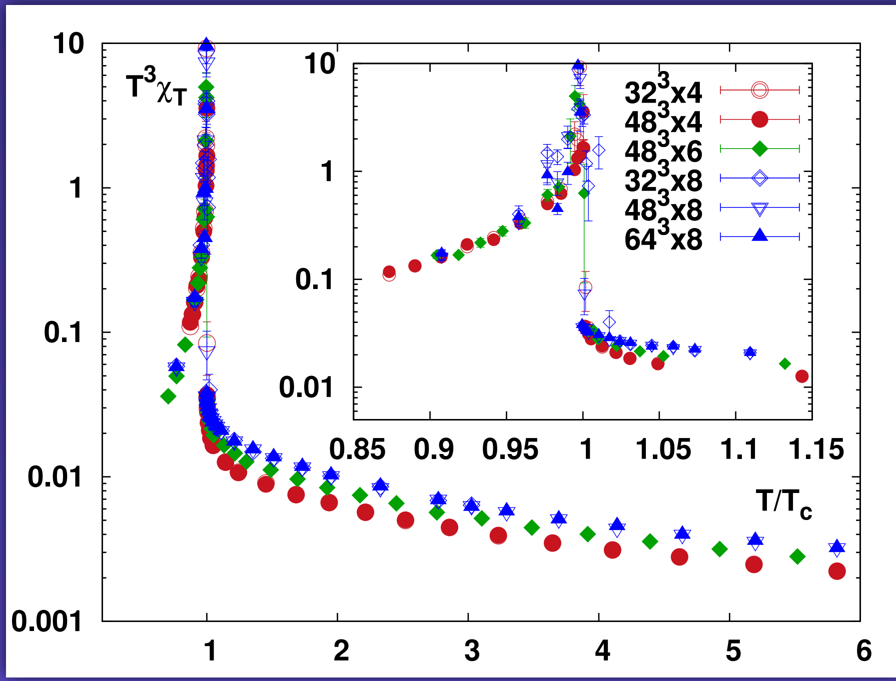
χ_A



χ_R



χ_I



Ratios of susceptibilities

- Less sensitive to finite size and lattice effects; Signals gluon deconf.

- $T > T_c$

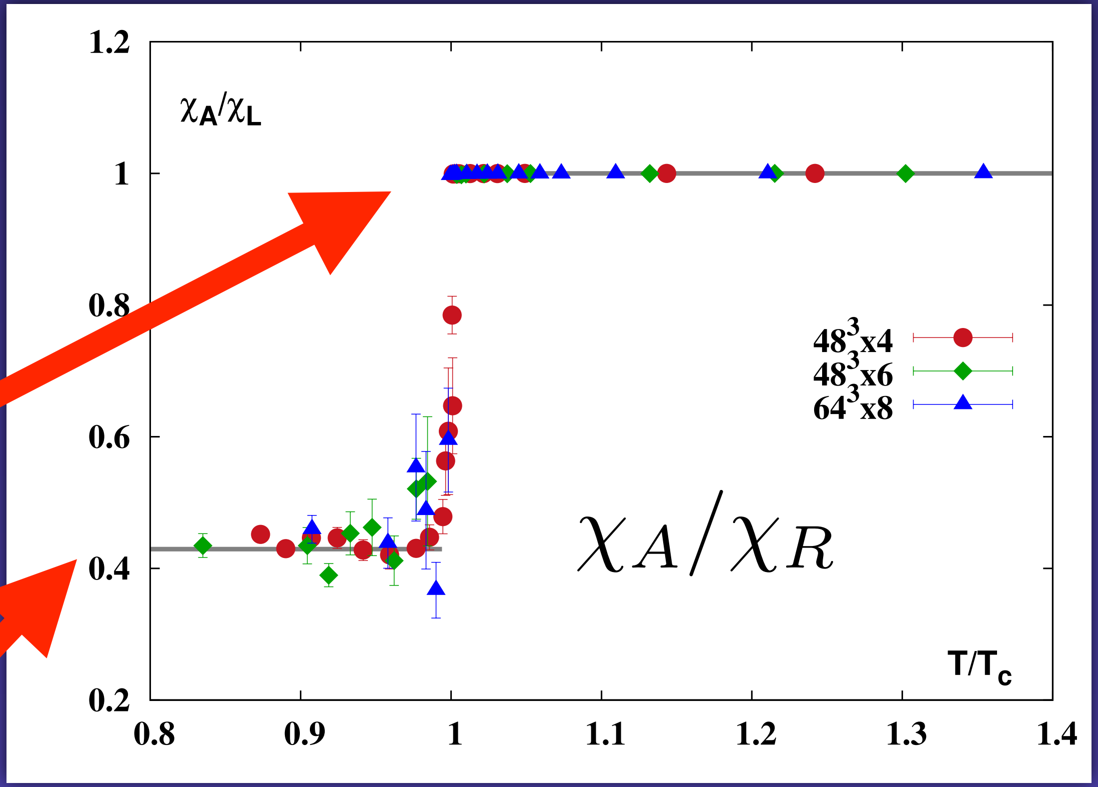
$$\chi_A = \chi_R + \mathcal{O}(\chi_I^2)$$

