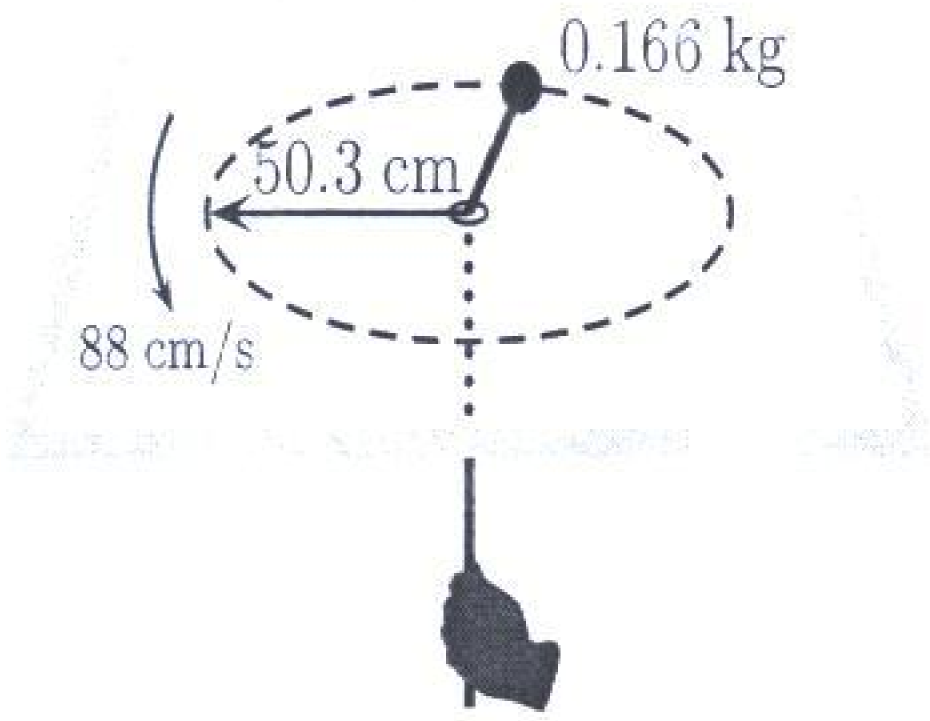
Exam A

The puck in the figure has a mass of 0.166 kg. Its original distance from the center of rotation is 0.5 m , and the puck is moving with a speed of 0.88 m/s. The string is pulled downward 0.13 m through the hole in the frictionless table.



1. Find the velocity of the puck after it has been pulled in.
2. Find the work done on the puck.

Angular Momentum Consv.

(1)

