Exam B - Solutions

(1) Increasing

$$a = \frac{\Delta V}{\Delta t} = \frac{434\,m/s}{2.63\,s} = 165.0\,m/s^2 \approx 16g$$

$$a = \frac{\Delta v}{\Delta t} = \frac{0 - 434\,m/s}{3.22s} = -134\,m/s^2 \approx -13g$$

(2) i) Find the velocity at $t = 5s$:

- At $t = 3s$, $V = at + \cancel{v_0}^{0}$ so $V = -1\,m/s^2 \cdot 3s = -3\,m/s$

- At $t = 5s$

$$V = V_0 + a\Delta t \quad \text{so}$$

$$V = -3m/s + 1\,m/s^2 \cdot 2s = -1\,m/s$$

So $\boxed{v = -1\,m/s}$

- Find position at $t = 3$ $\quad x(t=3) = \cancel{x_0}^{0} + \cancel{v_0}^{0} t + \frac{1}{2}at^2$

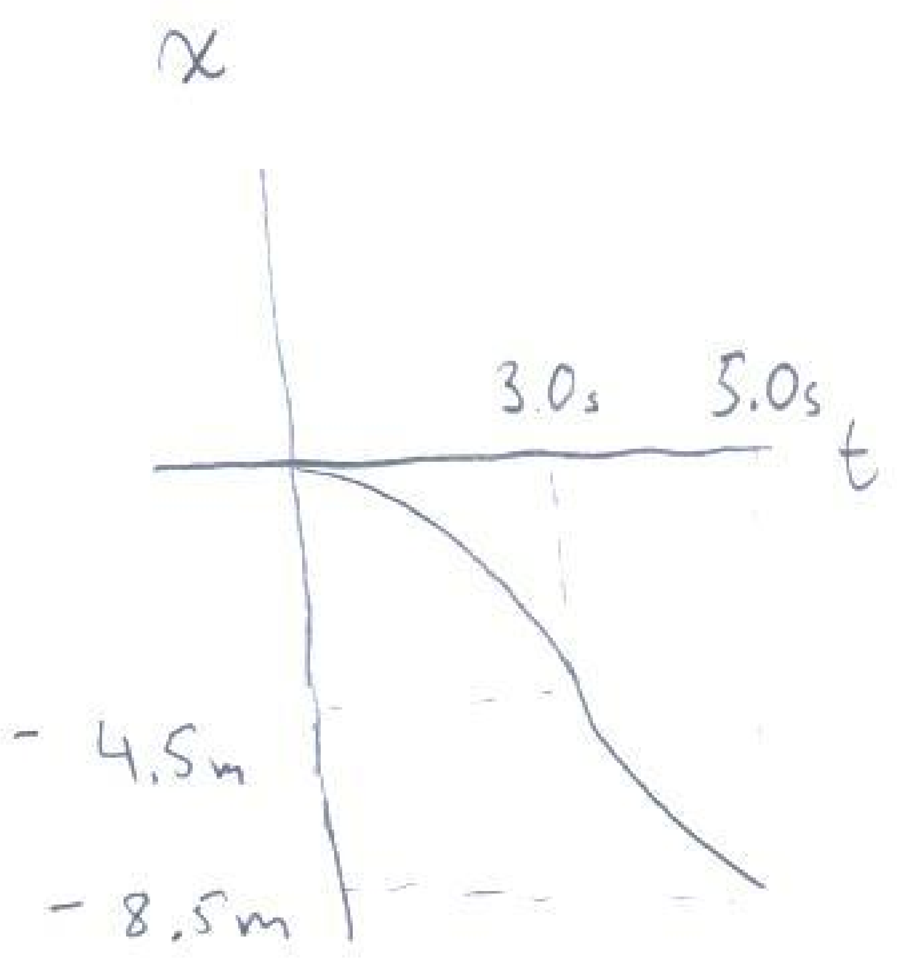
$$= -\frac{1}{2}(1\,m/s^2)(3s)^2$$

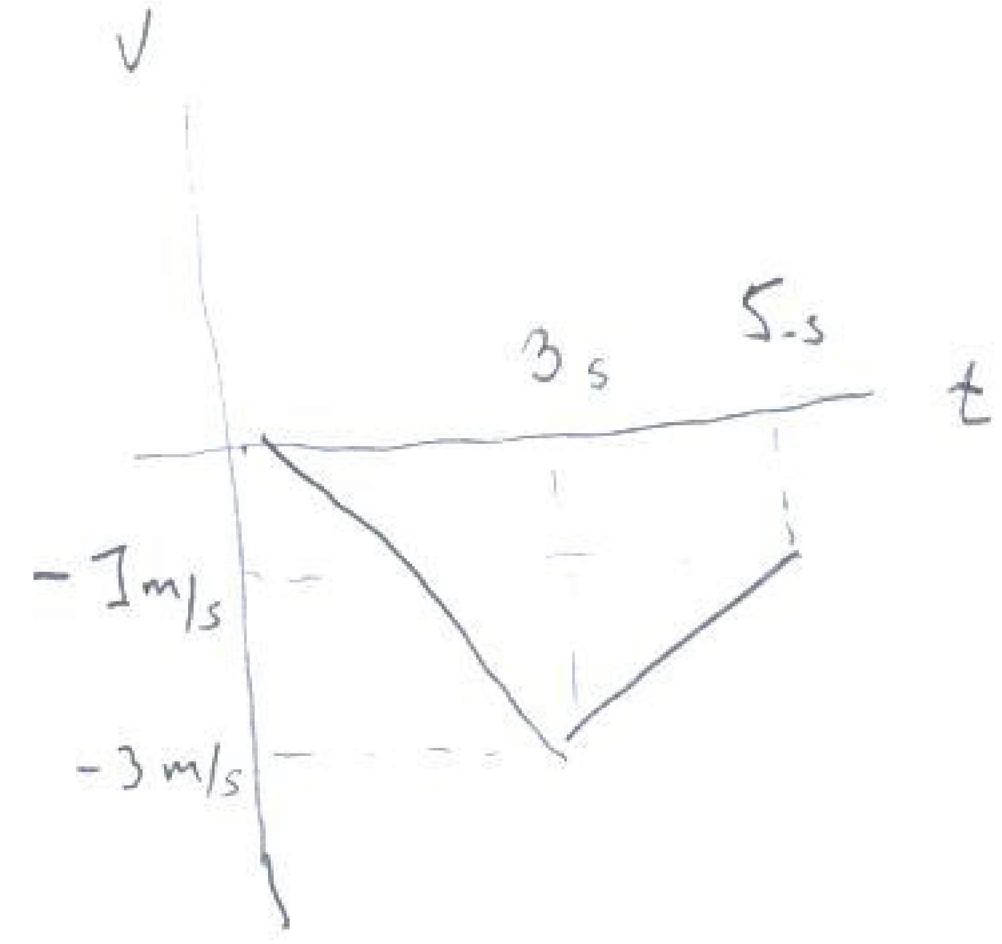
$$= -4.5m$$

Then at $t = 5$ the

$$x = x_0 + v_0\,\Delta t + \frac{1}{2}a(\Delta t)^2$$

$$x = -4.5m + (-3m/s)\,2s + \frac{1}{2}(1m/s^2)(2s)^2 \Rightarrow \boxed{x = -8.5m/s}$$





(3) First resolve $\vec{v}_0$ in to components

$$\vec{V}_0 = \begin{pmatrix} 10\,m/s \cos 49^\circ \\ 10\,m/s \sin 49^\circ \end{pmatrix} = \begin{pmatrix} 6.56\,m/s \\ 7.54\,m/s \end{pmatrix}$$