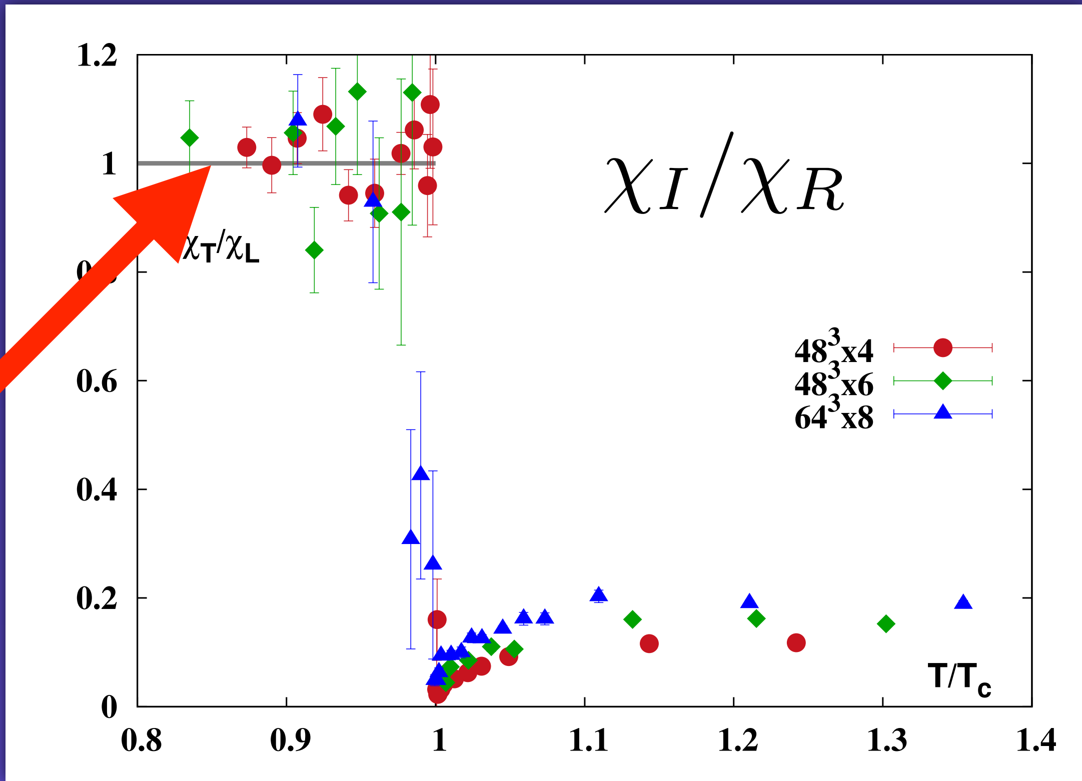
- $T < T_c$ consistent with Gaussian fluct.

