

Feb.2,2004,IAS talk

**The Quark-Gluon Plasma**  
at strong (QCD) and superstrong (CFT) coupling

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**Why this talk?**

- Results obtained via AdS/CFT duality for strong coupling helped already, **by a moral support and the strong coupling results**
- Some ideas why CFT and QCD at  $T = 1 - 3T_c$  are similar were suggested, but much more has to be clarified. **We need your help**

- The phases of QCD and the RHIC project (the **Little** vs the **Big Bang**)
- **QGP at RHIC  $\neq$  weakly interacting quasiparticle gas:** “parton cascades” do not work but **hydrodynamics does**, so  $l \ll L$  and in fact **QGP at RHIC is the most perfect liquid known** (viscosity/entropy  $\eta/s \sim .1$  is much less than for e.g. water!
- **QGP at RHIC ( $T = (1 - 3)T_c$ ) is in a strongly interacting regime**  $\alpha_s \sim 1, \lambda = g^2 N \sim 10 - 20 \gg 1$
- $\mathcal{N} = 4$  **SUSY YM=CFT in strong coupling  $g^2 N_c \gg 1$ :** both the thermodynamics and transport (viscosity) results from the AdS/CFT (Maldacena) duality seem to hold in QCD (why?)

- I will emphasize the role of **bound states in 3 strong coupling problems:**

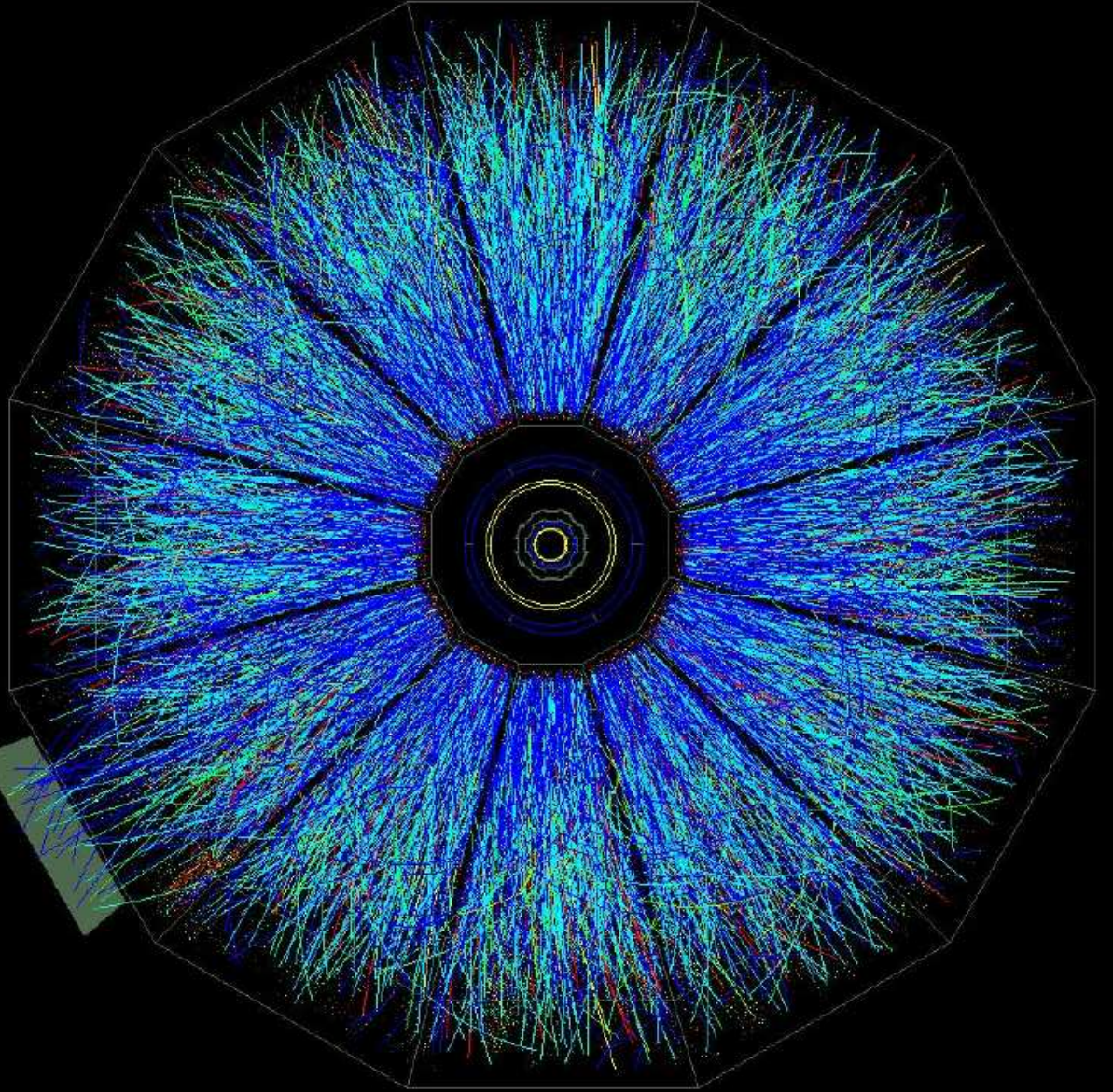
- Trapped ultracold atoms at scattering length  $a \rightarrow \infty$

- In QCD: exotic  $qq, gg$  plus also “old”  $\bar{q}q$  bound states: **zero binding lines** inside the QGP phase

- In CFT a large set of light composites with  $M \sim T$  (Zahed and ES, hep-th/0308073)



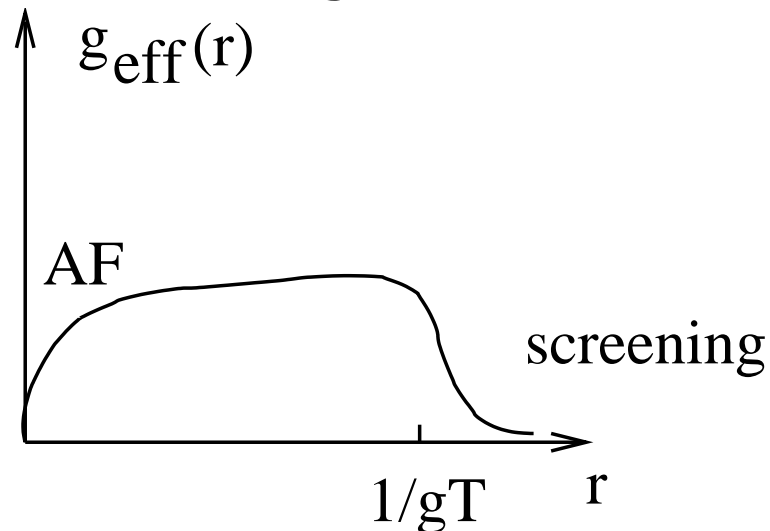
The RHIC project (as it is seen from space):  $\sqrt{s}=250+250$  GeV pp,  $100+100$  GeV/N AuAu  
only factor 4 smaller than Tevatron, but 2 accelerators not one.