$$l_i = l_f$$

$$\cancel{m} v_i r_i = \cancel{m} v_f r_f$$

$$v_i \frac{r_i}{r_f} = v_f \qquad v_f = 1.19 \, m/s$$

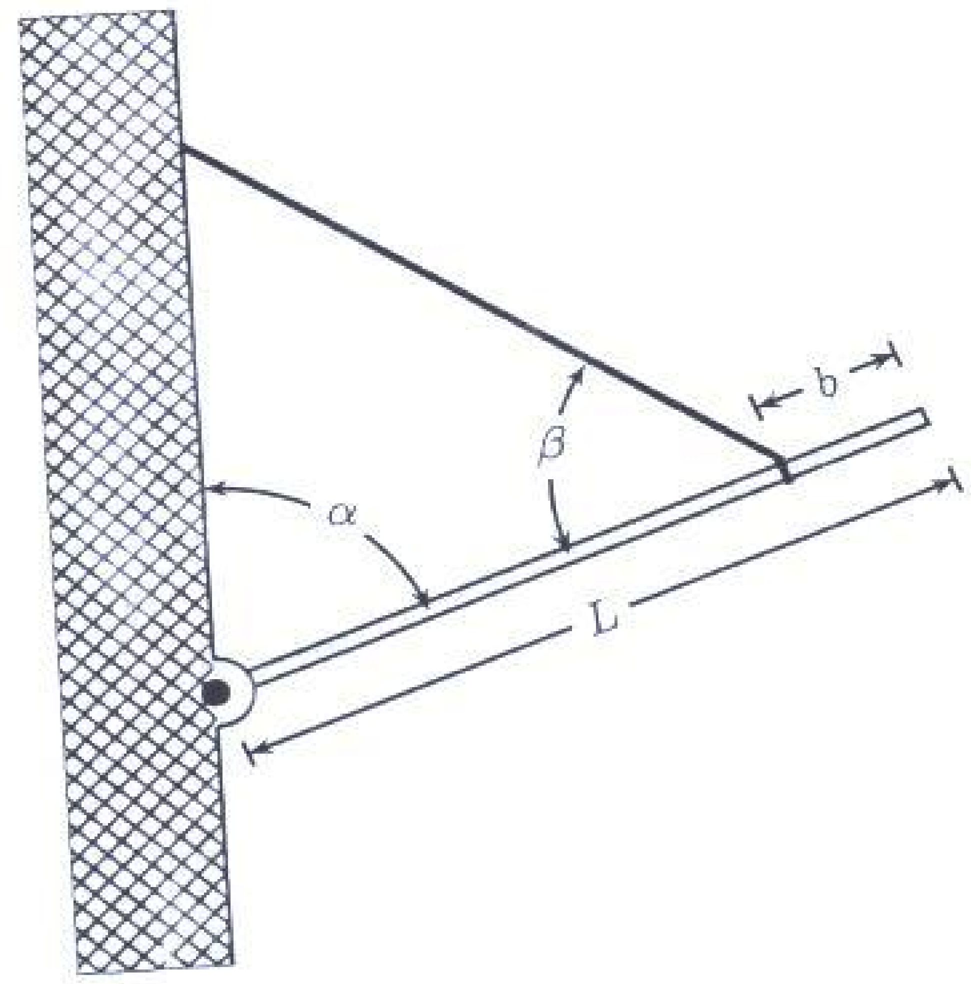
(2)

$$W = \Delta KE + \cancel{\Delta PE}$$

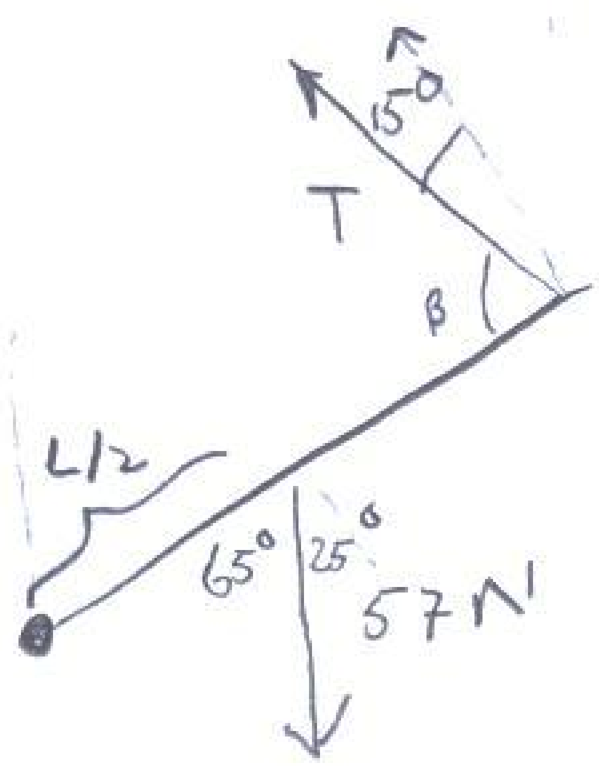
$$W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

$$W = \underbrace{\frac{1}{2} m v_i^2}_{0.064 J} \underbrace{\left(\left(\frac{r_i}{r_f}\right)^2 - 1 \right)}_{0.83} = 0.053 J$$

Consider a lever rod of length L=7.57 m, with weight $W = 57N$ and uniform density. As shown on the picture below, the rod is pivoted on one end and is supported by a cable attached at a point $b = 1.33$ m from the other end:



The lever rod is in equilibrium at angle $\alpha = 65°$ and angle $\beta = 75°$. What is the tension in the supporting cable.