$$\chi_A/\chi_R = 2 - \frac{\pi}{2}$$

$$Z = \int dL_R\, dL_I\, e^{V T^3 [\alpha(T)(L_R^2 + L_I^2)]}$$

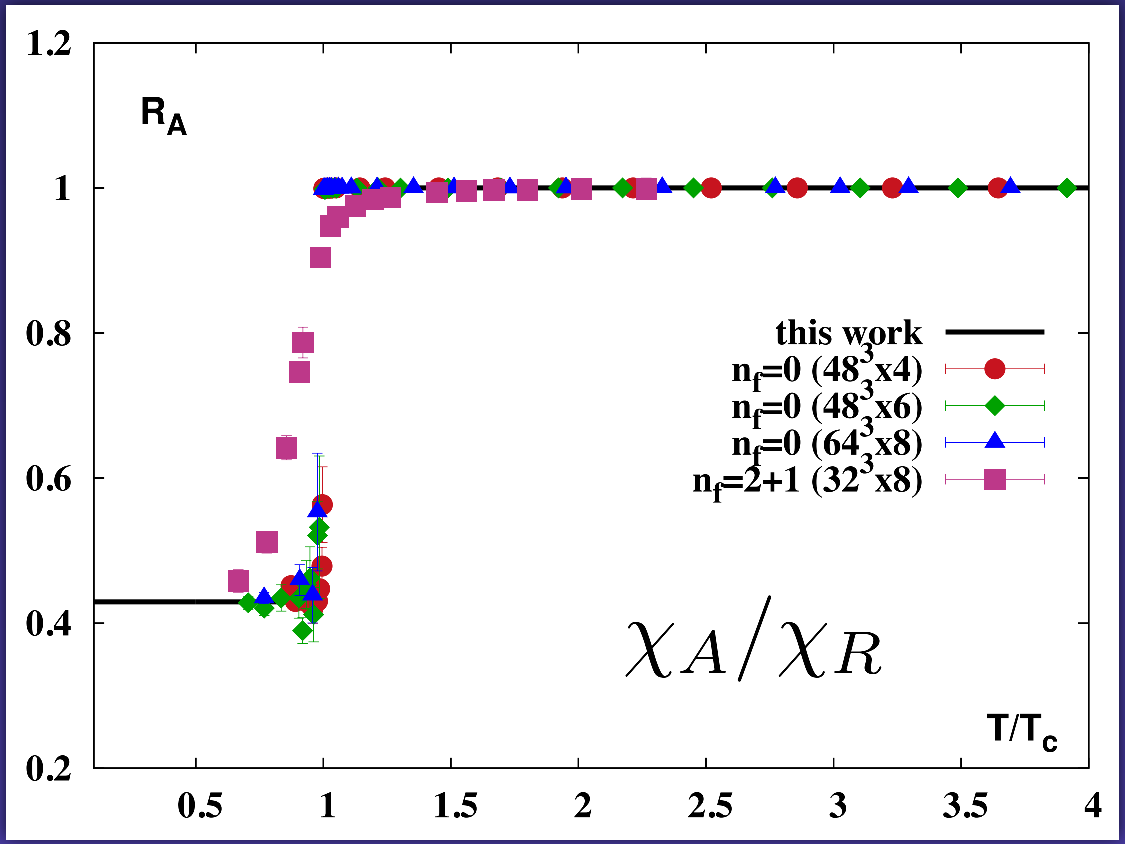
Good signature for deconf.!