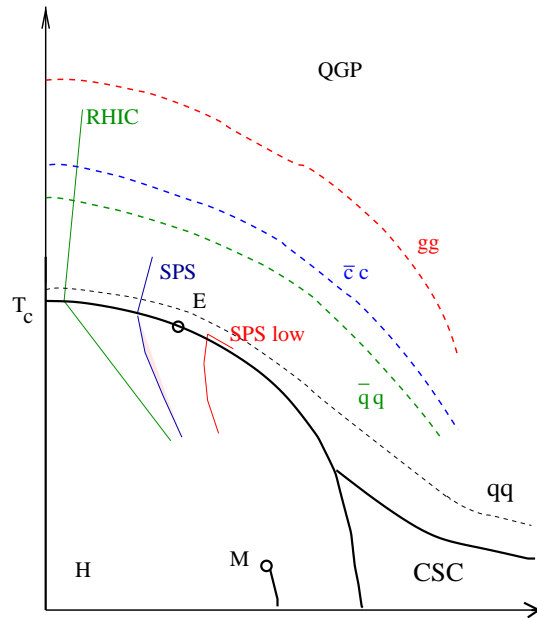
## Why Quark Gluon Plasma is a Plasma?

- In QED vacuum the charge is **screened**
- In QCD vacuum the charge at **small distances** is **anti-screened** - Asymptotic Freedom (**Politzer, Gross, Wilczek, 1973**)
- In the QCD heat bath at high enough  $T$  – **the QGP phase** – the charge is **screened** at large distances (**ES, 1977**)

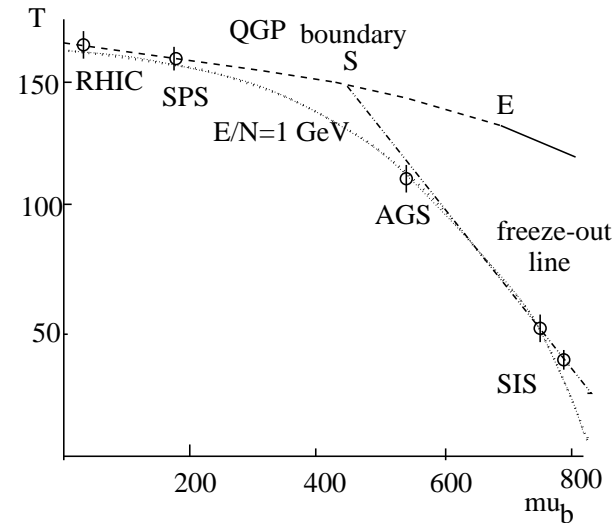


Debye radius  $R_D \sim 1/gT$   
and the charge  $g_{\text{eff}} \sim 1/\log(gT/\Lambda_{\text{QCD}}) \ll 1$  **is small at high  $T$**

- **No confinement or chiral symmetry breaking!**



The QCD Phase Diagram: H is hadronic phase, QGP is quark gluon plasma, CSC is Color superconducting phases, E is the endpoint of the first order line, M (multifragmentation) is the end of nuclear liquid-gas line.



The **chemical freezeout line** extracted from heavy ion collisions: it reproduces the abundances of all species such as  $\pi, K, N, \Lambda, \Sigma, \Xi, \Omega, \phi, K^* \dots$

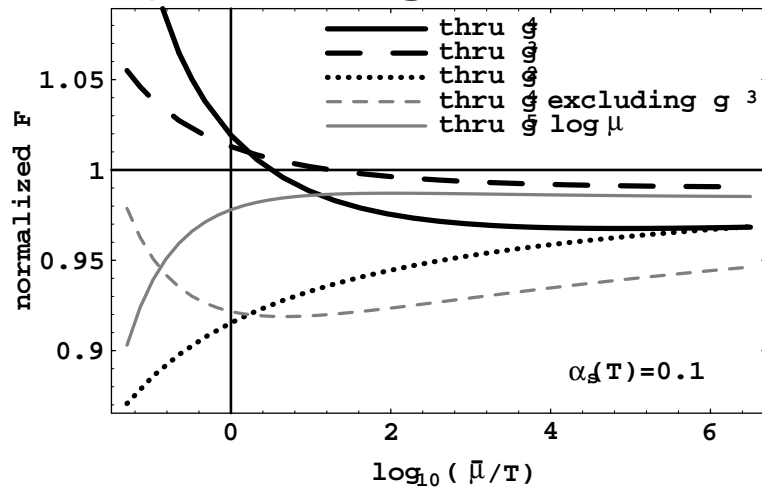


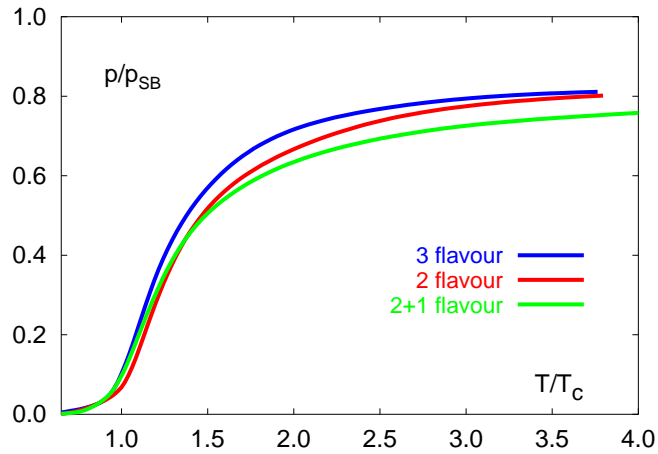
# The thermodynamics of gauge theories

- High-T: Weak coupling QCD via diagram resummation, done by many people (mine are  $g^2, g^3$  terms). Numerically for QCD with  $n_f$  quark flavors the series are

$$\begin{aligned}
 F = - \frac{8\pi^2 T^4}{45} & \left\{ 1 + \frac{21}{32} n_f - 0.09499 g^2 \left( 1 + \frac{5}{12} n_f \right) + 0.12094 g^3 \left( 1 + \frac{1}{6} n_f \right)^{3/2} \right. \\
 & + g^4 \left[ 0.08662 \left( 1 + \frac{1}{6} n_f \right) \ln \left( g \sqrt{1 + \frac{1}{6} n_f} \right) - 0.01323 \left( 1 + \frac{5}{12} n_f \right) \left( 1 - \frac{2}{33} n_f \right) \ln \frac{\bar{\mu}}{T} \right. \\
 & \left. \left. + 0.01733 - 0.00763 n_f - 0.00088 n_f^2 \right] + O(g^5) \right\}. \quad (1)
 \end{aligned}$$

