Oscillations:

\[ -kx = ma \]
\[ -kx = m \frac{d^2x}{dt^2} \]
\[ \omega^2 = \frac{k}{m} = \frac{1}{s} \]

\[ \omega_0 = \text{Angular Frequency} \]
\[ A = \text{Amplitude} \]
\[ f = \frac{1}{T} \]

Vertical Spring:

Everything still applies around the equilibrium position.
A hanging spring stretches 35.0 cm when an object of 450 g is hung on it at rest. The object is pulled down an additional 18 cm.

(1) What is position 84.4 s later?

(2) When does it reach the top of its arc?

**Solution**

(1) First determine the spring constant.

\[ kx_0 - mg = 0 \]

\[ k = \frac{mg}{x_0} \]

\[ k = 12.6 \text{ N/m} \]

(2) Next determine the angular frequency:

\[ \omega_0 = \sqrt{\frac{k}{m}} = 5.29 \text{ rad/s} \]

\[ T = \frac{2\pi}{\omega_0} = 1.9 \text{ s} \]
Then

\[ x(t) = A \cos(\omega t + \phi) \]

\[ v(t) = -A \omega \sin(\omega t + \phi) \]

**Position**

Need IC

\[ x(t=0) = -18 \text{ cm} = A \cos \phi \]

\[ v(t=0) = 0 = -A \omega \sin \phi \]

\[ \phi = 0 \quad A = -0.18 \text{ m} \]

\[ x(t) = (-0.18 \text{ m}) \cos \left( 5.29 \text{ rad} \cdot t \right) \]

**Tip:** be sure to use radians

\[ x(t=84.4 \text{ s}) = -16.8 \text{ s} \quad \# \text{ periods} = \frac{84.4 \text{ s}}{1.19 \text{ s}} = 70.9 \]

\[ \text{70 times} \quad \text{almost 71 times} \]
What is the speed as the block passes through the equilibrium position?

Solution: \( PE_g = 0 \)

Use Energy Consrvi:

\[ KE_i + PE_i = KE_f + PE_f \]

\[ \frac{1}{2} k(x_0 + A)^2 - mgA = \frac{1}{2} mV_f^2 + \frac{1}{2} kx_0^2 \]

\[ \frac{1}{2} kx_0^2 + 2 \frac{1}{2} kx_0 A + \frac{1}{2} kA^2 - mgA = \frac{1}{2} mV_f^2 + \frac{1}{2} kx_0^2 \]

\[ kx_0 = mg \]

\[ \frac{1}{2} kA^2 = \frac{1}{2} mV_f^2 \]

\[ \frac{1}{2} \left( \frac{12,6}{2} \right) \]

\[ \text{just like horizontal} \]
Pendulum

\[ F = m \ddot{a} \]

\[-mg \sin \theta = m \frac{d^2 \Delta \theta}{dt^2} = m L \frac{d^2(\Delta \theta)}{dt^2} \]

\[ \sin \theta = \Delta \theta \]

\[-\frac{mg}{L} \Delta \theta = m \ddot{C} \Delta \theta \]

\[ \frac{d^2 \Delta \theta}{dt^2} = \left( \frac{1}{\ell} \right)^2 \times \left( \frac{d^2x}{dt^2} \right) \]

\[ \Delta \theta = A \cos(\omega t + \phi) \]

\[ T = \frac{2\pi}{\ell \omega} \text{ etc.} \]

\[ \omega_0 = \sqrt{\frac{g}{L}} \]
Another Example

$I\ d = \text{distance from pivot to cm}$

$I = I\ \alpha$

$-mg\sin\theta = I\ \frac{d^2\theta}{dt^2}$

$-mg\Delta\theta = I\ \frac{d^2(\Delta\theta)}{dt^2}$

$-\left(\frac{mgd}{I}\right)\Delta\theta = \frac{d^2(\Delta\theta)}{dt^2} = \frac{d^2x}{dt^2} = -w^2x$

$w = \sqrt{\frac{mgd}{I}}$

$T = \frac{2\pi}{\frac{\omega}{w}}$
Last Time

Oscillations:

\[-kx = ma\]
\[-kx = m \frac{d^2x}{dt^2}\]

\[x = A \cos(\omega_o t + \phi)\]

\[\omega_o^2 = \frac{k}{m} = \frac{1}{s}\]

\[\omega_o = \text{Angular Freq}\]

\[A = \text{Amplitude}\]

\[f = \frac{1}{T}\]

Vertical Spring:

- Everything still applies around the Equilibrium position