

**RHIC initial conditions with the QCD sphalerons:
a reduced shadowing and enhanced jet quenching**

Edward Shuryak, Stony Brook

- **“Small gluonic spots”** in DIS and diffraction
- **The instanton/sphaleron mechanism** of high energy collisions (pp, DIS γ^*p , double DIS $\gamma^*\gamma^*$)
- AA collisions (at RHIC): **a CGC with a topological structure?**
 - \Rightarrow prompt quark production
 - \Rightarrow **reduced** shadowing due to “lumpy glue”
 - \Rightarrow **enhanced** jet quenching due to stronger field

The remaining puzzle of the “initial state” at RHIC
($\tau \sim .5 - 1 \text{ fm}$):

$\frac{dN}{dy}$ “minijets” ($y = 0$) ~ 1000 estimated from the jet quenching,
which matches the **total entropy at the end**
 \Rightarrow ideal hydro with the conserved entropy.

All fits together well

But where does such large parton density come from?

HIJING and other perturbative parton cascades (analytic and numeric) show that hard collisions lead to 1/10th of that, and time of several fm is needed to get to the QGP with such entropy. This seems to contradict to the elliptic flow data!

Classical Glue in pp and AA (at RHIC?)

- **Color Glass Condensate (CGC)**: due to high parton density at small $x \Rightarrow$ large occupation numbers \Rightarrow classical YM field \Rightarrow nonperturbative dynamics

McLerran-Venugopalan model: a saturated random Gaussian field.

Venugopalan, Krasnitz et al: explosive behavior, 1/2 gluons gets physical

small perturbative quark production

- The CGC can also be made of **topological clusters** – sphalerons, or interrupted instantons. (A jump from **under** the barrier **on** the barrier.) **Leads to prompt production of quarks**: $\bar{u}u\bar{d}d\bar{s}s$ produced per cluster, plus few gluons: Maybe this is the **Rapid Entropy Generator** we badly need

Semiclassical Theory of High Energy Collisions based on **Instantons and Sphalerons**

- ‘‘pomeron from instantons’’:

ES, Zahed PRD62:085014,2000 hep-ph/0005152

D. E. Kharzeev, Y. V. Kovchegov and E. Levin Nucl. Phys. A **690**, 621 (2001) [hep-ph/0007182].

M. A. Nowak, ES and Zahed, PRD 64, 034008 (2001) [hep-ph/0012232].

G.W.Carter,D.Ostrovsky and ES, Phys. Rev. D **65**, 074034 (2002) [hep-ph/0204224]

- the turning states and their explosion

D.Ostrovsky, G.W.Carter and ES, hep-ph/0204224,PRD

- Landau method for cross section - rescaled YM sphalerons are produced

D. Diakonov and V. Petrov, Phys. Rev. D **50**, 266 (1994) R. A. Janik, ES and Zahed, hep-ph/0206005.

- Explicit solution of the Dirac eqn in the exploding field background.
an end of ‘‘the fermion puzzle’’?

ES and Zahed, hep-ph/0206022.

- Gluonic cluster production in double-Pomeron processes (e.g. $pp \rightarrow pp\eta'$; $pp f_0(1600)$, $pp + \text{cluster}$) compared to data; ES and Zahed, 2002
- Instanton-induced Double DIS ($\gamma^* \gamma^*$) ES and Zahed, 2003

Earlier developments:

- (Baryon-number violating) instanton-induced processes in electroweak theory A. Ringwald, Nucl. Phys. B330 (1990) 1, O. Espinosa, Nucl. Phys. B343 (1990) 310; L. McLerran, A. Vainshtein V.I. Zakharov, A. Muller, M. Maggiore and M. Shifman, : extremely insightful works, but the effect is too small to be seen!

- Instanton is the only nontrivial field for which straight Wilson lines can be easily analytically calculated

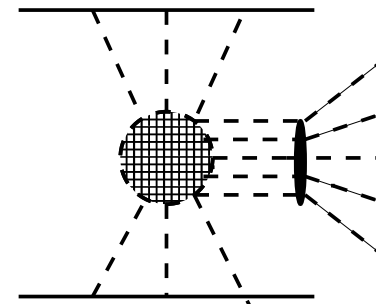
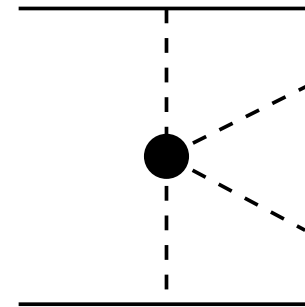
$$\int \tau^a A_\mu^a dx_\mu \sim (\tau^a \eta_{\mu\nu}^a e_\nu^t e_\mu^l) \int F(x^2) dt$$

- The trick with rotating cross section to Minkowski seem to work perturbatively and in general (Meggiolaro): we tried it for instanton

- Soft Pomeron with ‘‘instanton ladder’’ provides reasonable intercept and slope

- **No Odderon** SU(3) allows for a colorless combination of 3 gluons.

Perturbatively, the odderon/pomeron ratio is $O(\alpha_s)$ and not as suppressed as the data shows. Instantons are SU(2) beasts and do not allow it.



Double-DIS $\gamma^*\gamma^*$ (Next linear collider?)

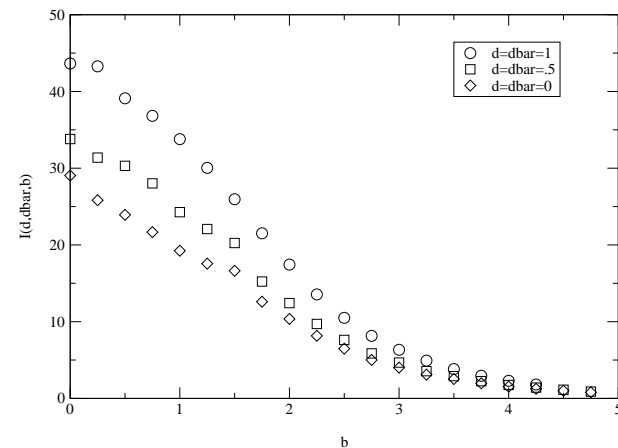
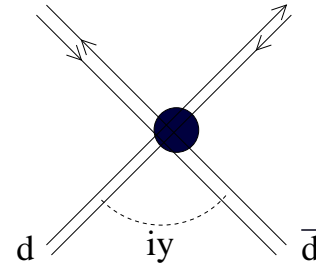
About the same small gluonic spot without any nucleon