B. First find the time when it reaches the maximum ($v_y =$

$$\vec{V} = \vec{V}_0 + \vec{a}\,t \implies V_y = V_{oy} + a\,t$$

$$0 = 7.54\,m/s - 9.8\,m/s^2\; t$$

$$\boxed{t = \frac{-7.54\,m/s}{-9.8\,m/s^2} = 0.769\,s}$$

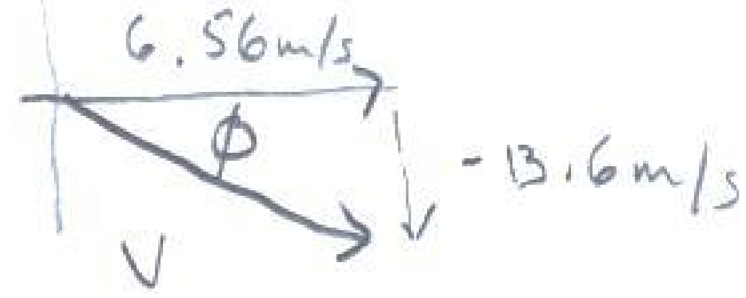
A. Then find the height from $\vec{r} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2}\vec{a}\,t^2$

$$y(t) = y_0 + v_{oy}\,t + \frac{1}{2} a_y\,t^2$$

D. $\vec{v} = \vec{v}_0 + \vec{a}\,t$

$$\begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} 6.56\,m/s \\ 7.54\,m/s \end{pmatrix} + \begin{pmatrix} 0 \\ -9.8\,m/s^2 \end{pmatrix} 2.161\,s$$

$$\begin{pmatrix} v_x \\ v_y \end{pmatrix} = \begin{pmatrix} 6.56\,m/s \\ -13.64\,m/s \end{pmatrix} \Longrightarrow$$



