I. ANGULAR AND LINEAR QUANTITIES

- We have angular velocities and accelerations

$$\theta = \theta_r \cdot \text{rad} \qquad \theta_r = \frac{\Delta x_t}{R} \tag{1}$$
$$\omega = \omega_r \cdot \text{rad} \qquad \omega_r = \frac{v_t}{R} \tag{2}$$
$$\alpha = \alpha_r \cdot \text{rad} \qquad \alpha_r = \frac{a_t}{R} \tag{3}$$

For example a_t means the tagential acceleration

- For constant acceleration we have

$$\omega = \omega_o + \alpha t \tag{4}$$
$$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2 \tag{5}$$
$$\omega = \omega_o^2 + 2\alpha\Delta\theta \tag{6}$$

- The full acceleration is a "vector sum" of the tangential and centripetal acceleration

$$a_{\text{tot}} = \sqrt{a_t^2 + a_c^2} \tag{7}$$

with the centripetal acceleration

$$a_c = \frac{v^2}{r} = \omega^2 r \tag{8}$$

II. CALCULATING MOMENTS OF INERTIA

- The moment of inertia is

$$I = M \left\langle r^2 \right\rangle_M \tag{9}$$
$$= \sum_i m_i (r_\perp^2)_i \tag{10}$$

$r_\perp$ is the distance from the object to the axis of rotation.

- Moment of Inertia depends upon the axis of rotation. To find the moment of inertia around a new axis wich is shifted by an amount $r_{\perp,cm}^2$ from the center of mass use the parallel axis theorem

$$I_A = I_{cm} + M r_{\perp,cm}^2 \tag{11}$$

- The moment of inertia of a bunch of objects is the sum of the moment of inertias

$$I_{\text{tot}} = I_1 + I_2 + \ldots \tag{12}$$

III. TORQUE – THE THING MAKES YOU ROTATE

- Torque is given by

$$\vec{\tau} = \vec{r} \times \vec{F} \tag{13}$$

The magnitude is

$$|\vec{\tau}| = |rF\sin(\theta)| = |r_\perp F| \tag{14}$$

- Torque is generally a vector pointing out of or into the page as given by the right hand rule. We will indicate this direction with a sign. Out page torque is counter clockwise and is positive for positive rotations (counter-clockwise) and into the page for negative (clockwise) rotations. Thus

$$\vec{\tau} = |rF\sin(\theta)| \cdot (\text{sign}) \tag{15}$$
$$= |r_\perp F| \cdot (\text{sign}) \tag{16}$$

IV. NEWTON LAWS

- A positive torque try to create a positive angular acceleration (counter-clocwise). The sum of all torques gives

$$\sum \tau = I\alpha_r \tag{17}$$

- If the problem does not have a fixed pivot point, you must use the center of mass when writign $\sum \tau = I\alpha$

V. ENERGY IN ROTATIONAL MOTION

- For rotations about fixed axis A the kinetic energy is

$$KE = \frac{1}{2} I_A \omega_r^2 \tag{18}$$

- For an object which is rotating and moving the kinetic energy is

$$KE = \frac{1}{2} M v_{cm}^2 + \frac{1}{2} I_{cm} \omega_r^2 \tag{19}$$

- If an item is rolling without slipping then the bottom point of the wheel is stationary.

$$v_{cm} = \omega_r R \tag{20}$$

You can calculate the kinetic energy using either the rotation around the fixed axis Eq. (18) or the center of mass Eq. (19). The moment of inertia in these two formulas are different but are related by the parallel axis theorem. The answer for the KE is the same.

- One can still use formulas like

$$W_{ext} = \Delta KE + \Delta PE_g \tag{21}$$

But the kinetic energy must include the rotation and the potential energy is

$$(\Delta PE_g) = Mg\, y_{\text{cm}} \tag{22}$$

with y_{cm} the height of the center of mass.

- The work done by an external torque is

$$W_{ext} = |\tau \Delta\theta| \cdot (\text{sign}) \tag{23}$$

where the sign is positive (negative) if the torque and angle are in the same (opposite) direction (counter clockwise vs. clockwise).

VI. STATIC EQUILIBRIUM

1. Draw a free body diagram
2. Pick a pivot point
3. Write

$$\begin{aligned} \sum F_x &= 0 \qquad (24)\\ \sum F_y &= 0 \qquad (25)\\ \sum \vec{\tau} &= 0 \qquad (26) \end{aligned}$$

4. Solve for what you want to know

VII. ANGULAR MOMENTUM

- The angular momentum of a single particle is

$$\begin{aligned} \vec{\ell} &= \vec{r} \times \vec{p} = \vec{r} \times (m\vec{v}) \qquad (27)\\ &= |r_\perp m v| \cdot (\text{sign}) \qquad (28) \end{aligned}$$

- The angular momentum of a solid body is

$$\vec{\ell} = I\omega_r \tag{29}$$

- Generally we have

$$\sum \vec{\tau}_{\text{ext}} = \frac{\Delta(\vec{\ell})}{\Delta t} \tag{30}$$

- If there is no external torques we have

$$\sum \vec{\ell}_{\text{before}} = \sum \vec{\ell}_{\text{after}} \tag{31}$$

- For a single solid body (ice skater for example) this means

$$I_i\omega_i = I_f\omega_f \tag{32}$$

in this last equation we implicitly mean ω_r.