ES, I. Zahed hep-ph/0307103,

PRD in press

Two dipoles cross the instanton, the angle is changed to iy at the end of the calculation

$\sigma \sim d_1^2 d_2^2$ times the dimensionless function of the three 2-d vector variables $I(\vec{d}_1/\rho, \vec{d}_2/\rho, \vec{b}/\rho)$ explicitly given and tabulated: see b-profile



**Semiclassical Double-Pomeron Production
of Glueballs, η' and clusters**

ES and I.Zahed, Phys.Rev. D68 (2003)
034001

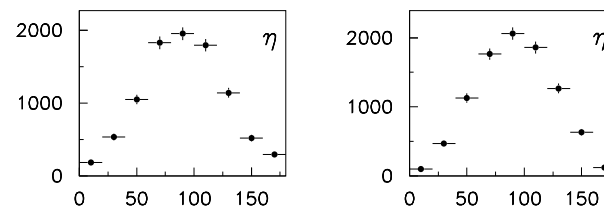
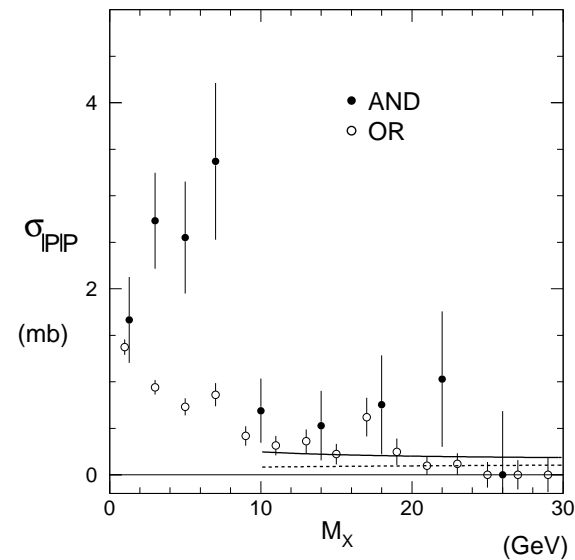
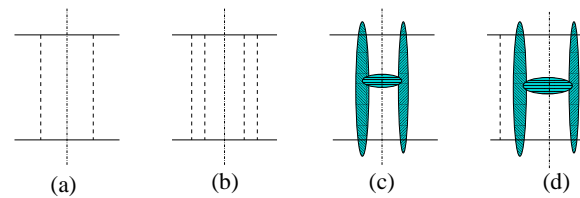
Pomeron-Pomeron into cluster, cross section from UA8 collaboration: heavy gluonic clusters with isotropic decay.

What are they?

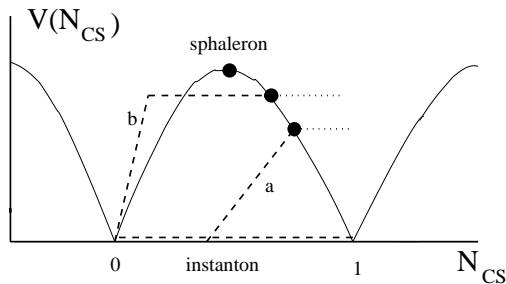
Note: a cross section that is an order of magnitude larger than the one predicted by Pomeron factorization

WA102 collaboration at CERN, pp Double-Pomeron into identified central hadron: strong dependence of the cross section on the azimuthal angle ϕ (between two kicks to two protons), not expected from standard Pomeron phenomenology.

we get $\cos^2(\phi)$ for $P=+$ and $\sin^2(\phi)$ for $P=-1$.



Quantum Mechanics of the tunneling glue



- The energy of Yang-Mills field versus the Chern-Simons number $N_{CS} = \int d^3x K_0$ is a periodic function, with zeros at **integer** points.

- The *instanton* (the lowest dashed line) describes tunneling between vacua. It is a path at $E=0$, it starts and ends at no field strength
- If energy is deposited, the paths (the dashed lines) goes up and emerge from ‘‘under the barrier’’ into real (Minkowskian) world at the **turning points**, where momenta (in the $A_0 = 0$ gauge) ‘‘ \vec{p} ’’ = $\frac{d\vec{A}}{dt} = \vec{E} = 0 \Rightarrow$ the field is **magnetic**
- Real time motion outside the barrier (shown by horizontal dotted lines) \Rightarrow **explosions**
- The maximal cross section corresponds to the top of the barrier, called the **sphaleron** = ‘‘ready to fall’’ in Greek, according to **Klinkhammer and Manton**

The Turning States from Constrained Minimization

• What is the minimal potential energy of static Yang-Mills field, consistent with the constraints:

(i) the given value of (corrected) Chern-Simons number.

(ii) the given value of the r.m.s. size $\langle r^2 \rangle = \int d^3x r^2 \mathcal{B}^2 / \int d^3x \mathcal{B}^2$

• Solution (found by D.Ostrovsky) is a ball made of three magnetic gluon fields (out of 8 in SU(3)) rotated around x,y,z axes

$$B^2/2 = 24(1 - \kappa^2)^2 \rho^4 / (r^2 + \rho^2)^4$$

$$E_{stat} = 3\pi^2(1 - \kappa^2)^2 / (g^2 \rho) \quad \tilde{N}_{CS} = \text{sign}(\kappa)(1 - |\kappa|)^2(2 + |\kappa|)/4.$$

Eliminating κ one gets the topological potential energy,

$\kappa = 0$ gives the sphaleron

Explosion of the Turning States

- Solved both **numerically** (G.Carter) and **analytically** (by D.Ostrovsky based on work by Luescher and Schechter from 1977 which can also be via conformal transformation -Zahed)

- Sphalerons at $t = 0 \Rightarrow$ (at large t) into a spherical transverse

wave
$$4\pi e(r, t) = \frac{8\pi}{g^2 \rho^2} (1 - \kappa^2)^2 \left(\frac{\rho^2}{\rho^2 + (r-t)^2} \right)^3$$