$$\boxed{\text{Speed} = \sqrt{v_x^2 + v_y^2} = 15.13\,m/s}$$

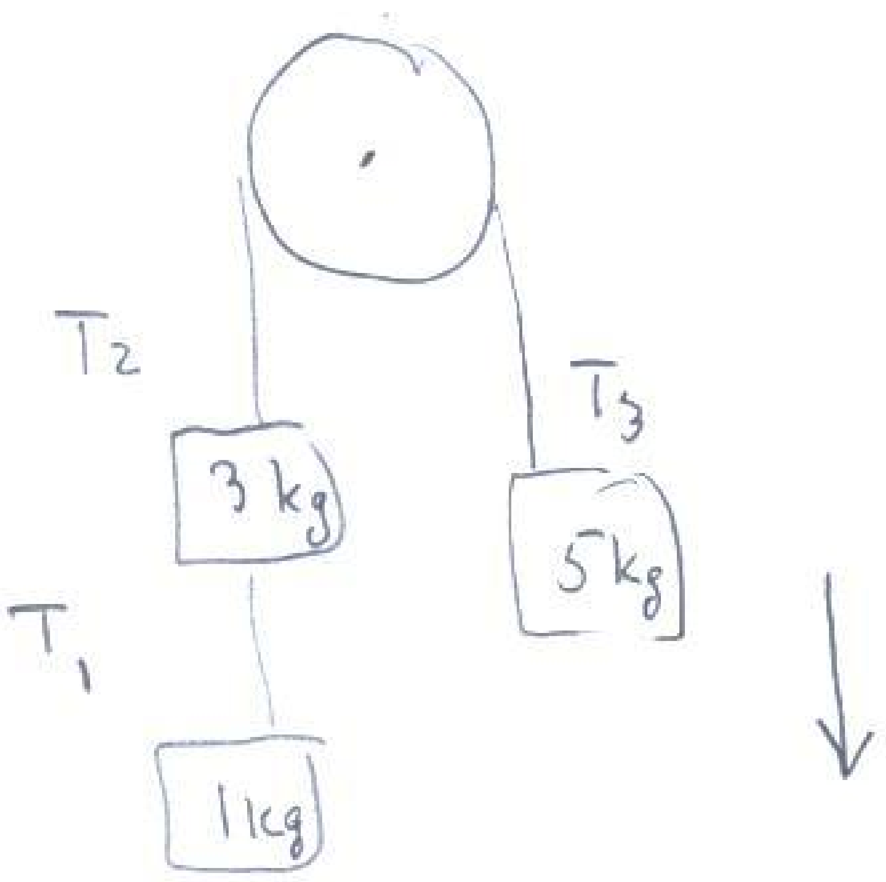
E. Angle

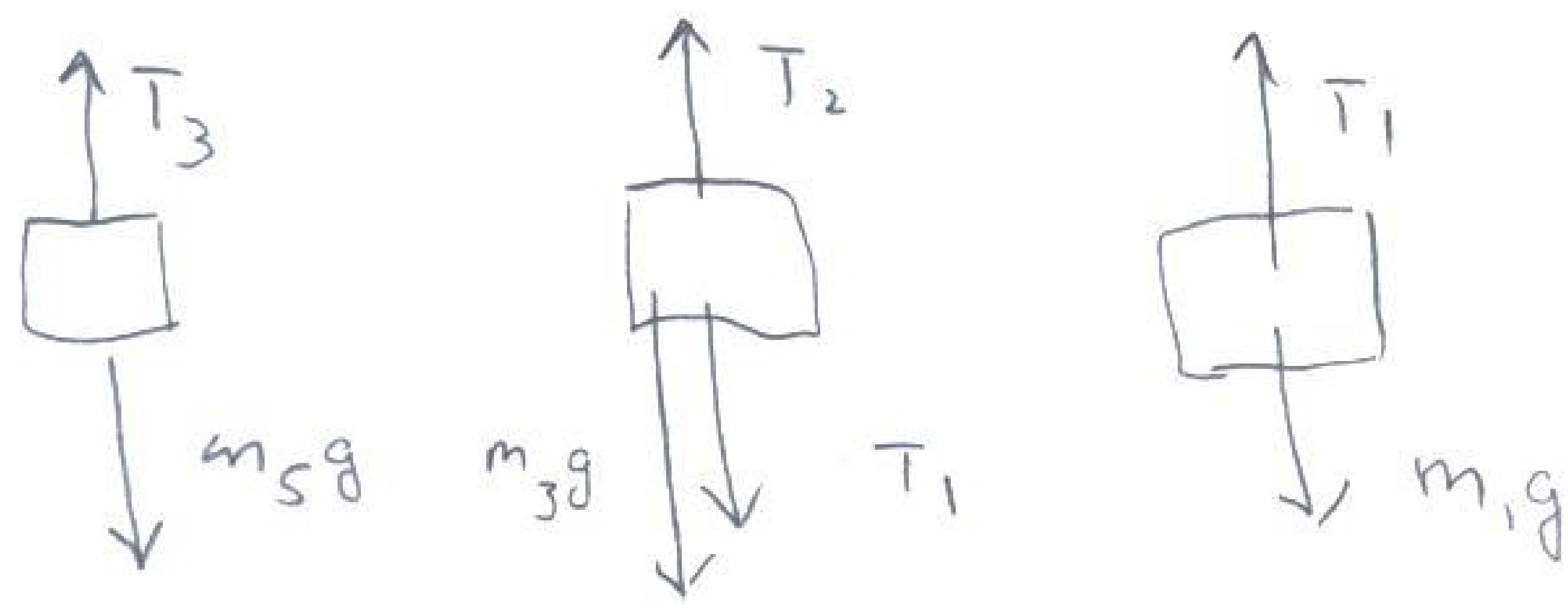
$$\tan\phi = \frac{v_y}{v_x} \Rightarrow \boxed{\phi = -64.3^\circ \text{ below horizon}}$$

F. Range:

$$x = \cancelto{0}{x_0} + v_{0x} t + \frac{1}{2}\cancelto{0}{a_x} t^2$$

$$x = (6.56\,m/s)(2.161\,s) \Rightarrow \boxed{x = 14.17\,m}$$





- Writing newton laws :

$$T_3 - m_5 g = m_5 a_5 \qquad a_1 = a_3 \equiv a$$

$$T_2 - T_1 - m_3 g = m_3 a_3 \qquad a_5 = -a$$

$$T_1 - m_1 g = m_1 a_1$$

- So the equations become :