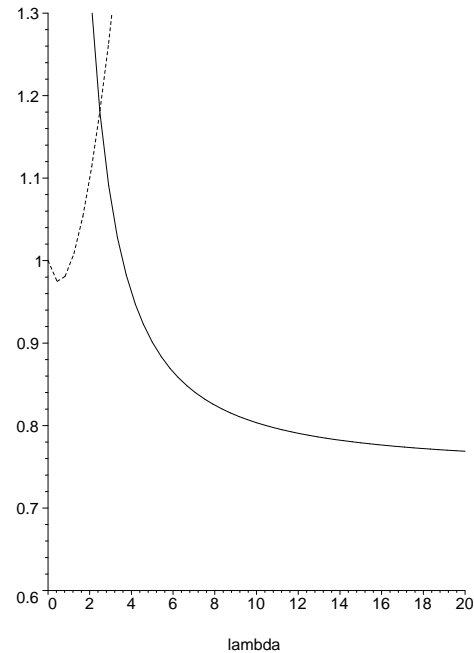
Only converges if  $T \sim 10^6 \text{ GeV}$  or so





- Lattice results (Bielefeld group) for QCD thermodynamics: pressure normalized to Stephan-Boltzmann value

**Strong coupling is not unreasonable!**



- Weak (5 terms) vs. strong ( $3/4 + const/\lambda^{3/2}$ ) coupling for the CFT: the ratio of the pressure to Stephan-Boltzmann value vs the 't Hooft coupling  $\lambda = g^2 N$ .

Three explosions:

the Big Bang versus the Little one

plus mini-explosions of classical glue we will not really discuss

Both Bangs rapidly create and then nicely conserve the entropy (at early time  $dN/dy(\text{gluons}) \approx 1100$  same as pions at the end)

	Big Bang	Little Bang
eqns expansion The <i>final</i> velocities status acceleration history conclusion angular harmonics harmonics <i>fluctuations</i> ,	Einstein's Hubble law $v \sim r$ Hubble constant, recently fixed to 10% distant supernovae <i>acceleration now</i> l=1: solar system direction l ~ 100	hydro anisotropic final radial flow up to $v_t = .7$ RHIC $\Omega^-$ flow acceleration in all phases $v_2$ , the <i>ellipticity</i> not seen so far

“Dark energy” = Cosmological constant = “bag constant” B  
in  $p = \text{const}T^4 - B$  works *against* the QGP explosion, unlike  
in the Big Bang

## Overview/Summary

- Are AuAu central collisions similar to pp?

**No:** Even  $dN/dy$  and  $dN/dp_t^2$  are different. But the main thing is it is a **bang** not a **fizzle** (as predicted by “parton cascades” )

- How do we know that? From **collective flow** effects, especially **elliptic**

- Has the Quark-Gluon Plasma been actually produced?

**Sure:** the energy density is high, equilibration is good, and its **EoS works**.

- At what time equilibration occurs? **Quite early,  $\tau \sim 1/2 fm/c$**  (from elliptic flow)

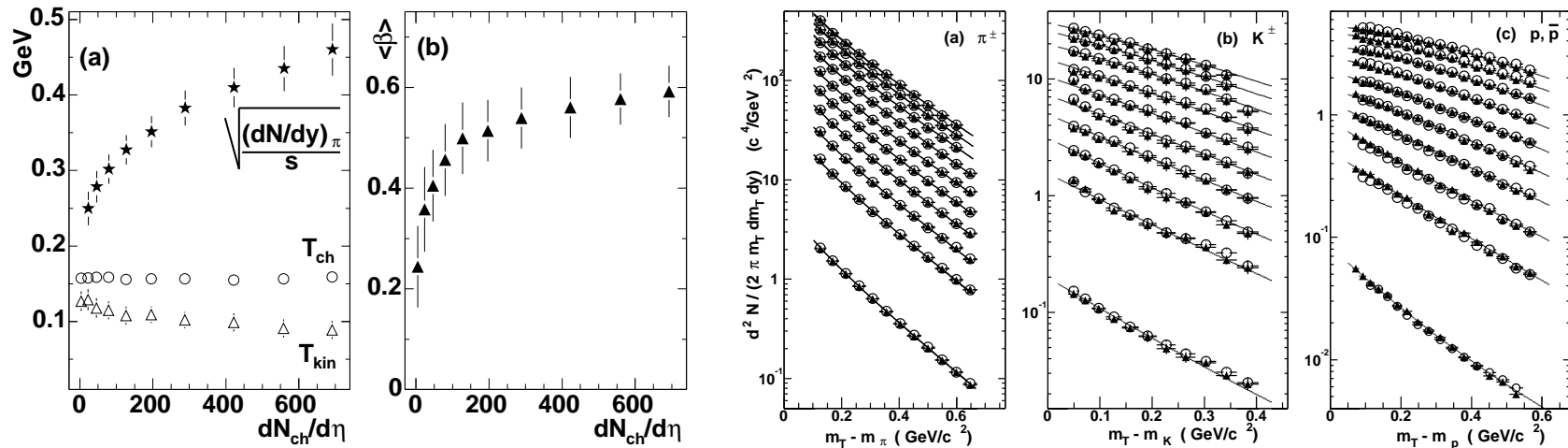
- Do we understand why? **No**

- Does the parton model work at large  $p_t$ ?

**No:** unexpectedly strong **jet quenching** of .8-.9 jets is observed, till  $p_t \sim 15 \text{ GeV}$  we can reach so far

- Are we sure it is not a modification of Au structure functions? **absolutely yes:** photons, dAu collisions etc.

# Radial Flow



- Blast wave fits to RHIC spectra (STAR): **all species** can be well fitted with the same velocity and  $T_{kin}$ .
- The chemical freezeout  $T_{ch}$  is **independent** on centrality, while  $T_f$  decreases to about 90 MeV.
- **Hung+ES,96**: **larger systems cool further**.