(i) happens in short time

$$\sim \rho \sim 1/3 \text{ fm}$$

(ii) about half gluons

survive

(iii) quasi-thermal

spectrum

all like in Venugopalan et
al transverse lattice
solution for random glue

Quark production puzzle

- The puzzle - from electroweak sphaleron decay

J. Zadrozny, Phys. Lett. **B284** (1992) 88; M. Hellmund and J. Kripfgger, Nucl. Phys. **B373** (1992) 749.

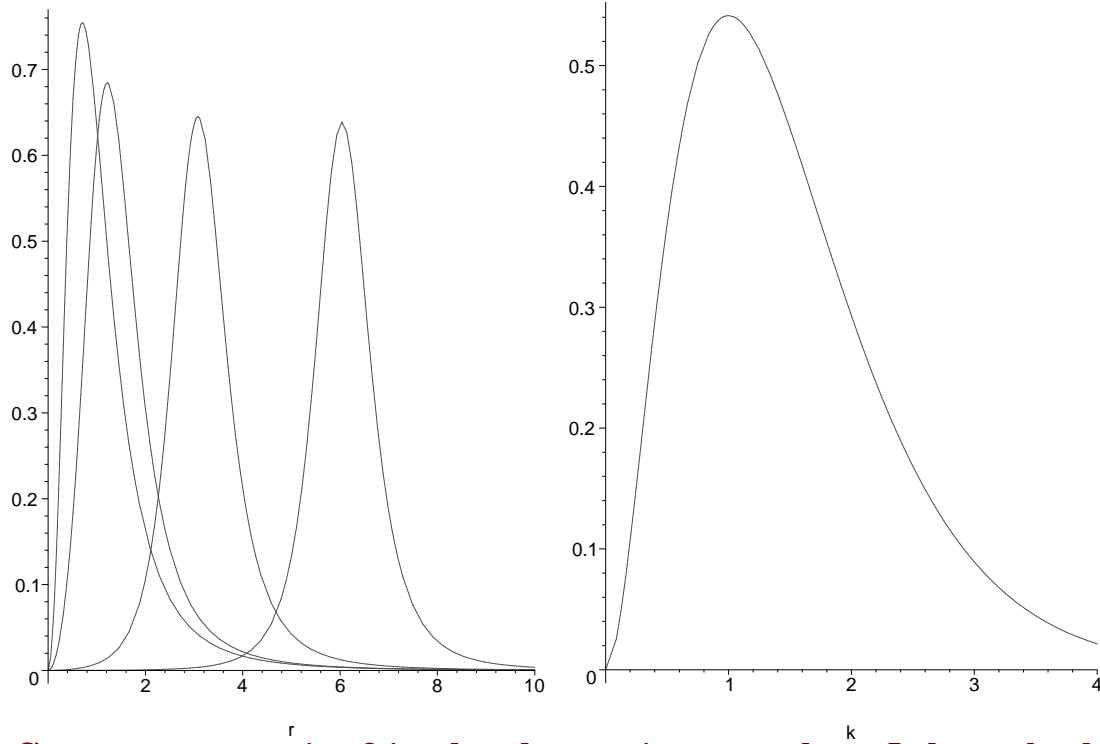
Chern-Simons number change is **NOT an integer**, while quark level motion can only happen in integers.

Few papers by well known people, not much progress..

- ES+Zahed: new solution to the Dirac equation in exploding background obtained by **inversion of the fermionic $O(4)$ zero mode of $O(4)$ symmetric solution.**

It explicitly shows **how the quark acceleration occurs**, starting from zero energy at $t=0$ to the final spectrum

- The sphalerons produce **one** level crossing $N_F \bar{L}R$ quarks, and the antisphaleron-like clusters the chirality opposite.



(a) The radial density distribution $\rho_+(t, r)$ for $t/\rho = 0, 1, 3, 6$:
 (b) The quark spectrum versus k in units of $1/\rho$.

Some unsatisfied theorists should ask how one can generalize index theorems, so that the number of level crossing would be calculable directly from the gauge field itself, without the explicit solution of the Dirac eqn. We dont know how.

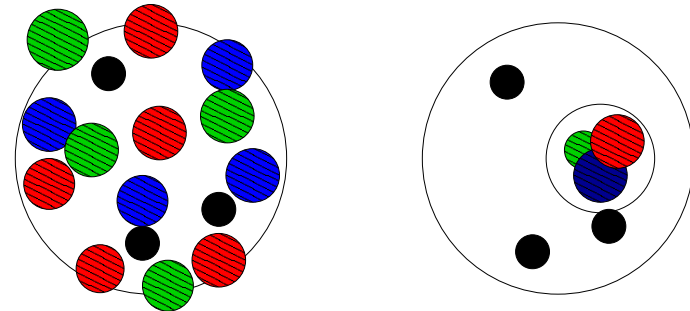
‘‘Small gluonic spots’’ in DIS and diffraction

● myths of 1970’s: all glue in the nucleon is radiated off the valence quarks, by DGLAP/BFKL pert.process

● Not true at all:

(i) there is a lot of glue even at low normalization $\mu \sim 1 GeV$

(ii), most important, its $R_{r.m.s.} \sim .3 - .4 fm$ is smaller than for quarks. Obviously a glue cannot be radiated from quarks as size never decreases in a random walk



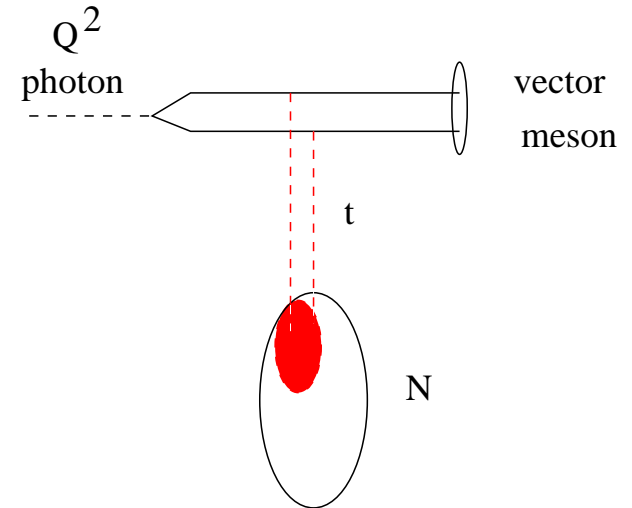
Snapshots of the nucleon: ‘‘large pizza’’ vs ‘‘small gluon spot’’

The empirical sources of this information:

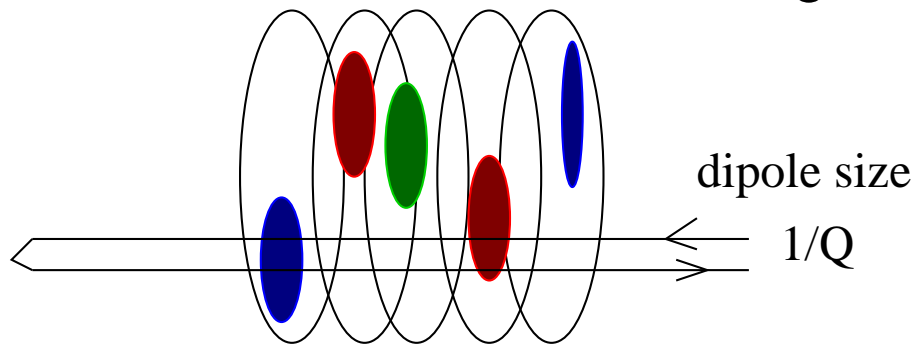
- The pomeron is a small-size object, its formfactor is found to be **harder** than F_{em} (Lanshoff et al fits in 1980's)

- the profile $T(b)$ was also determined from HERA diffractive DIS $\gamma^*p \rightarrow J/\psi + \dots$
Kopeliovich 2001, Kowalski and Teaney 2003 ($T(b)$ is derived from the Fourier transform over t)

The size is $R_g \sim 1/3 fm \approx \rho_{instanton}$



Consequences for Heavy Ion Collisions:
enhanced shadowing in pp
but reduced nuclear shadowing

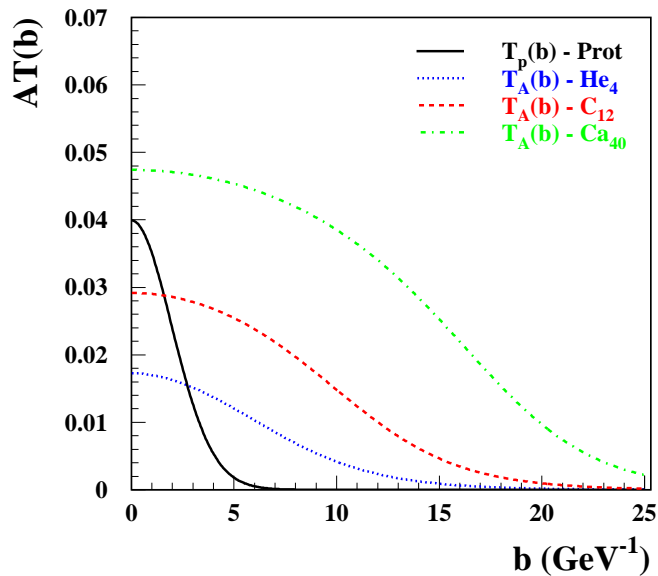


Kowalski and Teaney, hep-ph/0304189:

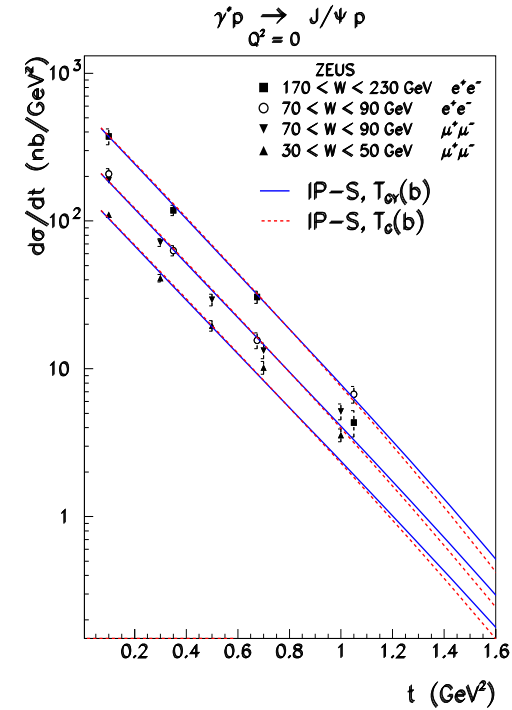
diffractive vector-meson t -distributions

can be described by $\frac{d\sigma_{VM}^{\gamma^*p}}{dt} \propto \exp(-B|t|)$.

$$\Rightarrow T_G(b) = \frac{1}{2\pi B_G} \exp(-b^2/2B_G), \quad B_G = 4.25 \text{ GeV}^{-2}$$



The transverse density $AT_A(b)$ for several light nuclei compared to the proton transverse profile, $T_p(b)$.



The differential cross-section for exclusive diffractive J/ψ production as a function of t for representative bins in W [?]. The solid (dashed) lines show the results of the IP saturation model assuming 2 parameterizations

Small gluon spot is also needed to get the nonlinear evolution going, and to get the needed deviations from BFKL: Levin, Lublinsky et al fitted HERA pdfs with the best fitted $R_N^g \approx .35 fm$

Levin, Lublinsky hep-ph/0308279: a modified evolution eqn:

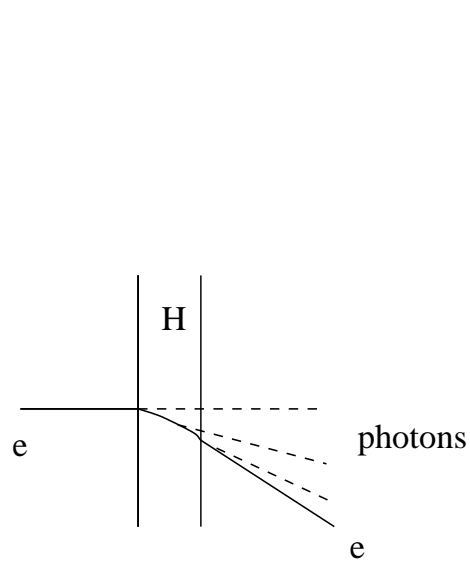
$$\kappa_A(b) = \pi R_N^2 S_A(b) \quad \frac{dN_A}{dY} = Ker \otimes \left[N_A - \frac{1 + \kappa_A}{\kappa_A} N_A^2 \right]$$

Large $\kappa_A(b)$ is the same, small $\kappa_A(b)$ enhances the second term, $N_A = \kappa_A N_N$ with N_N being a solution of BK eqn for a nucleon target