(1) $T_3 - m_5 g = -m_5 a$ $\qquad T_3 = T_2$ newtons laws

(2) $T_3 - T_1 - m_3 g = m_3 a$

(3) $T_1 - m_1 g = m_1 a$

Subtracting Eq (1) from Eq (2)

$$(-T_1 - m_3 g) + m_5 g = m_3 a + m_5 a$$

$$T_1 = -m_3 g + m_5 g - m_3 a - m_5 a$$

$$(-m_3 g + m_5 g - m_3 a - m_5 a) - m_1 g = m_1 a$$

$$(m_5 g - m_3 g - m_1 g) = (m_1 + m_3 + m_5) a$$

$$\left(\frac{m_5 - (m_3 + m_1)}{(m_1 + m_3) + m_5} \right) g = a$$

Substituting #s :

$$\frac{5kg - (3kg + 1kg)}{(4kg + 3kg + 5kg)} g = a$$

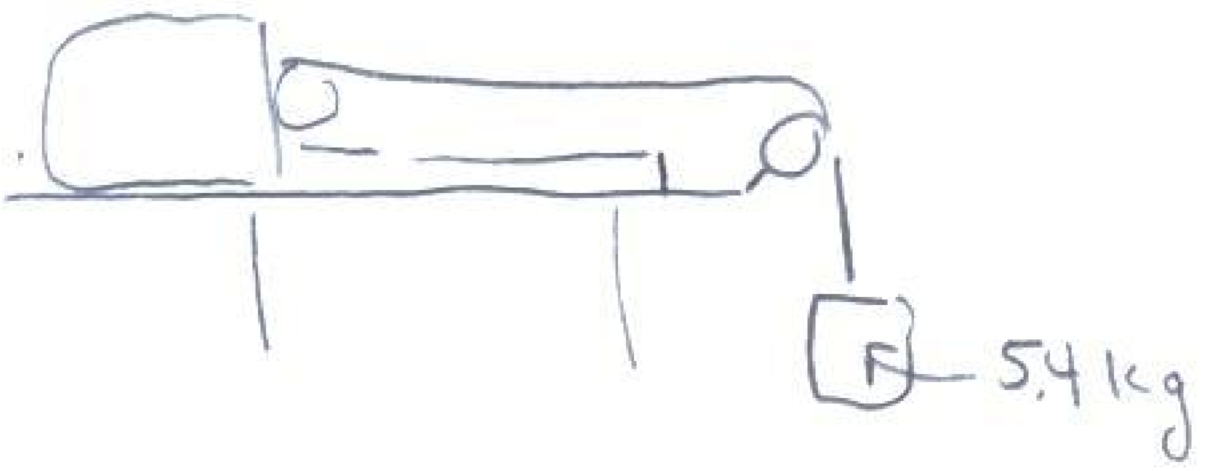
$$\frac{1}{9} g = a$$

So the tension is :

$$T_1 = m_1 g + m_1 a$$

$$T_1 = (1kg) g + (1kg)\left(\frac{1}{9} g\right) = \frac{10}{9} \cdot kg \cdot g$$

$$T_1 = 10.89 \text{ N}$$

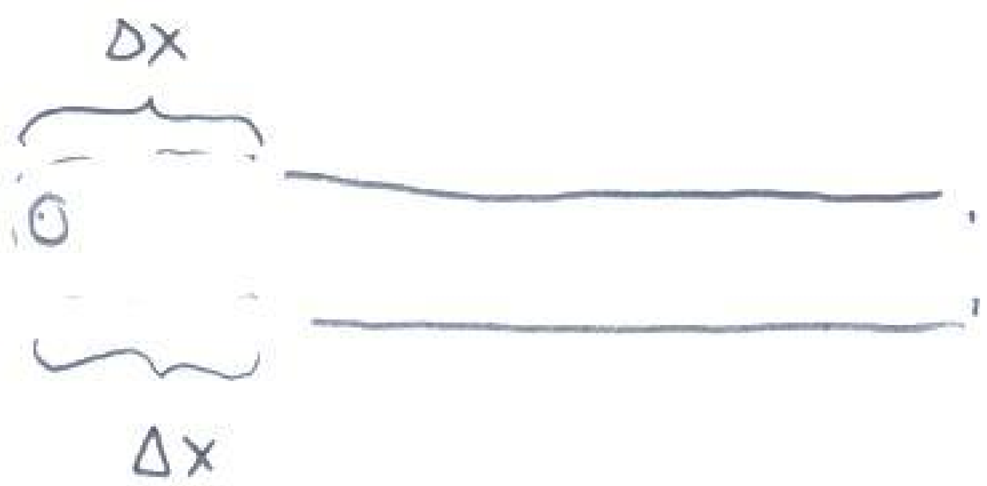


① Imagine moving the 5.4 kg block up an amount Δy. Then the "slack" in the rope is taken up by both ropes so its change in position is $\Delta x = \frac{\Delta y}{2}$