$$\sum F^x = 0$$

$$\sum F^y = 0$$

$$\sum \tau = I \cancel{\alpha}^{0}$$

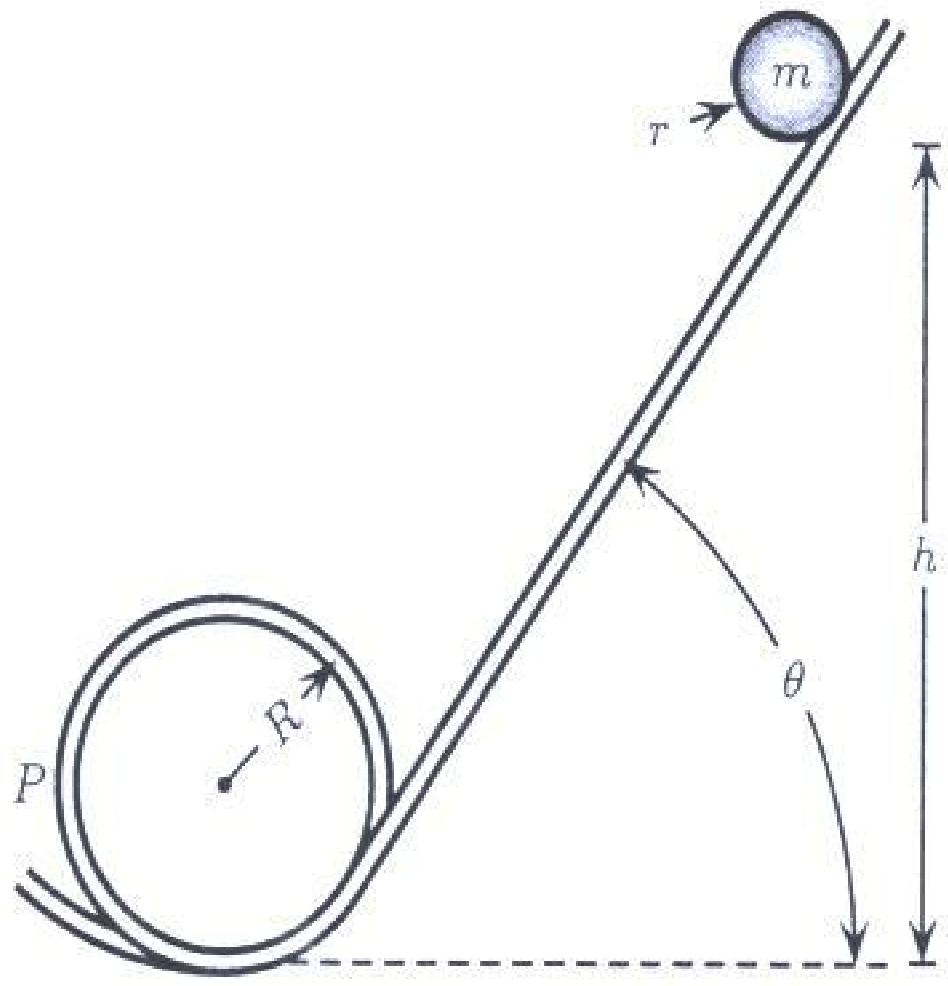
$$\vec{\tau}_g + \vec{\tau}_T = 0$$

$$-W\frac{L}{2}\sin\alpha + T(L-b)\sin\beta = 0$$

$$T = W\frac{\sin\alpha}{\sin\beta}\left(\frac{L/2}{L-b}\right)$$

$$T = 32.4\,N$$

A small solid sphere of mass m and radius r rolls without slipping along a track consisting of a slope and loop-the-loop with radius R at the end of the slope. It starts from rest near the top of the track at a height h, where h is large compared to r. The height h is large enough to make it to the top of the loop-the-loop.



1. What is the speed at the bottom of the loop-the-loop. Express your answer in terms of the givens – h, r, m, g
2. What is the speed at the top of the loop-the-loop.