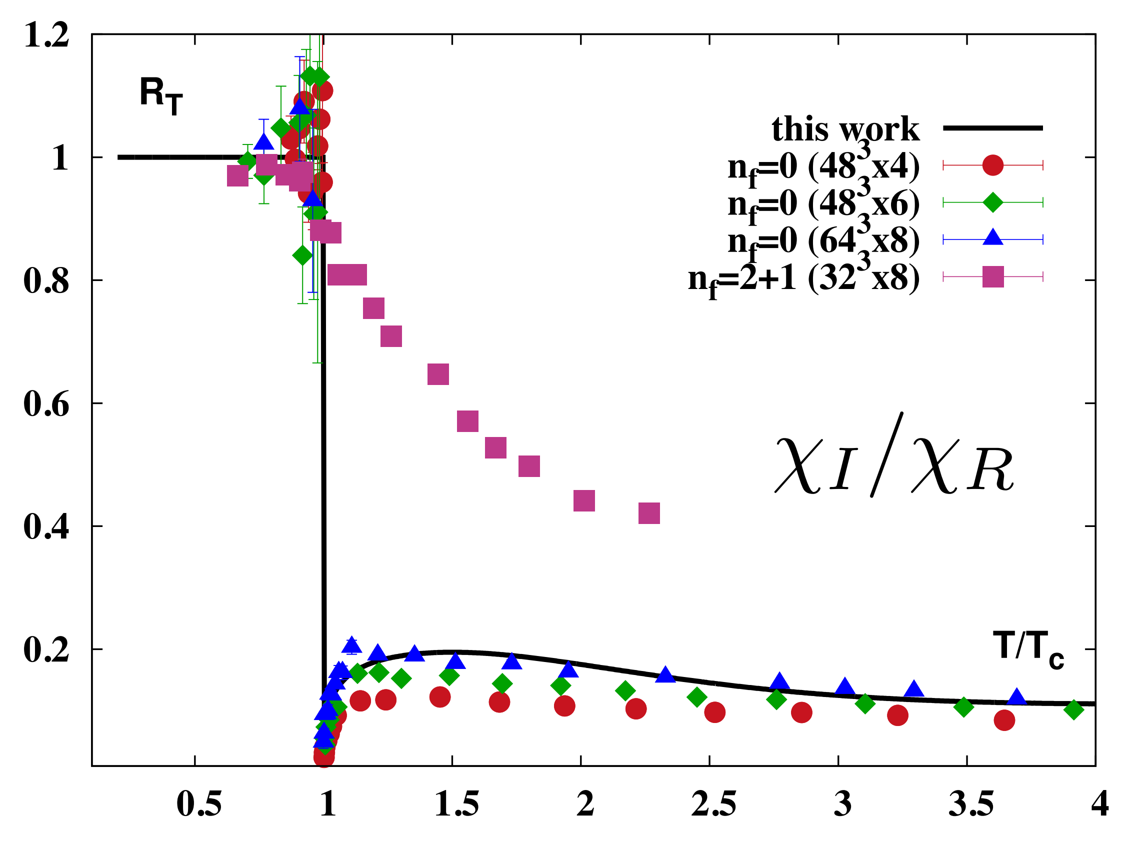




Adding quarks

- Quarks break Z(3) symmetry explicitly
- Expect smooth transition between “pure glue” ratios.
- Lo, B.F., Kaczmarek, Redlich, Sasaki, PRD 88, 074502
QCD results: HotQCD