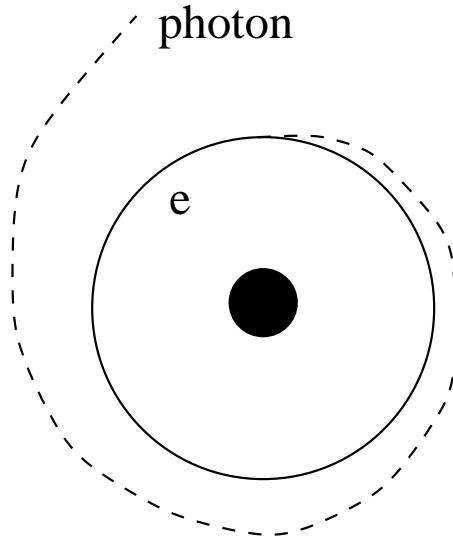
Enhanced Jet Quenching by
QCD synchrotron-like radiation

ES and Zahed, Phys.Rev.D67:054025,2003, hep-ph/0207163

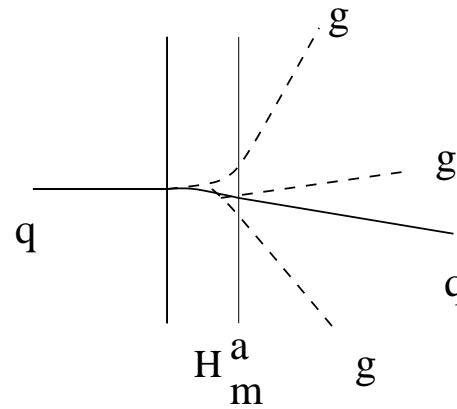
- synchrotron-like radiation in QCD generalizing Schwinger's treatment of quantum synchrotron radiation in QED



the usual magnet



a charge rotating ultra-relativistically in a gravity field (e.g. around a black hole)



the gluonic magnet

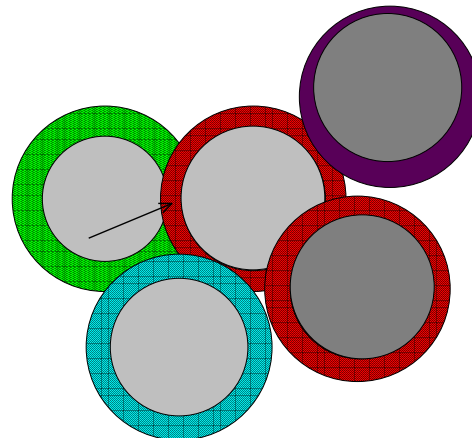
Jet quenching in Color Glass Condensate the energy loss of a quark during the CGC time period Δt_{CGC}

$$\frac{\Delta E_{CGC}}{E} \approx .3 \left(\frac{H}{1 \text{ GeV}^2} \right)^{2/3} \left(\frac{\Delta t_{CGC}}{.5 \text{ fm}} \right) \left(\frac{1 \text{ GeV}}{E} \right)^{1/3}$$

The gluon loss is about twice that.

● Jet Quenching on the Exploding Sphalerons

At $t=0$ sphalerons form a dilute gas and thus cannot affect most of the jets. This however must happen later: exploding shells \Rightarrow foam-like structure at $t = .6 - .8 \text{ fm}$

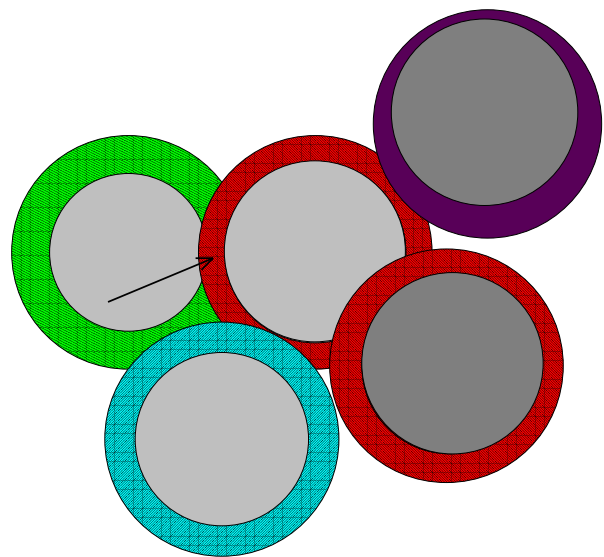


$$\frac{\Delta E_{clust}}{E} \approx .21 \left(\frac{H}{.2 \text{ GeV}^2} \right)^{2/3} \left(\frac{1 \text{ GeV}}{E} \right)^{1/3}$$

As usual, the gluon energy loss is about twice larger

Conclusions

- The glue in the nucleon, a ‘‘small gluon spot’’, is due to prompt excitation of a topological glue in the vacuum, instantons which dominate the quark condensate and a lot of hadronic physics
- Semiclassical theory of high energy pp, DIS etc is being developed, complementing pQCD approaches (DGLAP/BFKL) at the scale $Q^2 < \text{several } G$
=> new soft pomeron
- Heavy Ions: CGC with a topological structure?
- => prompt quark production may generate enough entropy
- => saturation in pp but reduced nuclear shadowing
- => strongly enhanced jet quenching?



New “explosive” scenario in brief

- **Tunneling** under the topological barrier is described by instantons. Leads to non-zero $\langle \bar{q}q \rangle \sim 5 \text{ pairs}/\text{fm}^3$ compare to $\sim 0.5 \text{ quarks}/\text{fm}^3$ in nuclear matter
- High energy collisions interrupt tunneling and make virtual fields **real**, in form of specific **gluomagnetic** objects or the **Turning States** (generalizing **sphalerons** of electroweak theory)
- Those clusters have $M \sim 3 \text{ GeV}$ and are very **explosive**: they decay into a thin spherical shell of coherent E and B.
- In hh collisions they hadronize, but in AA they produce $\sim 3 \text{ gluons} + \bar{u}u\bar{d}d\bar{s}s$
- Many such clusters (up to 400) are produced in (central) AuAu – **(early entropy)**. Decay products have quasi-thermal spectrum and nearly exactly match the equilibrium QGP composition
- Jets fly through those spherical shells and get a kick from its **coherent field** and radiate: **early jet quenching**