No external work

$$\cancel{KE_i}^{\,0} + PE_i = KE_f + \cancel{PE_f}^{\,0}$$

$$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$$

$$mgh = \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v}{r}\right)^2 + \frac{1}{2}mv^2$$

$$\cancel{m}gh = \frac{1}{5}\cancel{m}\cancel{r^2}\frac{v^2}{\cancel{r^2}} + \frac{1}{2}\cancel{m}v^2$$

$$gh = \frac{7}{10}v^2$$

$$\sqrt{\frac{10}{7}gh} = v$$

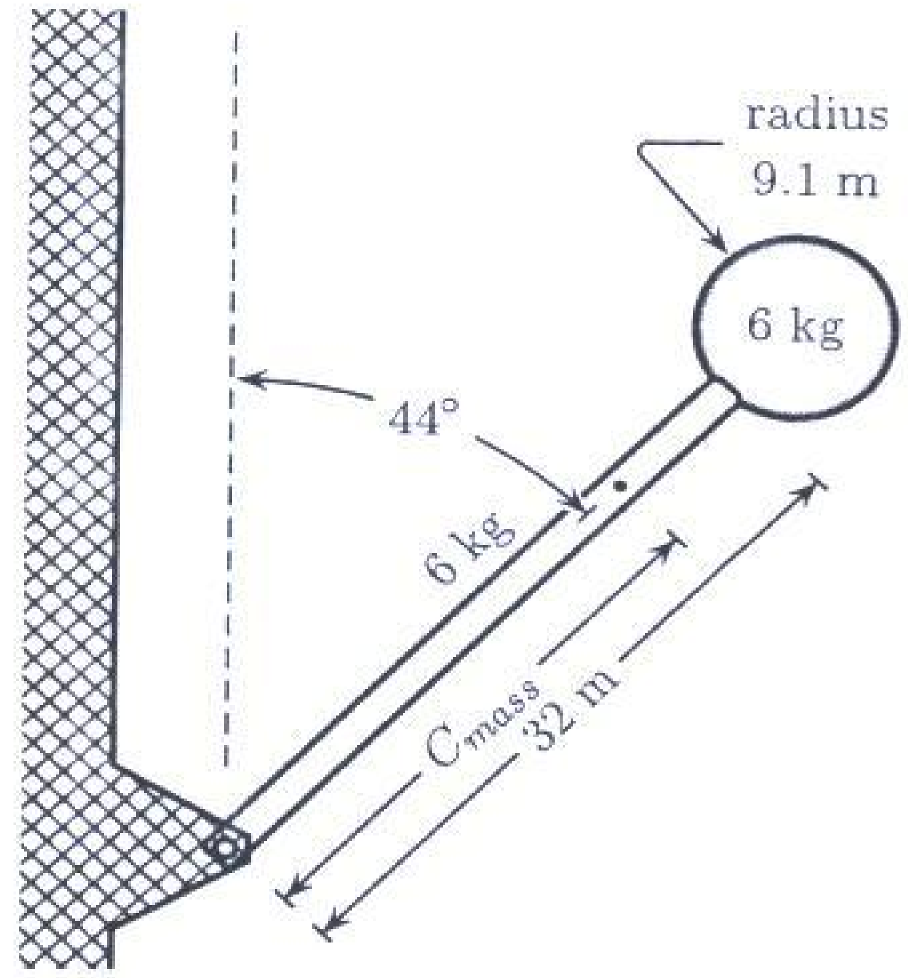
$$mgh = KE_f + PE_f$$

$$mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2 + mg2R$$

$$g(h-2R) = \frac{1}{2}\left(\frac{2}{5}mr^2\right)\frac{v^2}{r^2} + \frac{1}{2}mv$$

$$\sqrt{\frac{10}{7}g(h-2R)} = v$$

Consider a 32 m rod (That's big!) pivoted at one end. A uniform spherical object (whose mass is 6 kg and radius is 9.1 m) is attached to the free end of the rod. The moment of inertia of a rod about an end is $I_{\text{rod}} = \frac{1}{3}ML^2$, the momentum inertia of a sphere about its center-of-mass is $I_{\text{sph}} = \frac{2}{5}mr^2$.



1. What is the moment of inertia of this object around the pivot.
2. What is the angular acceleration of this object immediately after it is released?