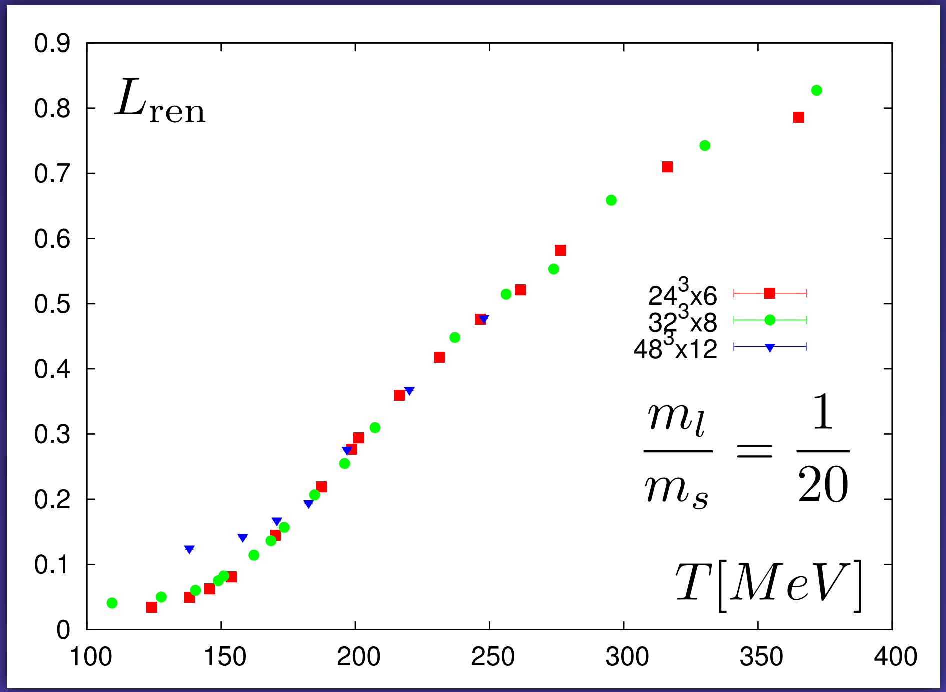


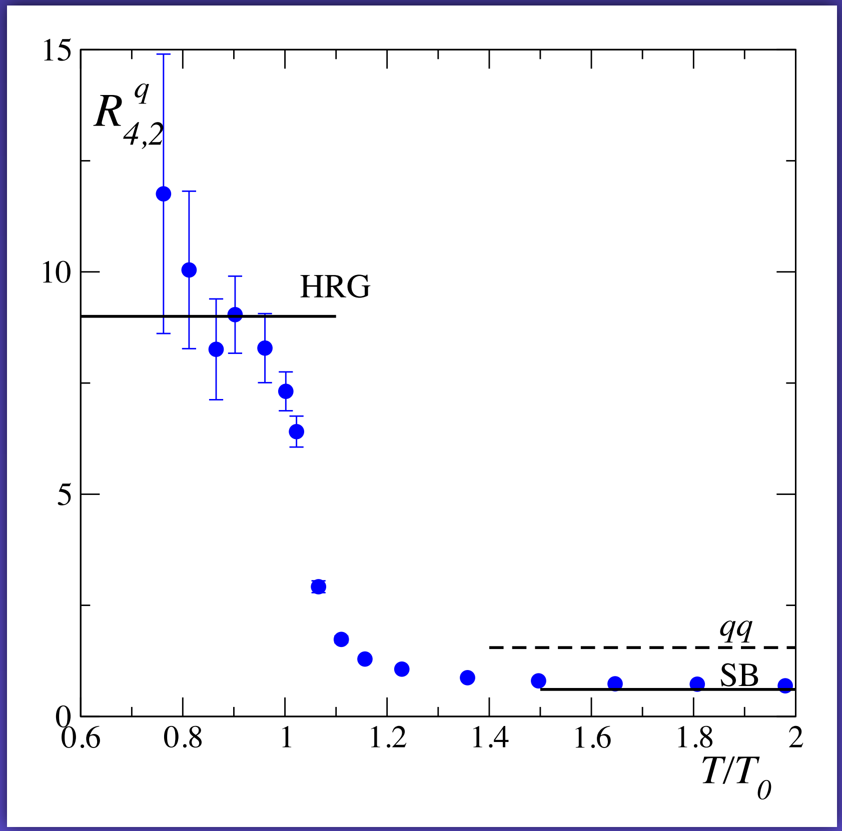


Deconfinement with light quarks

HotQCD

- Quarks breaks Z(3) symmetry: very smooth cross over?
- What do fluctuations imply?
- χ_4/χ_2 sensitive to baryon #
Ejiri et al., PLB 633, 275

