Last Time:

\[ E \rightarrow \text{sum of all potential and kinetic energies.} \]

\[ Q \rightarrow \text{heat flow of Energy from one system to another.} \]

\[ T \rightarrow \text{temperature, loosely \textit{energy/dot}} \]

Ex: \[ \frac{Q}{m} \sim \frac{\text{Energy}}{\text{# of dot}} \]

\[ Q = mc \Delta T \]

Sometimes it doesn't work that way.

\[ T \]

\[ \text{Steam} \rightarrow \text{Water} \rightarrow \text{Ice} \rightarrow \text{mixed} \]

\[ Q \]
Energy in an Ideal mono-atomic gas

Energy in gas

\[ \frac{E}{3N} = \frac{1}{2} k_B T \]

This can also be written

\[ E = \frac{3}{2} N k_B T = \frac{3}{2} n R T \]

For an Ideal diatomic gas

\[ \frac{E}{5N} = \frac{1}{2} k_B T \]

\[ E = \frac{5}{2} N k_B T = \frac{5}{2} n R T \]

\[ \text{number of moles} \quad \text{number of d.o.f} = \text{Number of molecules \times (3 directions each molecule can move)} \]

\[ 5 = 3 + 2 \]

3 ways to move

two ways to spin
Roughly

\[ T \sim \frac{\text{Energy}}{\text{degrees of freedom}} \]

\[ T \sim \text{constant} \]

Specifically, take a hunk of ice mass \( m \)

\[ Q = m \cdot L \]

Heat required to melt this hunk

Latent Heat for water to ice \( Q = m \cdot L \_f \)

(2) "heat of vaporization" water to steam

\[ Q = m \cdot L \_v \]
P21

a) In an insulated vessel 250g of ice at 0°C is added to 600g of H₂O at 18°C. What is the final temperature?

b) How much ice remains?

Sol

\[ L_f = 3.33 \times 10^5 \frac{J}{kg} \]

\[ C_w = 4.186 \frac{J}{kg \cdot ^\circ C} \]

Heat required to cool water

\[ Q = m c_w \Delta T \]

\[ Q = (600 \text{ kg})(4.186 \frac{J}{kg \cdot ^\circ C}) (-18^\circ K) \]

\[ Q = -45,208 \]
So

\[ Q_{in} = m_{melt} L_f \]

\[ \frac{Q_{in}}{L_f} = m_{melt} \]

\[ + \frac{4.15 \times 208}{3.33 \times 10^5} = m \]

\[ 135 \text{ g} = m \rightarrow \frac{135}{250} \text{ g} = \frac{54}{\%} \text{ of ice melted} \]

Liquid Helium has a boiling point of 4.2K and a low heat of vaporization.

\[ 2 \times 10^4 \frac{J}{kg} \]

How low does it take a 10W electric heater to boil away 5kg of this stuff

\[ Q = m L \]

\[ Q = (5 \text{ kg}) \left( 2 \times 10^4 \frac{J}{\text{kg}} \right) = 10^5 J \]
\[ P = 10 \text{ kW} = 36 \frac{kW}{h} \]

\[ P = \frac{\text{Energy}}{\text{time}} \Rightarrow \text{time} = \frac{\text{Energy}}{\text{Power}} = \frac{10^9 \text{s}}{10} = 10^8 \text{s} = 2.77 \text{ h} \]

**Work:**

\[ P = \frac{F}{A} = \frac{\text{Force}}{\text{Area}} \]

\[ PA = F \]

\[ \frac{PA \Delta x}{\Delta v} = F \Delta x = W \text{ by gas} \]

\[ P \Delta v = W \text{ by gas} \]

Pressure is a function of \( V \)

History: relation to steam to work

~James Watt~ 1785
Examples:

\[ \begin{align*}
W_{\text{by gas}} & < 0 \quad \text{Volume of gas decreasing} \\
W_{\text{by gas}} & > 0 \quad \text{Your doing work}
\end{align*} \]

Putting it All together, E-conservation:

\[ E = Q - W_{\text{by gas}} \]

\[ \begin{align*}
\text{Sum of} & \quad \text{Heat flow into gas} \\
\text{pot + KE} & \quad \text{Work done by gas} = -W_{\text{on gas}}
\end{align*} \]
I use an opposite sign convention from the book; see below.

\[ W_{\text{by gas}} = \int V_f P(v) \, dv \]

\[ = \text{Area under } P \text{ vs } V \text{ curve} \]

What is this "by gas", "on gas" nonsense?