(1)

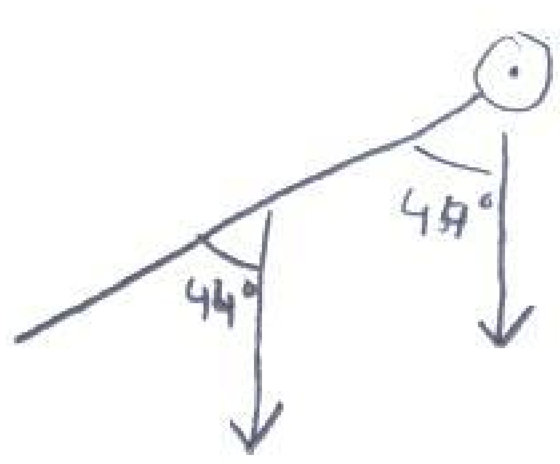
$$I_{TOT} = I_{rod} + I_{sphere}$$

$$= \frac{1}{3} m_{rd} L^2 + \underbrace{m_{sp} d^2 + I_{sphere,cm}}_{I_{sphere}}$$

$$I_{TOT} = \frac{1}{3}(6kg)(32m)^2 + (6kg)(32m + 9.1m)^2 + \frac{2}{5}(6kg)(9.1m)$$

$$I_{TOT} = 12382 \ kgm^2$$

(2) $\tau_{Net} = I\alpha$



$$\vec{\tau}_{rod} + \vec{\tau}_{ball} = I\alpha$$

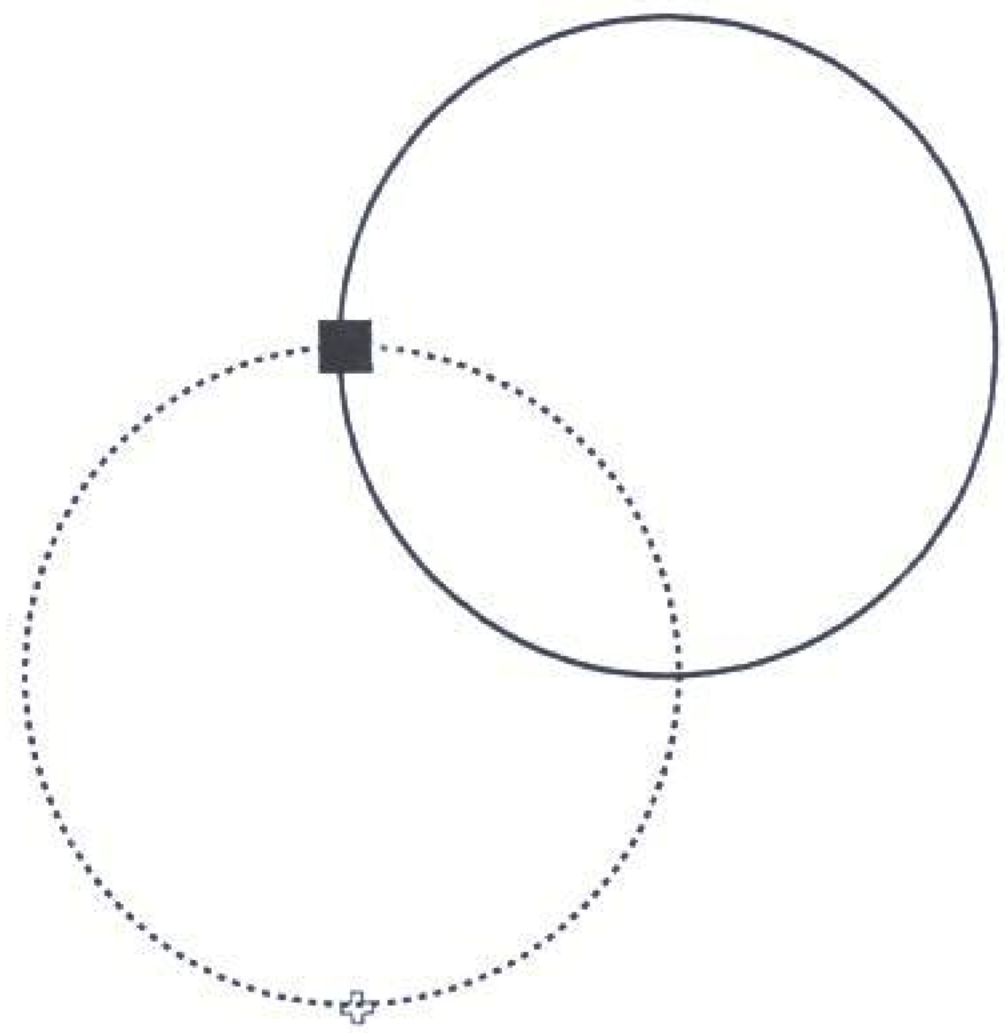
$$\alpha = 0.188 \ \frac{rad}{s^2}$$

$$m_{rd} g \frac{L}{2} \sin 44^\circ + m_{sp} g (L + r) \sin 44^\circ = I\alpha$$

$$(6kg)(9.8 m/s^2)\left(\frac{32m}{2}\right)\sin 44^\circ + (6kg)(9.8 m/s^2)(32m + 9.1m)\sin 44^\circ = 2332 \ kgm^2/s^2 = I\alpha_r$$