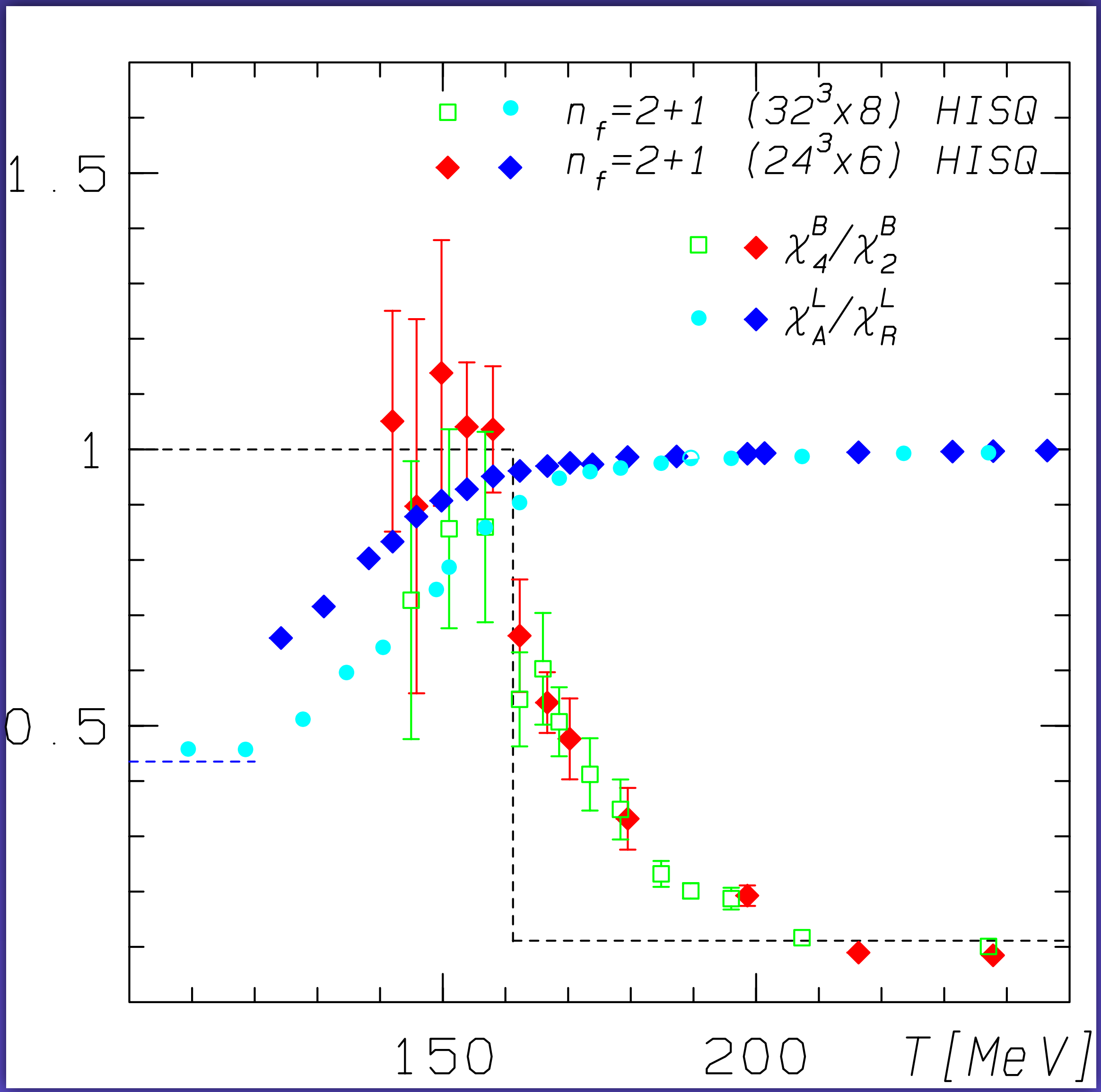




Deconfinement with light quarks

- Quarks breaks Z(3) symmetry: very smooth cross over?
- What do fluctuations show?
- χ_4/χ_2 sensitive to baryon # Ejiri et al., PLB 633, 275
- χ_A/χ_R Polyakov loop susc.