At RHIC we are making progress in the proverbial “rocket science” getting as strong as possible conversion of the internal energy into flow

In pp all species have  $m_t$  scaling

$$dN/dp_t^2 \sim \exp(-m_t/T) \quad (m_t^2 = p_t^2 + m^2)$$

while in AA all secondaries have different shapes at small  $m_t$

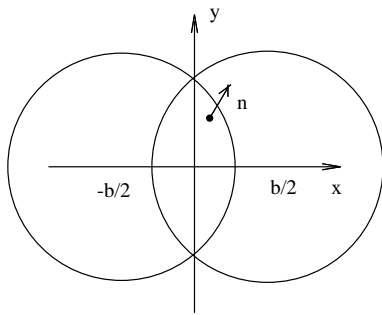
Only the mass matters:  $\rho, K^*, f_0(980), N, \phi$  have similar shapes and slopes

The rescatterings result in a pressure which causes the system to expand collectively. If all the particles freeze-out simultaneously and the system is in local thermal equilibrium their momentum spectra can be characterized by only two parameters, the temperature and the transverse collective flow velocity.

The slope is a **blue-shifted** temperature, hydro-like fit for RHIC gives  $v_t \approx 0.7, T_f \approx 100 \text{ MeV}$

## Elliptic flow

Sorry, I have no time to discuss the “directed flow”  $v_1$ , or higher  $v_4$  which are both recently measured at RHIC

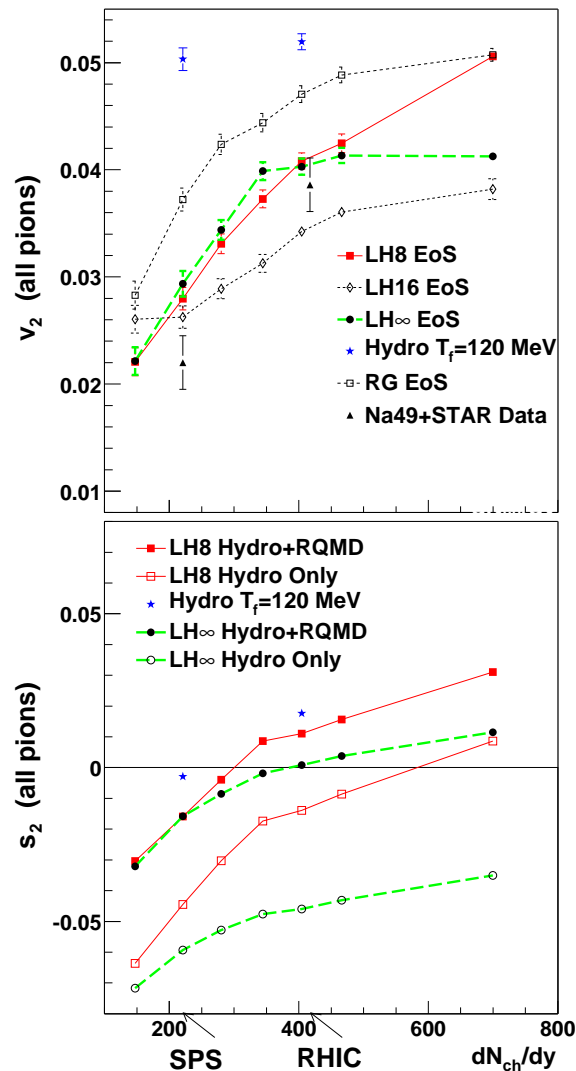
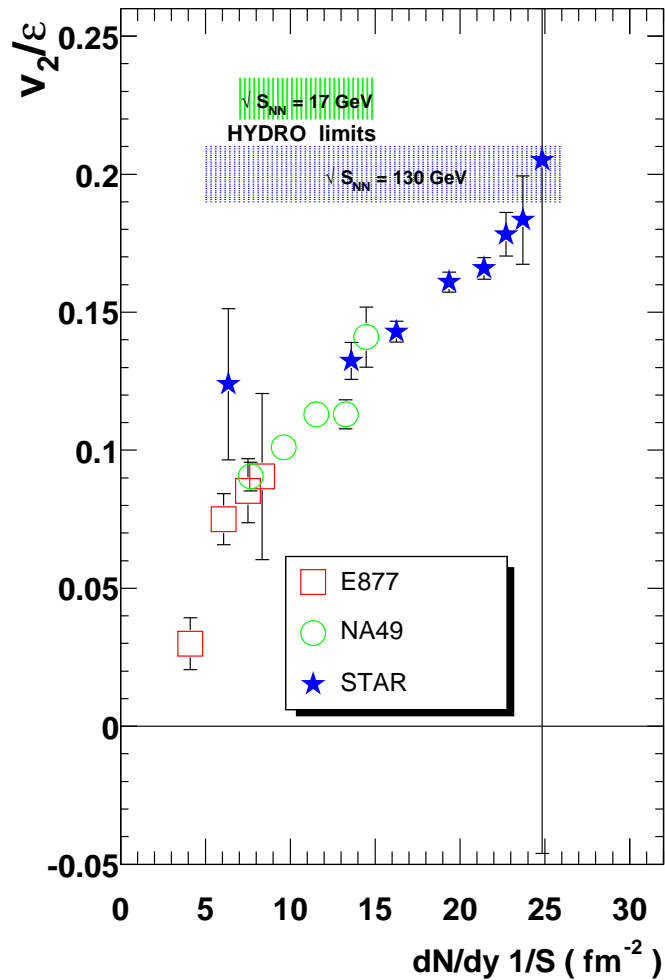