The following diagram may help:

Before:

After:



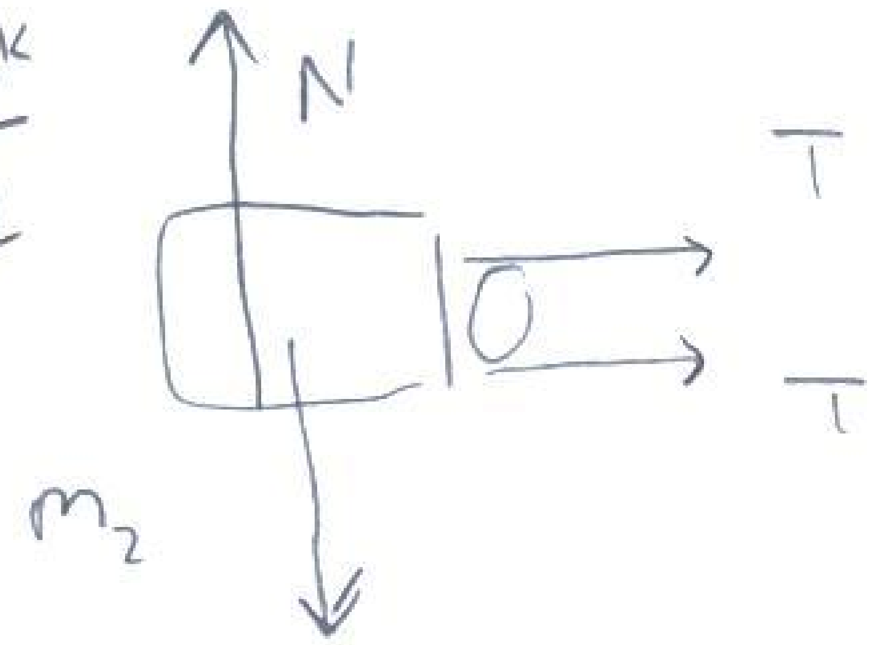
clearly $2|\Delta x| = |\Delta y|$, so if $\Delta y = 9.8$ cm

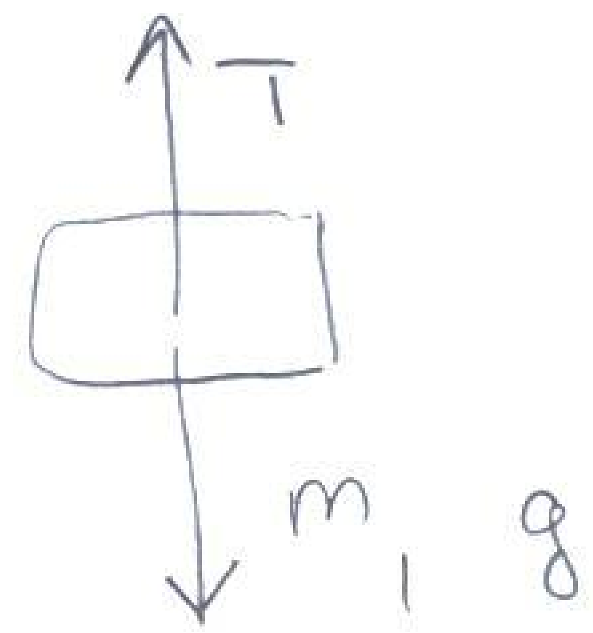
$\Delta x = 4.9$ cm

Now since, $V = \frac{\Delta(\text{distance})}{\Delta \text{ time}}$, $2|V_x| = |V_y|$