HotQCD



Deconfinement with light quarks

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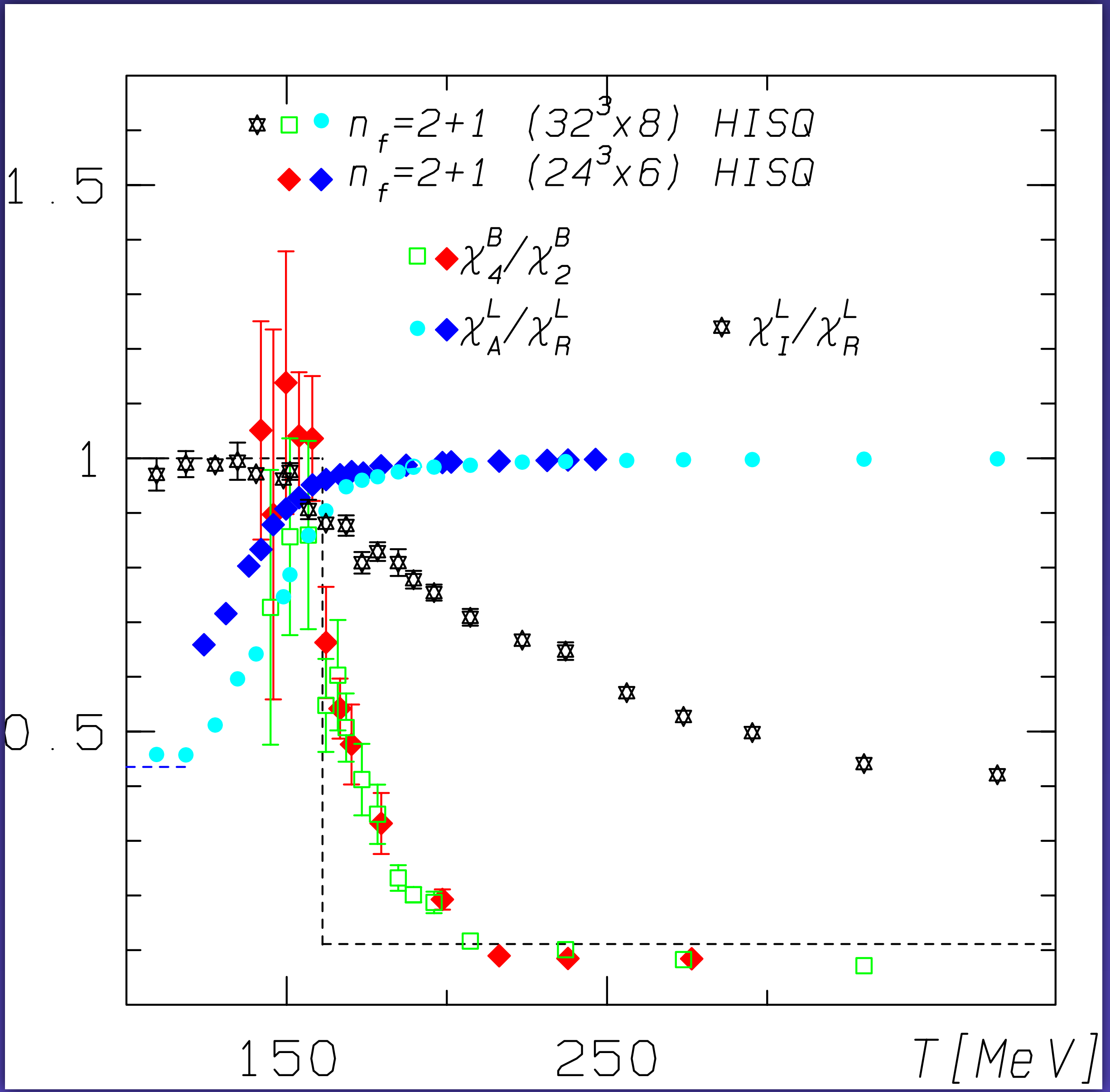
- χ_I/χ_R - " -