Forced Tunneling and Instanton-Antiinstanton configurations

- Two different views on Instanton-antiinstanton configurations. One: such fields occur in the YM vacuum and describe a **virtual path over the barrier but ends up in the same well** ($\delta Q = 0$).
- Another: the corresponding action would rather control the *probability* of transition

$$P \sim | \langle 0 | M | \text{turning state} \rangle |^2$$

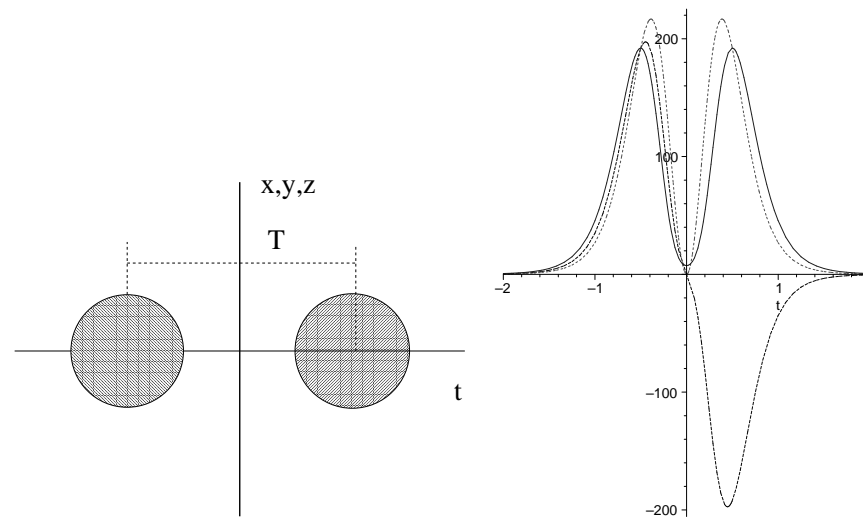
into *turning states* excited from the vacuum by some external force.

$D_\mu G_{\mu\nu} = j_\nu^{ext}$. • Simple sum of instanton and anti-instanton A_μ in singular gauge -- known as the **sum ansatz** -- has many bad qualities such as infinite fields at the centers • the so called *ratio ansatz*

(ES-1988) is better: for identical sizes and orientations is

$$gA_{a\mu}^{ratio}(x) = \frac{2\eta_{a,\mu,\nu}y_1^\nu\rho^2/y_1^2 + 2\bar{\eta}_{a,\mu,\nu}y_2^\nu\rho^2/y_2^2}{1 + \rho^2/y_1^2 + \rho^2/y_2^2}$$

- These trial functions are simple enough to have analytic expressions for the field strength, see fig.

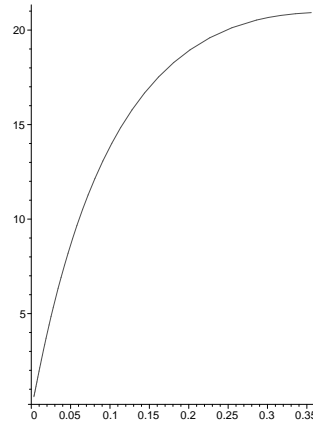


Instanton-antiinstanton configurations. (a) A schematic picture in the Euclidean space-time. The vertical thick line, $t=0$, corresponds

to the location of the turning state. It also display the definition of inter-center distance T . (b) Actual distribution along the time axis of $2\vec{B}^2, 2\vec{E}^2, 2\vec{B}\vec{E}$ for the ratio ansatz, $T=\rho$, shown by the solid, dashed and short-dashed lines respectively. The curve for $\vec{B}\vec{E}$ is the only one which is t -odd. above. • Due to $t \rightarrow -t$ symmetry, quantities which are odd under this transformation (like A_0 or electric field G_{0m}) should naturally vanish at $t=0$ 3-plane • the resulting purely magnetic configuration at this central 3-plane $t=0$ is the *turning points* of these paths we want to study. Their energy $E(T)$ and Chern-Simons numbers $N_{CS}(T)$ at $t=0$ can be calculated plotting $E(N_{CS})$ one can get the profile of barrier

• Alas, for the sum ansatz this idea does not produce reasonable results. When T decreases, the energy $E(T)$ of the turning state (as well as the action for the whole configuration) becomes very large, while $N_{CS}(T)$ no longer changes.

- The ratio ansatz turned somewhat better results, with finite (and even simple) field structure at all T including the coinciding IA centers ($T = 0$), but (see fig) it can only accomplish about 1/3 of the journey



The *Normalized* energy $E \cdot R$ versus the Chern-Simons number, for ratio ansatz.

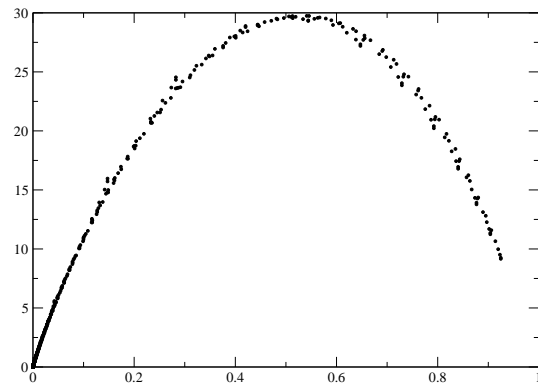
- Going uphill: the Yung ansatz (approximately a solution of the so called ‘streamline equation’ - Verbaarschot) The Yung ansatz for the field configuration is rather complicated, has no apparent t to $-t$ symmetry but accomplish everything

- As classic Yang-Mills theory has scale invariance one should evaluate the energy times the r.m.s. radius $E * R$ defined as

$$R^2 = \frac{\int d^3r r^2 B^2}{\int d^3r B^2}$$

Fig.shows indeed a parabolic-looking maximum near $N_{CS} = 1/2$.