Now since $a = \frac{\Delta V}{\Delta t}$, $2|a_x| = |a_y|$

Block #2





Block # 1

Newtons laws:

$$\sum F^x = m_2 a_2^x \qquad\qquad \sum F^y = m_2 a_2^y$$

$$2T = m_2 a_2^x \qquad\qquad \underbrace{N - m_2 g = m_2 \cancelto{0}{a_2^y} \qquad N = m_2 g}_{\text{This is clear -- the table supports the weights}}$$

$$\sum F^y = m_1 a_1^y$$

$$T - m_1 g = m_1 a_1^y$$

From part 1

$$a_2^x = a$$

$$a_1^y = -2a$$

So

$$2T = m_2 a \implies T = m_2 a/2$$

$$T - m_1 g = +m_1 (-2a)$$

Then

$$\frac{m_2 a}{2} - m_1 g = -2 m_1 a$$

$$\left(2m_1 + \frac{m_2}{2}\right) a = m_1 g \implies a = \frac{m_1 g}{2m_1 + \frac{m_2}{2}}$$

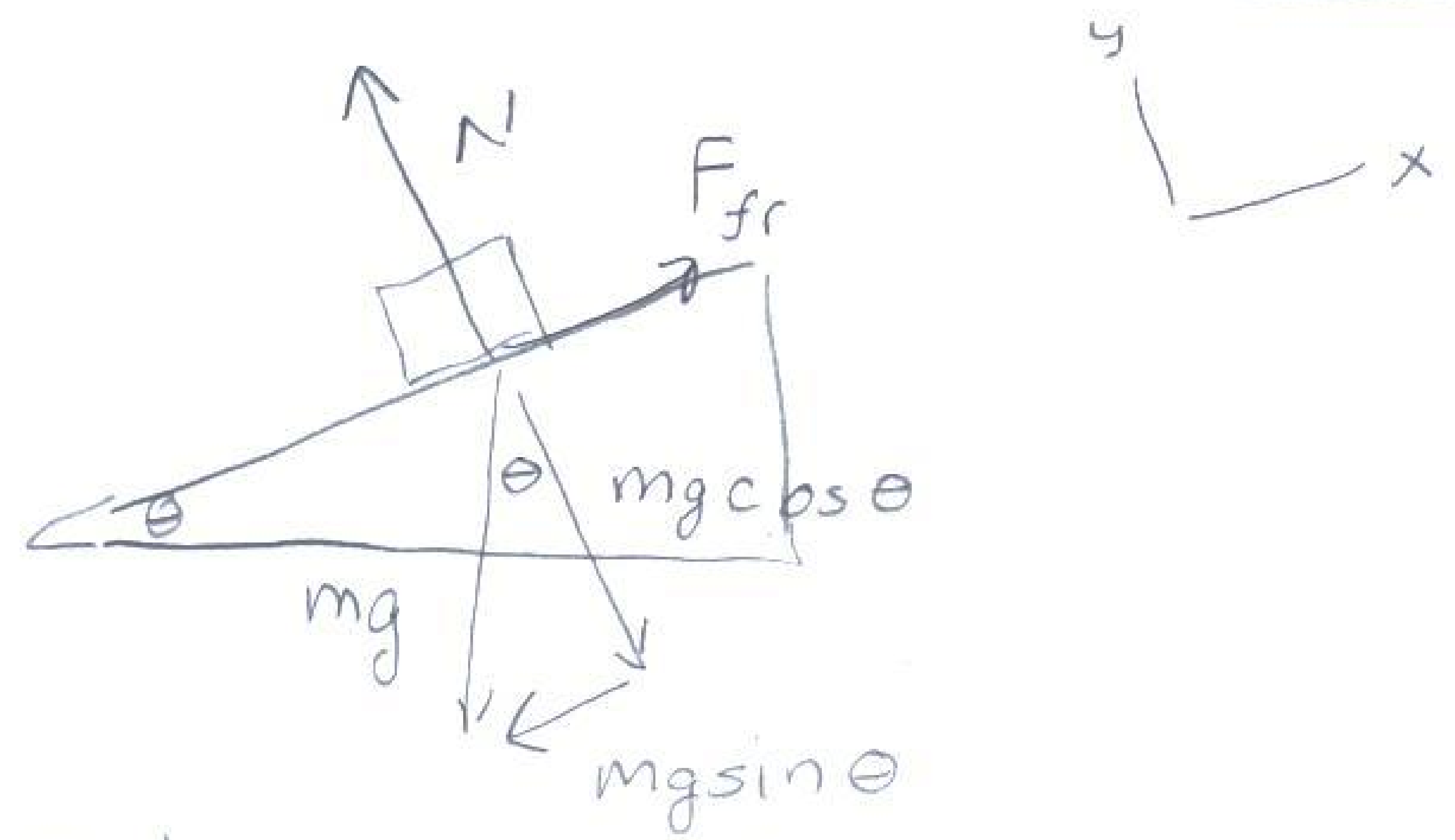
$$a = \frac{5.4\,kg}{2(5.4\,kg) + \left(\frac{20\,kg}{2}\right)} \cdot 9.8\,m/s^2$$