The picture of the “initial almond” for non-central collisions

$$\frac{dN}{d\phi} = \frac{v_2}{2\pi} + \frac{v_2}{\pi} \cos(2\phi) + \dots$$

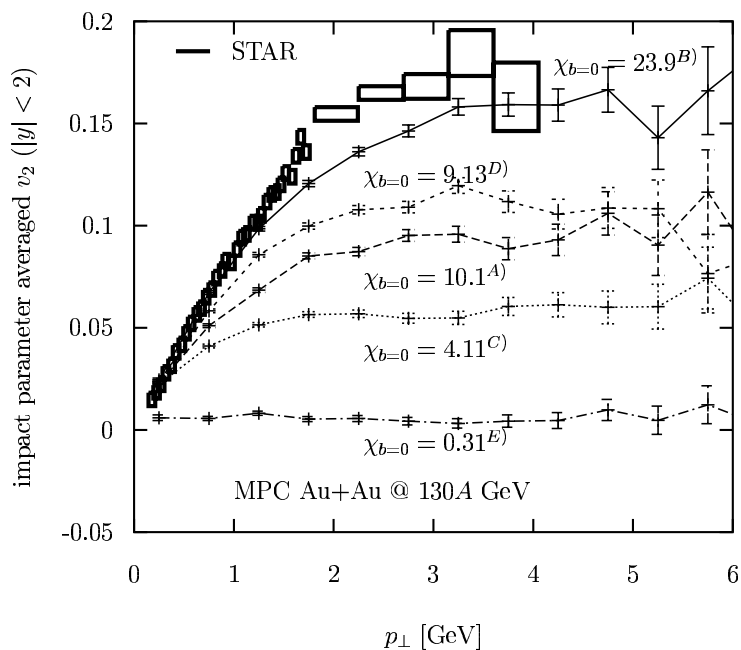


From Teaney, Lauret, ES (shown already at QM99, so it is well documented prediction). The rize is due to large QGP pressure. The red curve has the latent heat (LH) =800  $MeV/fm^3$ , it is is the closest to the lattice EoS is also the best fit to all flow data at SPS and RHIC. The “hydro limit” in the STAR fig is for wrong  $T_{freezeout} = const(b)$ , while the “theory” ones has a correct freezeout (via RQMD).



It is only possible to get elliptic flow if the quasiparticle **rescattering is increased by big factor  $\sim 50$**  relative to the pQCD expectations.

The Figure (from Gyulassy and Molnar) shows how the measured effect (boxes) can be reached if matter opacity of matter grows. The smallest value (gg scattering in pQCD) shows no collective effects whatsoever. **The rescattering cross section should be boosted by a big factor  $\Rightarrow$  small viscosity**

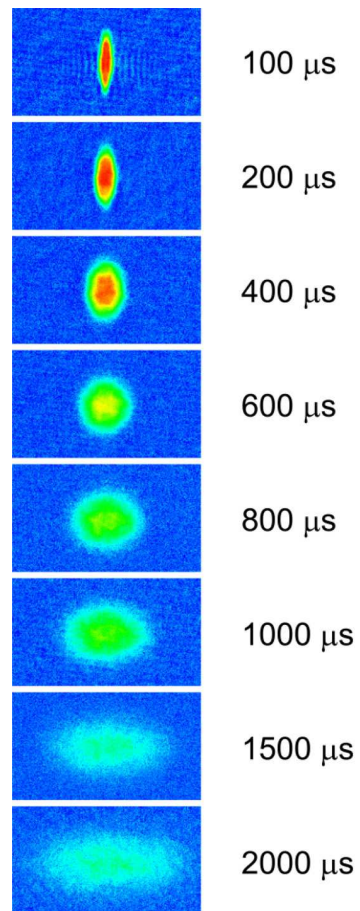


Gluon elliptic flow for Au+Au at  $\sqrt{s} = 130A$  GeV from Boltzmann eqn., with transport opacities  $\chi_{b=0} = 0.31, 4.11, 9.13, 10.1$  and  $23.9$ .)

## Elliptic flow with trapped $Li^6$ atoms:

K.M.O'Hara et al, Science 298,2179, 2002

T.Bourdel et al, PRL 91 020402 , July 11 2003



Magnetic field  $B \sim 800G$  shifts (via the Feshbach resonance  $|f = 1/2, m_f = 1/2 \rangle \Leftrightarrow |f = 1/2, m_f = -1/2 \rangle$ ) and makes the 38-th vibrational  $Li_2$  state to exactly **zero energy**  $\Rightarrow$  **infinite scattering length  $a$** , very large size and lifetime  $\sim 1$  sec.

Normally gas is transparent,  $l \ll L$ , and expands without collisions **isotropically**

But in the **strong coupling regime  $l \ll L$**  it explodes **hydrodynamically !**, see the figure

Cross section can be changed by many orders of magnitude, but the EoS changes by  $\sim 20\%$  only ! (like in QGP and CFT... why?)

## A crash course on viscosity

- Ideal hydro  $\Rightarrow$  effective theory in which all dynamics is local.

$$T^{\mu\nu} = (\epsilon + p)u^\mu u^\nu - pg^{\mu\nu}; \quad \partial_\mu T^{\mu\nu} = 0$$

- Viscosity is  $O(l_{m.f.p.}/L)$  correction due to **finite mean free path** (or other nonzero correlation lengths).

The corrected stress tensor:

$$T_{ij} = \delta_{ij}p - \eta(\partial_i u_j + \partial_j u_i - \frac{2}{3}\delta_{ij}\partial_k u_k) - \zeta\delta_{ij}\partial_k u_k$$

where  $u_i$  is the flow velocity,  $p$  is the pressure, and  $\eta$  and  $\zeta$  are, by definition, the shear and bulk viscosities respectively. In CFT  $T^\mu{}_\mu = 0$  and bulk viscosity vanishes identically,  $\zeta = 0$ .

Kubo relations

$$\eta = \lim_{\omega \rightarrow 0} \frac{1}{2\omega} \int dt d^3x e^{i\omega t} \langle [T_{xy}(t, \vec{x}), T_{xy}(0, 0)] \rangle = \lim_{\omega \rightarrow 0} \frac{1}{2\omega i} [G_A(\omega) - G_R(\omega)]$$

where the average is taken in the equilibrium and  $G_A$  and  $G_R$  are the advanced and retarded Green functions of  $T_{xy}$  computed at zero spatial momentum.

- Example: scaling boost-inv. (Bjorken) flow (axially symmetric)

$$\frac{1}{\epsilon + p} \frac{d\epsilon}{d\tau} = \frac{1}{s} \frac{ds}{d\tau} = -\frac{1}{\tau} \left( 1 - \frac{\Gamma_s}{\tau} \right); \quad \Gamma_s = \frac{4}{3} \frac{\eta}{\epsilon + p} = \frac{4}{3} \frac{\eta}{T_s}$$

one finds in the r.h.s. exactly **the combination** also known as **the sound attenuation length**. It is the micro scale which substitutes  $l_{m.f.p.}$  if **the quasiparticle language is inadequate**.

$$\Gamma_s \ll \tau$$

## Theory/Phenomenology of QGP viscosity

- Weak coupling or perturbative framework: large,  $\eta/T^3 \sim \text{const}/g^4 \log(1, \text{const} \sim 100$  at small  $g \ll 1$

If so,  $l \sim \text{few fm}$  and no hydro at RHIC ! (Recall the Gyulassy-Molnar plot here)

- However, in the strong coupling ( $\mathcal{N} = 4$  supersymmetric Yang-Mills or CFT) Polycastro, Son, Starinets, Phys. Rev. Lett. **87** (2001) 081601 (not yet in QCD!) RHS of the Kubo formula  $\leq$  classical absorption cross section of gravitons by black three-branes (Klebanov) calculated by solving the wave equation on the background metric.