Time dependent paths: E*R vs Ncs



Explosion of the Turning States

- Solved both **numerically** (G.Carter) and **analytically** (as it was found by D.Ostrovsky based on work by Luescher and Schehter) • (Witten -77) action for spherical YM

$$\mathcal{A}_j^a = A(r, t)\Theta_j^a + B(r, t)\Pi_j^a + C(r, t)\Sigma_j^a \mathcal{A}_0^a = D(r, t)\frac{x^a}{r}$$

with

$$\Theta_j^a = \frac{\epsilon_{jam} x^m}{r}, \quad \Pi_j^a = \delta_{aj} - \frac{x_a x_j}{r^2}, \quad \Sigma_j^a = \frac{x_a x_j}{r^2}$$

It is convenient to express functions $A, B, C,$ and D through the new set of r, t dependent parameters, which are related to the Abelian gauge ($A_{\mu=0,1}$) Higgs (ϕ, α) model on hyperboloid [?]

$$A = \frac{1 + \phi \sin \alpha}{r}, \quad B = \frac{\phi \cos \alpha}{r}, \quad C = A_1, \quad D = A_0.$$

$$\mathcal{A}_j^a = A(r, t)\Theta_j^a + B(r, t)\Pi_j^a + C(r, t)\Sigma_j^a \mathcal{A}_0^a = D(r, t)\frac{x^a}{r}$$

with

$$\Theta_j^a = \frac{\epsilon_{jam}x^m}{r}, \quad \Pi_j^a = \delta_{aj} - \frac{x_a x_j}{r^2}, \quad \Sigma_j^a = \frac{x_a x_j}{r^2}$$

It is convenient to express functions $A, B, C,$ and D through the new set of r, t dependent parameters, which are related to the Abelian gauge ($A_{\mu=0,1}$) Higgs (ϕ, α) model on hyperboloid [?]

$$A = \frac{1 + \phi \sin \alpha}{r}, \quad B = \frac{\phi \cos \alpha}{r}, \quad C = A_1, \quad D = A_0.$$

$$\mathcal{A}_j^a = A(r, t)\Theta_j^a + B(r, t)\Pi_j^a + C(r, t)\Sigma_j^a \mathcal{A}_0^a = D(r, t)\frac{x^a}{r}$$

with

$$\Theta_j^a = \frac{\epsilon_{jam}x^m}{r}, \quad \Pi_j^a = \delta_{aj} - \frac{x_a x_j}{r^2}, \quad \Sigma_j^a = \frac{x_a x_j}{r^2}$$

It is convenient to express functions $A, B, C,$ and D through the new set of r, t dependent parameters, which are related to the Abelian gauge $(A_{\mu=0,1})$ Higgs (ϕ, α) model on hyperboloid [?]

$$A = \frac{1 + \phi \sin \alpha}{r}, \quad B = \frac{\phi \cos \alpha}{r}, \quad C = A_1, \quad D = A_0.$$

$$\mathcal{A}_j^a = A(r, t) \Theta_j^a + B(r, t) \Pi_j^a + C(r, t) \Sigma_j^a \mathcal{A}_0^a = D(r, t) \frac{x^a}{r}$$

with

$$\Theta_j^a = \frac{\epsilon_{jam} x^m}{r}, \quad \Pi_j^a = \delta_{aj} - \frac{x_a x_j}{r^2}, \quad \Sigma_j^a = \frac{x_a x_j}{r^2}$$

It is convenient to express functions $A, B, C,$ and D through the new set of r, t dependent parameters, which are related to the Abelian gauge $(A_{\mu=0,1})$ Higgs (ϕ, α) model on hyperboloid

$$A = \frac{1 + \phi \sin \alpha}{r}, \quad B = \frac{\phi \cos \alpha}{r}, \quad C = A_1, \quad D = A_0.$$

$$S = \frac{1}{4g^2} \int d^3x dt [(\mathcal{B}_j^a)^2 - (\mathcal{E}_j^a)^2] = 4\pi \int dr dt \left((\partial_\mu \phi)^2 + \phi^2 (\partial_\mu - a_\mu)^2 + \frac{(1 - \phi)}{2r^2} \right)$$

- The Luscher-Schekhter solution can be found in a complicated coordinates obtained from $O(4)$ symmetric (non-zero E) one by a conformal transformation -Zahed item-- solution for large times becomes simple transverse wave with a simple profile

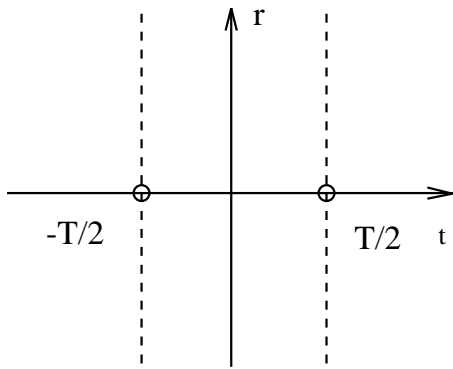
$$4\pi e(r, t) = \frac{8\pi}{g^2 \rho^2} (1 - \kappa^2)^2 \left(\frac{\rho^2}{\rho^2 + (r - t)^2} \right)^3$$

- the energy gluon distribution function defines the multiplicity
In pure YM a sphaleron of 3 GeV mass would decay into 3.5 gluons in average.

Paths maximizing cross section lead to rescaled sphalerons

in collisions the YM action is only one of the contributing factors. Another is the **overlap between the initial system of colliding gluons and the tunneling path**

One way to include *both* factors together is to use an adaptation of the Landau formula for overlapping matrix elements in terms of **singular** field configurations. (Khlebnikov, Diakonov-Petrov)



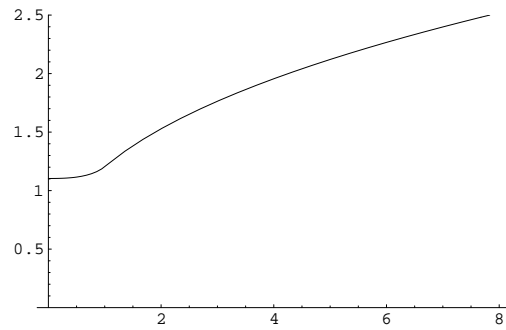
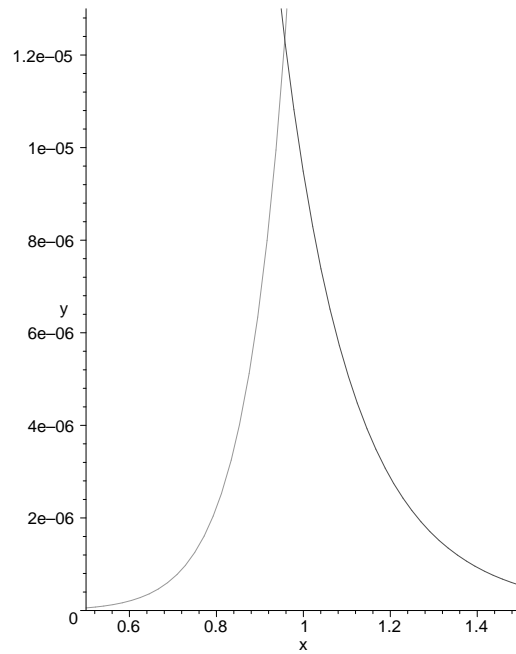
The space distance - time, r - t , plane. Small circles show positions of the singularities, the vertical dashed lines indicate lines at which two solutions with different energies

are joined together. The branch of the field that interpolates between the vacuum at $t = -\infty$ and the singularity at $t = -T/2$ with zero energy is singular instanton