A hollow ring of radius R and mass M is free to rotate on a frictionless pivot through a point on its rim (See Below). The disk is released from rest in the position shown by the solid circle:

1. Calculate the moment of inertia around the pivot.
2. What is the speed of the center of mass when the disk reaches the position indicated by the dashed circle (i.e. the bottom of the arc) ?
3. What is the speed of speed of the lowest point of the disk (indicated with the "x") when the disc reaches the bottom of the arc.



1. $I = I_{cm} + Md^2$

$I = MR^2 + MR^2 = 2MR^2$

2. $\overset{0}{\cancel{KE_i}} + PE_i = KE_f + \overset{0}{\cancel{PE_f}}$

$mgR = \frac{1}{2} I \omega_r^2$

$= \frac{1}{2} 2mR^2 \left(\frac{v}{R}\right)^2$

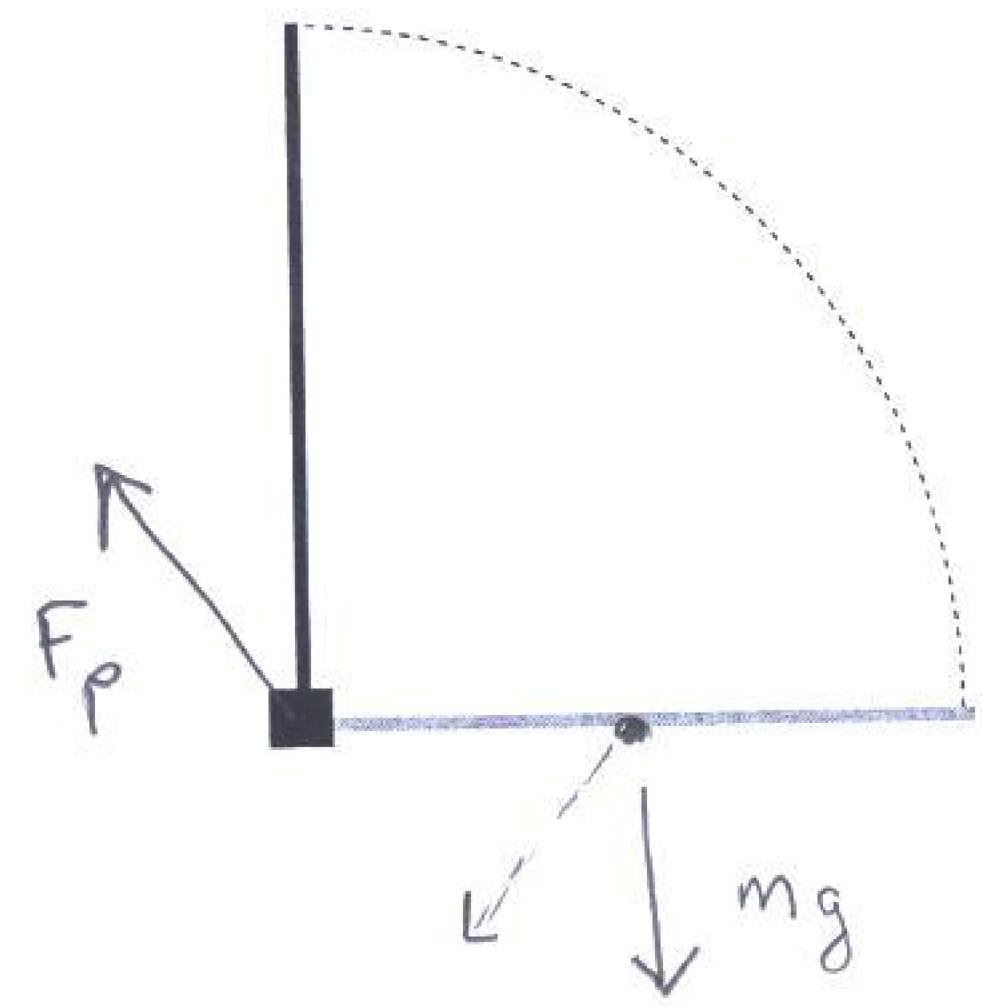
$\sqrt{gR} = v$

3. ω is constant $\qquad \omega = \frac{v}{R} = \frac{\sqrt{gR}}{R} = \sqrt{\frac{g}{R}}$

$V_{bottom} = \omega \cdot (2R) = 2\sqrt{gR}$

A long uniform rod of length L and mass M is pivoted about a frictionless pin though one end. The rod is released from rest at the top as shown below and falls. Consider the instant the rod is horizontal.