HotQCD

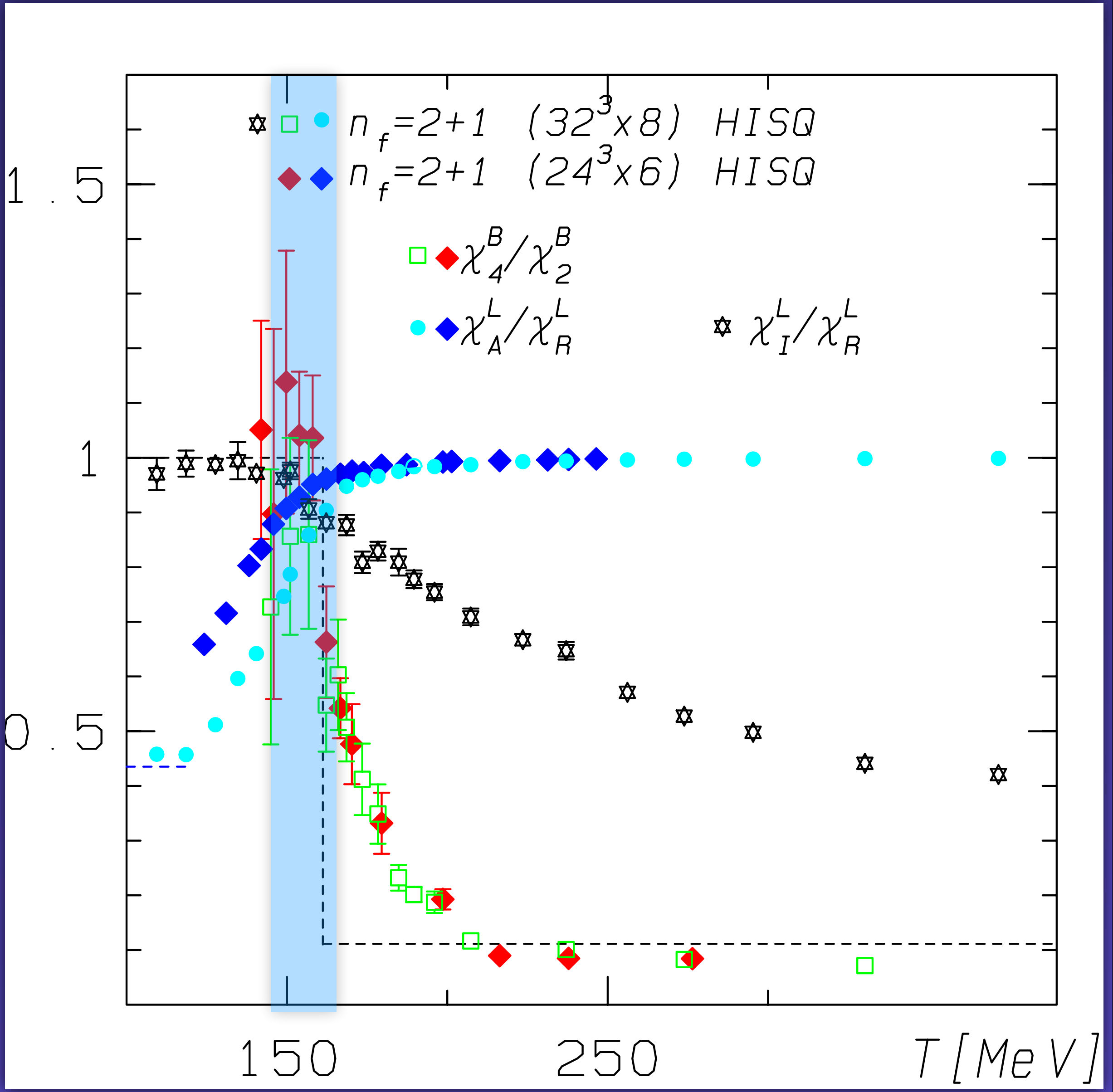


Lattice effects!

Deconfinement with light quarks

- Quarks break Z(3) symmetry: very smooth cross over?
- What do fluctuations show?
- χ_4/χ_2 sensitive to baryon # Ejiri et al., PLB 633, 275
- χ_A/χ_R Polyakov loop susc.
- χ_I/χ_R - " -
- Action focussed at $T \simeq T_\chi$

HotQCD



Narrow transition!

Conclusions

- Higher cumulants of conserved charges probe chiral critical fluctuations
  - Basically robust, but several open issues remain:
    - Sensitive to tail of distribution, cancellations
    - Overall conservation (Bzdak, Koch, Skokov)
    - Acceptance corrections (Bzdak, Koch)
    - Other sources of fluctuations (Skokov et al.)
    - …..

Conclusions II

- Fluctuations of Polyakov loop probe deconfinement
  - Indicate rather narrow transition, $T_{\mathrm{dec}} \simeq T_{\chi}$, not simply related to Polyakov loop
    - Consistent with analysis of strange d.o.f's across the QCD transition A. Bazavov et al. (HotQCD), PRL 111, 082301
    - also A. Dumitru et al., PRD 83, 034022
- Tune effective models (PNJL, PQM) to lattice results, including fluctuations

Darmstadt, June, 2006