The other brunch is the non-zero energy solution: we study it at high energy limit (of DP) and looked at $t=0$ or escape states: those are found to be just the YM sphalerons. with a simple rescaling of the size ρ and the energy density,

$$\rho \rightarrow \rho / (Q/M_S)^{1/5}$$

The same holds for explosion (Minkowski part of the path) a simple rescaling of the size ρ and the energy density. Gluon multiplicity gets $(Q/M_S)^{4/5}$ out of it



The inclusive gluon multiplicity $x^{4/5}\sigma(x)$ versus energy in units

of the sphaleron mass $x = Q/M_s$. (b) The ratio (??) of prompt gluons emitted as a function of $-t/M_S^2$.

●The Lienard expression for dipole radiation intensity

$$P = -\frac{2e^2}{3m^2} \left(\frac{dp_\mu}{ds} \right)^2$$

was derived in 1898, but is correct even for ultrarelativistic case. The acceleration appearing in it is proportional to field strength F and particle energy E , so classical energy losses depend quadratically on both of them, $P \sim e^4 F^2 E^2 / m^4$. the applicability condition on it (see textbooks e.g.[?]) is

$$e^3 (F/m^2)(E/m) \ll 1$$

where we have identified 3 dimensionless factors: the coupling constant, the field in m^2 units and the relativistic gamma factor. In cases when one uses classical QED such as synchrotron radiation in accelerators, this condition of course holds, in spite of the fact that γ can be as large as 10^5 (LEP): the reason is small couplin

and especially very small fields H/m_e^2 .

$$\frac{dN_B}{d\omega} \sim \frac{\alpha}{\pi} \frac{1}{\omega}$$

Synchrotron radiation has a different spectrum: total number of quanta per one circle is

$$\frac{dN_S}{d\omega} \sim \frac{\alpha}{\pi} \frac{(\omega/\omega_0)^{1/3}}{\omega},$$

The upper limit of this expression for synchrotron emission can be bounded by classical ‘‘characteristic frequency’’ $\omega_c = 3\gamma^3\omega_0$: substituting it in ?? we find that one circle of radiation emits $N_S \sim \alpha\gamma$ photons, while the total energy loss recovers what one can get directly from the Lienard expression mentioned above: it again grows quadratically with energy, or $\Delta E/E \sim E$. For synchrotron radiation a cut at $\omega_{\max} \sim E$ leads to

$$N_S \sim \alpha(E^2/eH)^{1/3}$$

and the energy loss per fixed* width of the field region Δz (rather than circle)

$$\Delta E_S/E \sim e^2(eH)^{2/3} \Delta z/E^{1/3}$$

which is *decreasing* with energy, although with a small power.

•The problem of quantum synchrotron radiation in QED was addressed in a fundamental way by Schwinger

$$-\frac{1}{E} \text{Im} M_{aa} = \int \frac{d\omega}{\omega} P_{aa}(\omega)$$

the chromomagnetic synchrotron emission by a scalar quark in the classical limit in the following form

$$P_{aa}(\omega) = -\frac{\alpha}{\pi} (T^A)_{ab} (T^A)_{ba} \times \omega \text{Im} \int_{0-i0}^{\infty-i0} \frac{d\tau}{\tau} \frac{e^{-i(E\omega_A)^2 \tau^3 / (24\omega)}}{\cos(E\omega_A \tau / (2\omega))} \times \left(\frac{m^2}{E^2} + \frac{1}{2} \omega_b^2 \right)$$

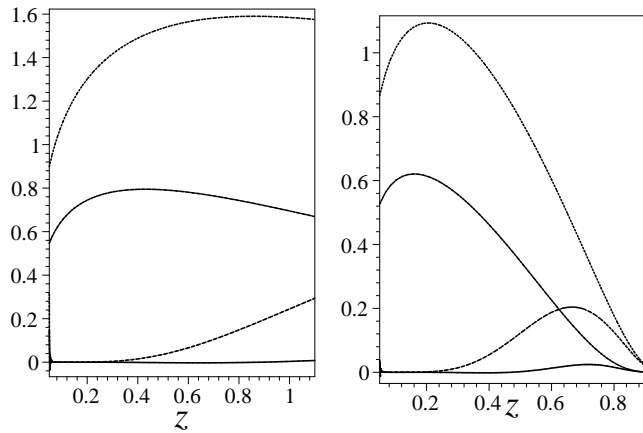
*It cannot be small compared to radiation length: $\Delta z > m/eH$.

where in () the $H = 0$ subtraction is not explicitly shown but implied. The quark synchrotron and gluon rescaled frequencies are $\omega_a = e_a H/E$ and $\omega_A = g_A H/E$ respectively.

In carrying out () the emitted gluon recoil effect on the jet was ignored, and so it is entirely classical. We have checked that the gluon recoil effect amounts in the first order to the shift

$$\frac{1}{\omega} \rightarrow \left(\frac{1}{\omega} - \frac{1}{E} \right)$$

in the combination P_{aa}/ω (the gluon multiplicity), thereby generalizing Schwinger's first quantum correction in QED to the QCD case. This substitution is not of course the complete quantum answer, but will be discussed below as an approximation.



The integral versus radiated energy fraction

$z = \omega/E$ for classical (upper figure) and quantum corrected (lower figure) cases. Four examples shown corresponds to all possible charge combinations, namely $e_a = 1, e_b = 1, e_A = 3$ (solid), $e_a = 1, e_b = -2, e_A = 3$ (dotted), $e_a = 1, e_b = 1, e_A = 0$ (dashed) and $e_a = 1, e_b = -2, e_A = 0$ dash-dotted lines, respectively.