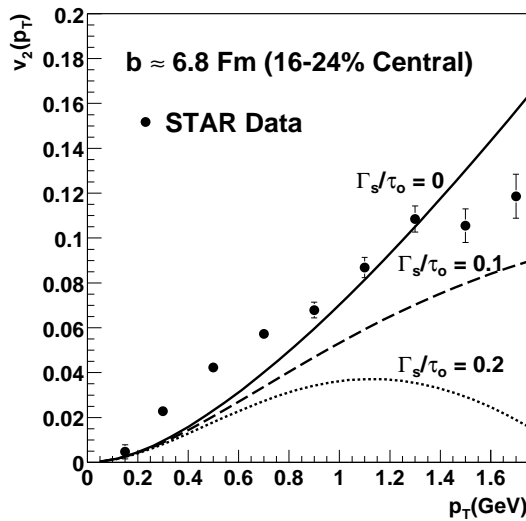
$$\sigma(\omega) = \frac{8\pi G}{\omega} \int dt d^3x e^{i\omega t} \langle [T_{xy}(t, \vec{x}), T_{xy}(0, 0)] \rangle$$

where  $G$  being the ten-dimensional gravitational constant. The result is  $\eta = \frac{\pi}{8} N^2 T^3$  or  $\eta/s = 1/4\pi$ . If used for RHIC QGP  $\Gamma_s \sim .1 \text{ fm}$  If so, excellent hydro, but no parton cascades...

Later several gravity backgrounds were found in which similar calculation is possible: all give **the same**  $\frac{\eta}{s} = \frac{1}{4\pi}$  conjectured to be the **lower bound** on the most perfect liquid.

- QGP viscosity: Teaney, hep-ph/0301099 from  $v_2(p_t)$  data  $\Rightarrow \Gamma_s \sim .1fm$  or  $\frac{\eta}{s} \sim 1/10$  the most perfect fluid ever
- The second best known liquid,  $He^4$  at high pressure, has  $\frac{\eta}{s} \sim 1$ , water at normal conditions about 40.

Elliptic flow  $v_2$  as a function of  $p_T$  for different values of  $\Gamma_s/\tau_0$ . The data points are four particle cummulant data from the STAR collaboration [?]. Only statistical errors are shown.



The results for different  $\Gamma_s/\tau$  is shown The experimental data deviate from ideal hydro curve at  $p_{\perp} \approx 1.6 GeV$  which indicates  $\Gamma_s/\tau \sim 0.05$  or so. Substituting here the

relevant time  $\tau \sim 3fm/c$  we get surprisingly small  $\Gamma_s \sim .15fm$ .

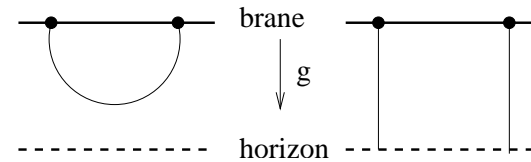


- The  $\mathcal{N}=4$  SUSY Yang Mills gauge theory is conformal (CFT) (the coupling does not run). At finite  $T$  it is a QGP phase at ANY coupling. If it is weak it is like high-T QCD  $\Rightarrow$  gas of quasiparticles. What is it like when the coupling gets strong  $\lambda = g^2 N_c \gg 1$ ?
- AdS/CFT correspondence by Maldacena turned the strongly coupled gauge theories to a classical problem of gravity in 10 dimensions
- Example: a modified Coulomb's

law (by Maldacena)

$$V(L) = -\frac{4\pi^2}{\Gamma(1/4)^4} \frac{\sqrt{\lambda}}{L}$$

- becomes a screened potential at finite  $T$

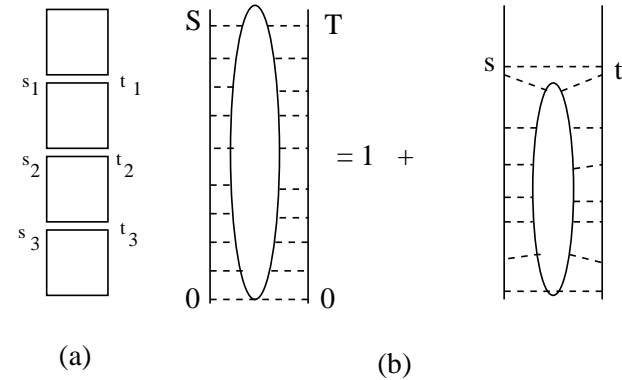


- Free energy at  $\lambda \gg 1$  is  $F = (3/4 + O(1/\lambda^{3/2}))F_{free}(T)$  while the quasiparticles are heavy  $m \sim \lambda^{1/2}T$ .

**What is the matter made of?**

- Zahed +ES, hep-th/0308073: of **light**  $m \sim \lambda^0 T$  binary composites  $gg, qg, qq$  with large angular momentum  $l \sim \sqrt{\lambda}$  whose spectrum can be calculated.

**G. Semenoff and K.Zarembo, hep-th/0202156.** have shown that a modified Coulomb law can be understood by the resummed gluonic ladder



$$\Gamma(S, T) = 1 + \frac{\lambda}{4\pi^2} \int_0^S ds \int_0^T dt \frac{1}{(s-t)^2 + L^2} \Gamma(s, t)$$

$$\frac{\partial^2 \Gamma}{\partial S \partial T} = \frac{\lambda/4\pi^2}{(S-T)^2 + L^2} \Gamma(S, T)$$

Change variables to  $x = (S - T)/L$  and  $y = (S + T)/L$

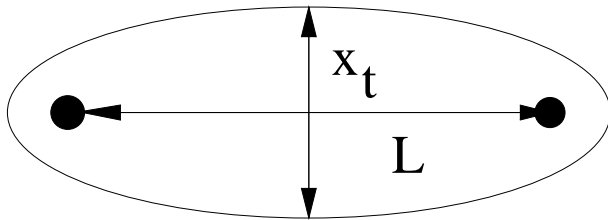
Expansion of the kernel in the first power of  $(S - T)^2$  leads to an oscillator potential and the problem is easily solved

$$\Gamma(x, y) \approx C_0 e^{-\sqrt{\lambda} x^2 / 4\pi} e^{\sqrt{\lambda} y / 2\pi}$$