Gas does work = \( P \Delta V \)

Work done by you = \(-P \Delta V\)

You do work on gas: \( W_{\text{on gas}} = P \Delta V \)

\( W_{\text{by gas}} = P \Delta V \)
Example Isothermal expansion

1 mole of mono atomic ideal gas undergoes an isobaric (constant expansion) from 2 L to 4 L pressure at 15.7°C

1. Calculate the work done and necessary heat flow

\[ \int P \, dV = P_0 \, \Delta V \]

\[ \Delta V = 4L - 2L = 2L \]

\[ W_{\text{by gas}} = 24 \times 10^5 \frac{N \cdot m}{m^2} \times \left( 2 \times 10^{-3} m^3 \right) = 4800 \, J \]
\( \Delta E = \frac{3}{2} n R \Delta T \)

\[ P \cdot V = n R \cdot T_b \]

\[(24 \times 10^5 \frac{N}{m^2}) (4 \times 10^{-3} m^3) = (2 \text{ mole}) \times (8.31 \frac{J}{\text{mol} \cdot \text{K}}) T_b \]

\[ 577 \text{ K} = T_b \]

\[ \Delta E = \frac{3}{2} n R (T_b - T_a) = \frac{3}{2} (2 \text{ mole}) (8.31 \frac{J}{\text{mol} \cdot \text{K}}) (288 \text{ K}) \]

\[ = 7200 \text{ J} \]

Then

\[ \Delta E = Q - W \]

\[ \Delta E + W = Q \]

\[ 12000 \text{ J} = Q \]
Now, suppose we make another expansion at constant temperature.

\[ W = \int_{V_b}^{V_c} p \, dV = \int_{V_b}^{V_c} \frac{nRT}{V} \, dV \]

\[ = nRT \log \frac{V_c}{V_b} \]

Work done in an isothermal expansion.

\[ W = 2 \times 8.31 \, \text{J/(mol\cdot K)} \log \frac{6 \, \text{K}}{4 \, \text{K}} \]

\[ W = 3.892 \, \text{J} \]
\[ \Delta E = \frac{3}{2} nR \Delta T \quad \Delta T = 0 \]

\[ \Delta E = 0 \]

\[ \Delta E = Q - W_{by \text{ gas}} \]

\[ W_{by \text{ gas}} = 0 = 3892 \text{ J} \]

Now suppose we make one more step

\[ \omega_{c \rightarrow a} = \int_{c}^{a} P \text{d}V = - \text{(Area under curve)} \]

First find the pressure at C

\[ P_c = \frac{nRT_c}{V_c} \quad T_c = 2T_0 \quad V_c = 2V_0 \]

\[ P_c = \frac{nR \cdot 2T_0}{3V_0} = \frac{2P_0}{3} = 16 \text{ bar} \]
Area = \( P_c (V_c - V_a) + \frac{1}{2} (V_c - V_a) (P_a - P_c) \)

\[
= \frac{2}{3} P_o (2V_o) + \frac{1}{2} (2V_o) \left( \frac{1}{3} P_o \right)
\]

Area = \( \left( \frac{4}{3} + \frac{1}{3} \right) P_o V_o \)

\[
= \frac{5}{3} P_o V_o
\]

\( W = -8000 J \)

Net Work = \( 41800 + 3892 - 8000 J \)

\[
= 692 J
\]

Intake of Heat = \( 12000 + 3892 = Q_{in} \)

\[
\text{Effic} = \frac{W}{Q_{in}} = 0.43\%
\]
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ex. $
\Rightarrow \text{See next page}
$

$$
\frac{Q}{m} \sim \frac{\text{Energy}}{\text{# of dot}} \propto \Delta T
$$

$$
Q = mc \Delta T
$$

Sometimes it doesn't work that way

T

\begin{center}
\includegraphics[width=0.5\textwidth]{chart.png}
\end{center}

\begin{itemize}
\item Ice
\item Mixed
\item Water
\item Steam
\end{itemize}