$$a = 2.54\,m/s^2$$

$$T = \frac{m_2}{2}a$$

$$T = \left(\frac{20\,kg}{2}\right)\left(2.54\,m/s^2\right)$$

$$T = 25.4\,N$$



To find the acceleration break up the forces into x and y directions:

$$\Sigma F^y = ma^y \qquad\qquad \Sigma F^x = ma^x$$

$$N - mg\cos\theta = \cancel{ma^y}^{\,0} \qquad\qquad -mg\sin\theta + \underbrace{\mu_k N}_{\text{friction}} = ma^x$$

So

$$N = mg\cos\theta$$

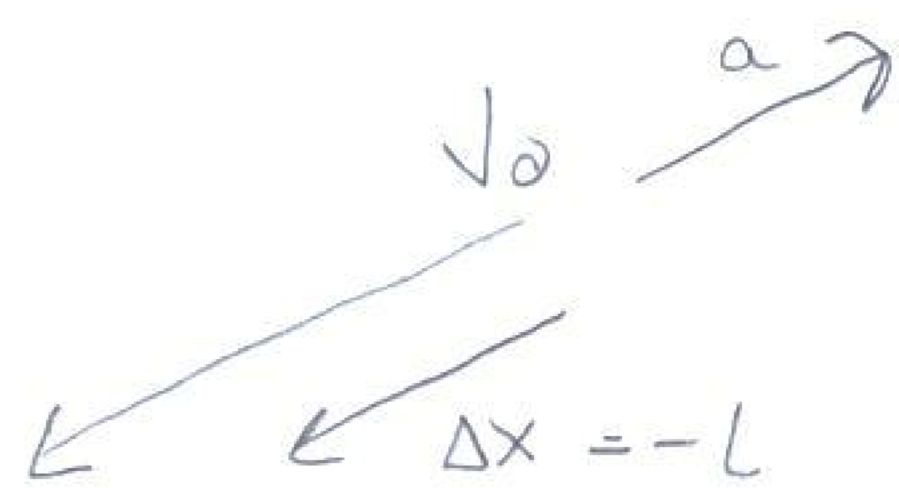
$$-\cancel{m}g\sin\theta + \mu_k \cancel{m}g\cos\theta = \cancel{m}a$$

$$\boxed{g(\mu_k\cos\theta - \sin\theta) = a}$$

Then to find how far it travels

$$V_f^2 = V_0^2 + 2a\,\Delta x$$

Sketch:



$$\cancelto{0}{V_f^2} = V_0^2 + 2a(-L)$$

$$\frac{V_0^2}{2a} = L$$

$$\boxed{\frac{V_0^2}{2g(\mu_k\cos\theta - \sin\theta)} = L}$$

we assume $\mu_k\cos\theta - \sin\theta > 0$

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- At $t = 5\,s$

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$$V = -3\,m/s + 1\,m/s^2 \cdot 2\,s = -1\,m/s$$

So $\boxed{v = -1\,m/s}$

- Find position at $t = 3$ $\quad x(t=3) = \cancel{x_0}^{\,0} + \cancel{v_0}^{\,0} t + \frac{1}{2}at^2$

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Then at $t = 5$ the

$$x = x_0 + v_0\,\Delta t + \frac{1}{2}a(\Delta t)^2$$

$$x = -4.5\,m + (-3\,m/s)\,2\,s + \frac{1}{2}(1\,m/s^2)(2\,s)^2 \Rightarrow \boxed{x = -8.5\,m/s}$$