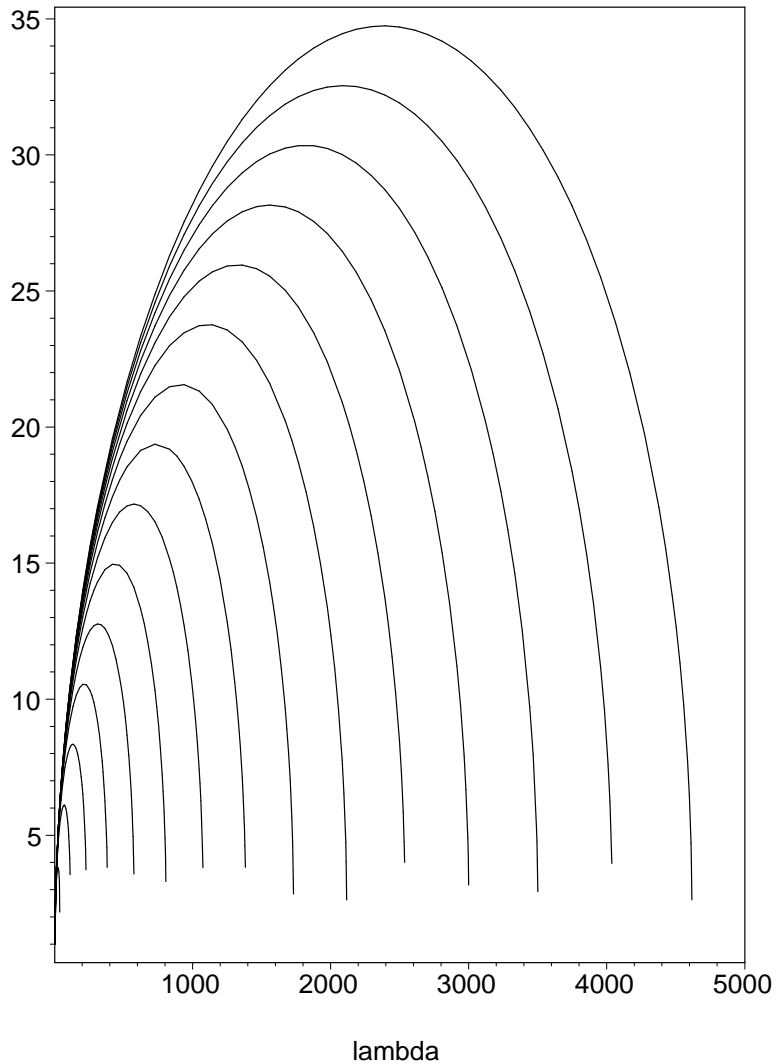
$$V_{\text{lad}}(L) = -\lim_{T \rightarrow +\infty} \frac{1}{T} \Gamma(T, T) = -\frac{\sqrt{\lambda}/\pi}{L}$$

**the same parametric form but different coefficient:**  $1/\pi = 0.318$  while the exact Maldacena value is 0.228.

- We observed that the exchanged gluons move with **the superluminal speed**  $v \sim \delta t/L \sim \lambda^{1/4} \gg 1$
- Some higher order diagrams have extra  $\lambda d^4x \sim O(1)$ : multi-body Bethe-Salpeter
- vertices distributed in a **quasi-string** regime



- The main idea: the modified Coulomb law can be used **even for relativistic bound states**, with  $v \sim 1$ .
- Using a Klein-Gordon eqn  $(E - V)^2 - m^2 = p^2$ ,  $V = -C/r$  with a Coulomb potential, by WKB or exactly, one can find the spectrum. (Known from about 1930).



$$V = -\frac{C}{r}$$

$$E_{nl} =$$

$$m \left[ 1 + \left( \frac{C}{n+1/2 + \sqrt{(l+1/2)^2 - C^2}} \right)^2 \right]^{-1/2}$$

Small  $C$  - nonrelat. atoms, Balmer series... **New regime at large  $C \gg 1$ : families of relativistic deeply bound states, with large orbital momentum balancing the supercritical Coulomb**

**At strong coupling only those composite states are ther-**

**mally excited,**

- Furthermore, the density of states can be shown **not to depend on  $\lambda$** .

The composites carry large angular momentum  $l \approx \text{const}1\sqrt{\lambda} + \text{const}2/\sqrt{\lambda}$  and  $\int_{l_{min}}^{l_{max}} dl^2 = l_{max}^2 - l_{min}^2 \approx \lambda^0$  thus the free energy  $F = CN_c^2 T^4$  has **no dependence on  $\lambda$**

