Lecture #5

Forces & Newton's Laws

What makes acceleration?

Forces \sim mass \times acceleration

Examples: gravity.

Earth

\text{Weight} = m g \_E

\text{Weight} = m \left(9.8 \text{ m/s}^2 \right)

\text{Mass}

Feather \sim 1 \text{ gram}

2 \text{ liter bottle}

20 \text{ lb body}

Moon

Weight \_E = m g \_E

Weight \_Moon = m g \_Moon

\text{Weight on Earth} = \text{Weight on Moon}

\text{Weight on Earth} = m \left(9.8 \text{ m/s}^2 \right)

\text{Weight on Moon} = m \left(1.2 \text{ m/s}^2 \right)

Feather \sim 1 \text{ gram}

2 \text{ kg}

20 \text{ lb}

\text{Weight on Earth} \sim 20 \text{ N}

\text{Weight on Moon} \sim 2 \text{ N}

\text{Weight on Earth} \sim 1000 \text{ N}

\text{Weight on Moon} \sim 100 \text{ N}
More Examples of Forces

- Electromagnetic
  - Chemistry
  - Biology

- Strong Nuclear Force
  - Nuclear Physics

- Normal Forces:
  - perpendicular to surface
  - molecules are like tiny springs

Forces is a vector:

Newton's Laws

1. A Body in motion (or at rest) stays in motion until acted upon by an "external" force.

2. \[ \sum \mathbf{F} = m \mathbf{a} \]

3. For every action there is an opposite and equal reaction
Book on a table

\[ F_{\text{tot}} \text{ on book} = m \ddot{a}_{\text{book}} \]

\[ N_B - W_B = m g_{\text{book}} \]

\[ N_B = W_B = m g_{\text{book}} \]

Now:

Look at Table:

\[ -N_B + N_T - W_T = m g_{\text{Table}} \]

\[ N_T = W_T + N_B = W_T + W_B \]

Agrees with intuition
Then where is the equal and opposite force to \( \vec{W}_B \)?

Forces on earth:

- \( \vec{W}_B \)
- \( \vec{W}_T \)
- \( \vec{N}_T \)

Pulling to blocks:

- \( M = 2 \text{ kg} \)
- \( m = 1 \text{ kg} \)

Write Newton law in all directions:

- \( \vec{N}_2 \)
- \( \vec{T} \)
- \( \vec{W}_2 = M \vec{g} \)
- \( \vec{F}_0 \)
- \( \vec{N}_1 \)
- \( \vec{mg} \)
\[ T = M_2 a \]
\[ -T + F_0 = m_1 a \]

\[ N_2 - M_2 g = M_1 \alpha \]
\[ N_1 - m_1 g = m_1 a_y \]

\[ N_2 = M_2 g \]
\[ N_1 = m_1 g \]

Then:

\[ \frac{I}{M_2} = \alpha \]
\[ -T + F_0 = m_1, \frac{I}{M} \]

\[ F_0 = \left( \frac{m_1 + 1}{M} \right) T \]

\[ \frac{F_0}{1 + m_1/M} \]

\[ \alpha = \frac{-1}{M_2 (1 + m_1)} \frac{F_0}{M + M_1} \]

\[ \alpha = \frac{F_0}{M + M_1} \]
Why is \( a = \frac{F_0}{M+m} \)?

\[ F_{\text{Net}} = M_{\text{Tot}} a_{\text{tor}} \]

\[ F_0 = (M + m) a \]

\[ F_0 = a \frac{M}{M+m} \]

**Dimensional Analysis**

\[ [F_0] \sim N \]

\[ [M] \sim \text{kg} \]

\[ [m] \sim \text{kg} \]

Dimensionless: \( \frac{m}{M} \)

\[ T \times F_0 \]

\[ T = F_0 f\left(\frac{m}{M}\right) \]

The tension must be proportional to \( F_0 \).
Atwood Machine

Problem

1) Find the acceleration of the blocks

Solution

- Write down Newton’s law for both blocks
- Agree on a coordinate system

Block 1

\[ \Sigma F_{net} = m_1 a_1 \]

\[ T - m_1 g = m_1 a_1 \quad (1) \]
Block 2

\[ T - M_2 g = M_2 a_2 \]

\[ \omega_2 = M_2 g \]

Since they are connected by a rope when block 1 goes up block 2 goes down so

\[ a_1 = -a_2 \]

We know that block 2 goes down -- \( a_2 \) is negative

\[ a_1 = \text{positive} \]

So Block 2 becomes

\[ T - M_2 g = -M_2 a_1 \] (2)

We now have two equations and two unknowns \( T \) and \( a_1 \)

From (2) \[ T = M_2 g - M_2 a_1 \]

From (1) \[ M_2 g - M_2 a_1 = m_1 g = m_1 a_1 \]

\[ M_2 g - m_1 g = m_1 a_1 + M_2 a_1 \]

\[ \left( \frac{M_2 - m_1}{m_1 + M_2} \right) g = a \]
So then

\[ T = M_2 g - M_2 \frac{g}{(M_2 + m_1)} \left( \frac{M_2 - m_1}{M_2 + m_1} \right) \]

\[ T = M_2 g \left( \frac{M_2 + m_1}{M_2 + m_1} - \frac{(M_2 - m_1)}{M_2 + m_1} \right) \]

\[ T = \frac{2M_2 m_1 g}{M_2 + m_1} \]
Lecture #5

Forces & Newton's Laws

→ What makes acceleration?

Forces \sim mass \times acceleration

Examples: gravity.

\[ \text{Weight} = mg \]

\[ \text{Weight} = m \times (9.8 \text{ m/s}^2) \]

\[ \text{Weight on Earth} = m \times 9.8 \text{ m/s}^2 \]

\[ \text{Weight on Moon} = m \times (1.2 \text{ m/s}^2) \]

Mass

Feather \sim 1 \text{ gram}

1 \text{ gram} \times 10 \text{ m/s}^2 = 0.01 \text{ N}

2 \text{ liter bottle} \sim 2 \text{ kg}

2 \text{ kg} \times 9.8 \text{ m/s}^2 \approx 20 \text{ N}

200 \text{ lb body} \sim 100 \text{ kg}

100 \text{ kg} \times 9.8 \text{ m/s}^2 \approx 1000 \text{ N}