1. Find its angular speed.
2. Draw a free body diagram.
3. Determine the x and y components of the total acceleration Show the acceleration vector on the free body diagram with a dashed line.
4. Determine the x and y components of the force on the pin.



① $PE_i + \cancel{KE_i}^{0} = \cancel{PE_f} + KE_f$

$mg\frac{L}{2} = \frac{1}{2}(I\omega_r^2)$

$mg\frac{L}{2} = \frac{1}{2}(\frac{1}{3}ML^2)\omega_r^2$

$\sqrt{\frac{3g}{L}} = \omega_r$

$\Sigma\tau = I\alpha_r$

$-mg\frac{L}{2} = \frac{1}{3}mL^2\alpha_r$

$\frac{3}{2}\frac{g}{L} = \alpha_r$

$a_y = \alpha_r R$

$a_y = \frac{3}{2}\frac{g}{L}\cdot$

$\boxed{a_y = \frac{3}{4}g}$ dow

③+④ $\Sigma F^x = ma^x$

$F_p^x = m(-\frac{3}{2}g)$

$\boxed{F_p^x = -\frac{3}{2}mg}$

$a_x = -v^2/R$

$a_x = -\omega^2 R$

$= -\frac{3g}{L}\cdot\frac{L}{2}$

$\boxed{a_x = -\frac{3}{2}g}$

$\Sigma F^y = ma_y$

$F_p - mg = m(-\frac{3}{4}g)$

$\boxed{F_p^y = \frac{1}{4}mg \text{ up}}$

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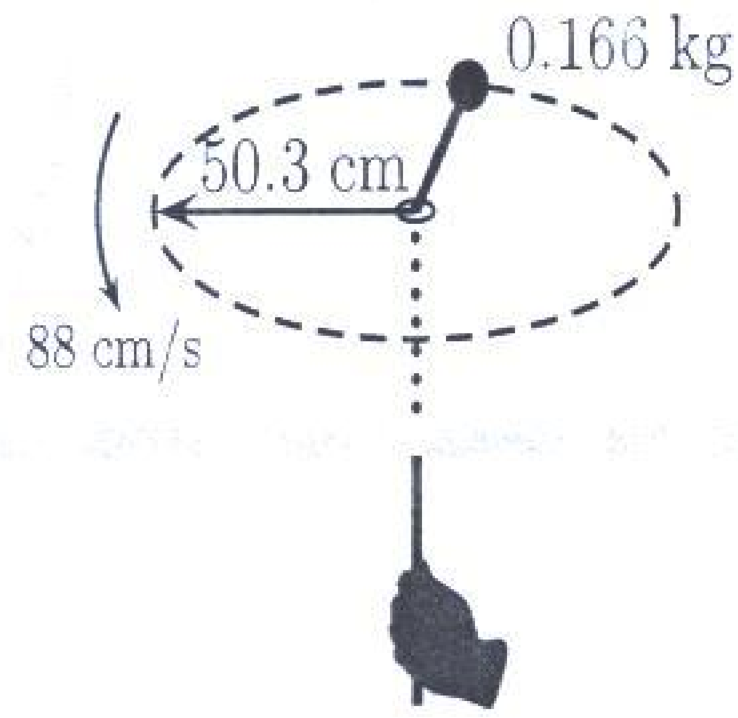

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