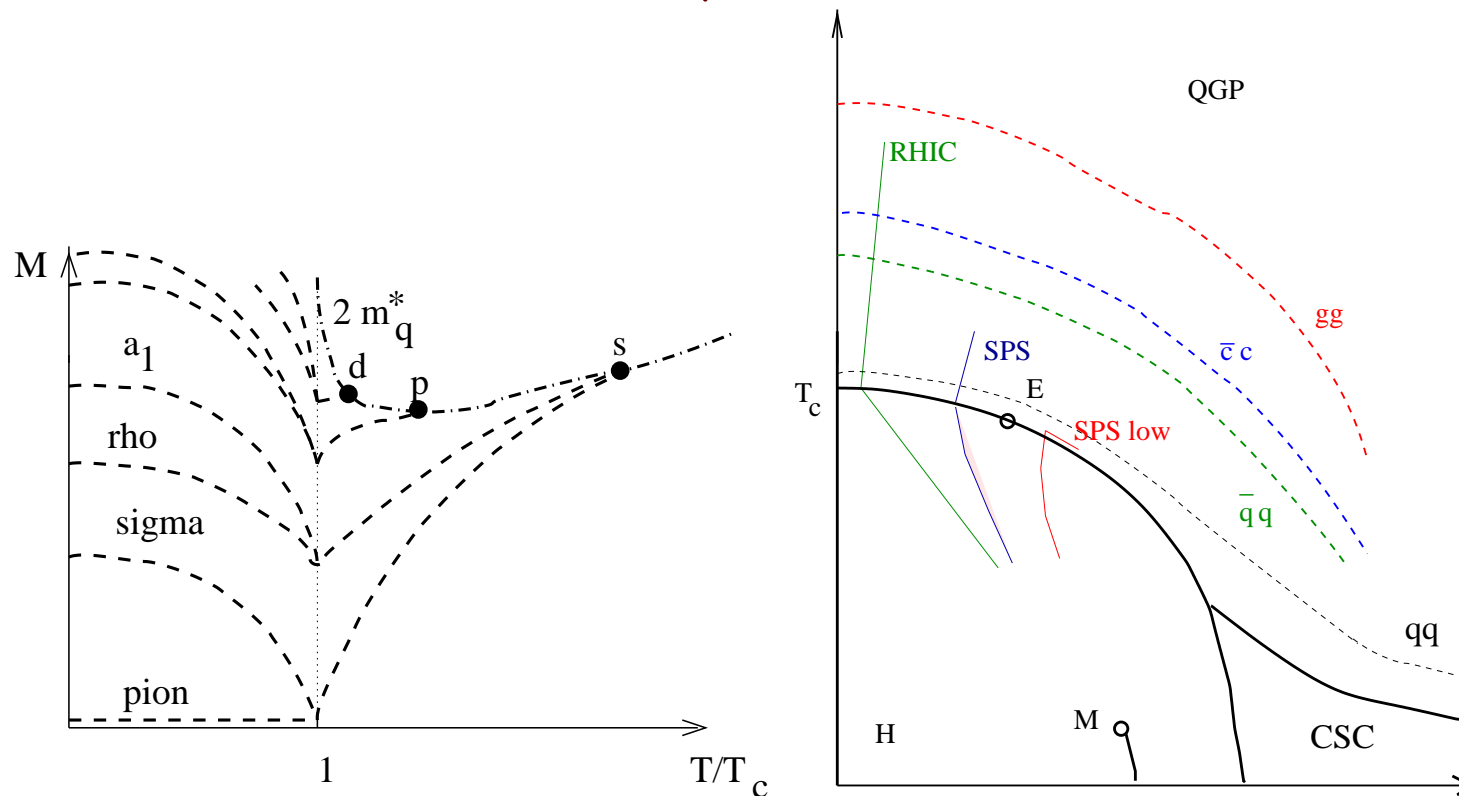
- (The oscillator-like spectrum  $M_n = \text{const}1 + \text{const}2nT$  (with complex constants) have been inferred by Starinets (and also Teaney, unpublished) before our work, by a direct calculation of the **graviton propagator  $\Rightarrow$  stress tensor correlator.**)

## Are there hadrons above $T_c$ , in Quark-Gluon Plasma?

- **Old point of view: most hadrons (including  $J/\psi = \bar{c}c$  but not  $\Upsilon = \bar{b}b$ ) melt there.**
- **Exceptions: the pion (+sigma etc chiral multiplet) was believed to survive as resonance e.g. NJL-based papers or T.Schafer+ES, PLB 356:147,1995**
  
- **New lattice results from Bielefeld group and Asakawa-Hatsuda (using the maximal entropy method) found bound  $\eta_c$  at  $1.5T_c$ , very recently Asakawa-Hatsuda (hep-lat/0309001) have argued that the de-binding point is  $1.6 < T_\psi < 1.9T_c$ .**
  
- **One important point: near  $T_c$  the q,g quasiparticles are not light, then their mass decreases and at high T grows again as predicted by pQCD  $M \sim gT$**



ES and I.Zahed, hep-ph/0307267



A general view of the endlines for  $\bar{q}q$  and also **colored composites** like  $qq$ ,  $gg$ . Another famous colored dimer is the  $qq$ , **the Cooper pair** of color superconductivity

## Why some hadrons may survive above $T_c$ ?

ES and I.Zahed, hep-ph/03072

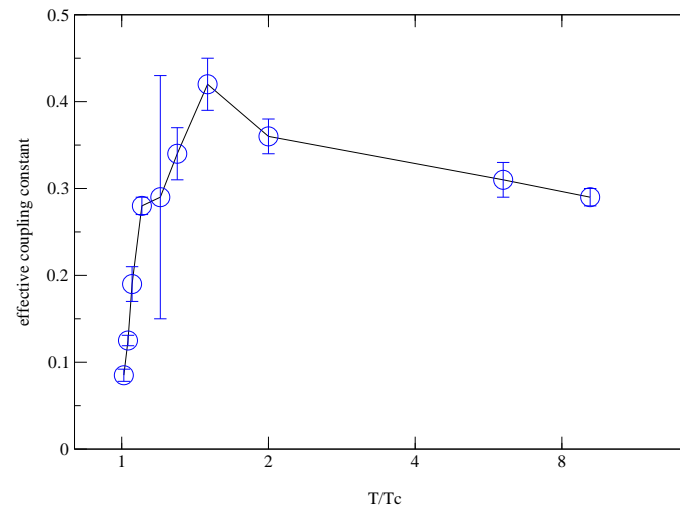
- The loophole in the old argument:  $\alpha_s$  kept frozen in the QGP as in-vacuum

- New idea: at  $T > T_c$  the charge continues to run to larger values, stopped by the Debye screening only.  $\alpha_s \sim 1/2$

is reached

G.Brown, C.H.Lee, M.Rho and ES, hep-ph/0312175  $\bar{q}q$  bound states for  $T_c < T < T_{zerobinding}$ : relativistic effects  $(1 - \vec{v}_1 \vec{v}_2)$  + spin-spin, plus the nonperturbative forces due to instanton-antiinstanton molecules  $\Rightarrow M_\sigma, M_\pi \Rightarrow 0$  at  $T \Rightarrow T_c$

- The main idea: large (unitarity limited) cross sections at the endlines  $\Rightarrow$  “sticky molasses”



## Summary

- QGP at RHIC is found to be a **surprisingly perfect fluid**, hydro works well with lattice EoS, and viscosity  $\eta/s \sim 1/10$  is smaller than ever seen before
- This **disagrees with pQCD (weak coupling)** but **does agree with the strong coupling predictions** as derived for CFT via AdS/CFT correspondence
- Can it be extended to other (reasonably) strongly coupled theories like QCD at  $T = (1 - 3)T_c$  with  $\lambda \sim 10$ ? ● Is it true that the matter is made of large- $l$  binary composites?
- In QCD it seem to be true that at  $T = (1 - 3)T_c$  all **s-wave mesons plus glueballs plus many exotic bound states  $qq, qg, gg$**  do exist, till the zero binding lines.
- Those generate about 1/2 pressure at RHIC and, presumably, **strong (unitarity limited) rescattering** e.g.  $\bar{q}q \rightleftharpoons \text{meson}$
- **Amazingly similar to trapped  $Li^6$  atoms.**