Exam B - Solutions

(1) Increasing

\[ a = \frac{\Delta v}{\Delta t} = \frac{4.34 \text{ m/s}}{2.63 \text{ s}} = 1.65 \text{ m/s}^2 \approx 16 \text{ g} \]

\[ a = \frac{\Delta v}{\Delta t} = \frac{0 - 4.34 \text{ m/s}}{3.22 \text{ s}} = -1.34 \text{ m/s}^2 \approx -13 \text{ g} \]

(2) i) Find the velocity at \( t=5 \text{ s} \):

- At \( t=3 \text{ s} \), \( v = at + v_0 \) so \( v = (-1\text{ m/s}^2)(3\text{ s}) = -3 \text{ m/s} \)

- At \( t=5 \text{ s} \)

\[ v = v_0 + a \Delta t \text{ so} \]

\[ v = -3 \text{ m/s} + 1 \text{ m/s}^2 \times 2\text{ s} = -1 \text{ m/s} \]

So \[ v = -1 \text{ m/s} \]

- Find position at \( t=3 \) \( x(t=3) = x_0 + v_0 t + \frac{1}{2} at^2 \)

\[ = -\frac{1}{2}(1 \text{ m/s}^2)(3 \text{ s})^2 \]

\[ = -4.5 \text{ m} \]

Then at \( t=5 \text{ s} \) the

\[ x = x_0 + v_0 \Delta t + \frac{1}{2} a(\Delta t)^2 \]

\[ x = -4.5 \text{ m} + (-3 \text{ m/s}) \times 2\text{ s} + \frac{1}{2}(1 \text{ m/s}^2)(2\text{ s})^2 \Rightarrow x = -8.5 \text{ m/s} \]
3. First resolve $\vec{v}_o$ into components

$$\vec{v}_o = \begin{pmatrix} 10 \text{ m/s} \cos 49^\circ \\ 10 \text{ m/s} \sin 49^\circ \end{pmatrix} = \begin{pmatrix} 6.56 \text{ m/s} \\ 7.54 \text{ m/s} \end{pmatrix}$$

8. First find the time when it reaches the maximum ($v_y = 0$)

$$v = v_o + at \Rightarrow v_y = v_{oy} + at$$

$$0 = 7.54 \text{ m/s} - 9.8 \text{ m/s}^2 \cdot t$$

$$t = \frac{7.54 \text{ m/s}}{9.8 \text{ m/s}^2} = 0.769 \text{ s}$$

A. Then find the height from $\vec{s} = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2$

$$y(t) = y_0 + v_{oy} t + \frac{1}{2} a_y t^2$$
D. \( \vec{v} = \vec{v}_0 + \vec{a}t \)

\[
\begin{pmatrix}
  v_x \\
  v_y
\end{pmatrix} = \begin{pmatrix}
  6.56 \text{ m/s} \\
  7.54 \text{ m/s}
\end{pmatrix} + \begin{pmatrix}
  0 \\
  -9.8 \text{ m/s}^2
\end{pmatrix} \times 2.161 \text{s}
\]

\[
\begin{pmatrix}
  v_x \\
  v_y
\end{pmatrix} = \begin{pmatrix}
  6.56 \text{ m/s} \\
  -13.614 \text{ m/s}
\end{pmatrix}
\]

Speed = \( \sqrt{v_x^2 + v_y^2} = 15.13 \text{ m/s} \)

E. Angle

\[ \tan \phi = \frac{v_y}{v_x} \Rightarrow \phi = -64.3^\circ \text{ below horizon} \]

F. Range:

\[ x = v_x t + v_{0x} t + \frac{1}{2} a_x t^2 \]

\[ x = (6.56 \text{ m/s}) (2.161 \text{s}) \Rightarrow x = 14.17 \text{ m} \]
Writing Newton's laws:

\[ T_3 - m_5 g = m_5 a_5 \]
\[ T_2 - T_1 - m_3 g = m_3 a_3 \]
\[ T_1 - m_1 g = m_1 a_1 \]

So the equations become:

1. \[ T_3 - m_5 g = -m_5 a \]
2. \[ T_3 - T_1 - m_3 g = m_3 a \]
3. \[ T_1 - m_1 g = m_1 a \]

\[ a_1 = a_3 = a \]
\[ a_5 = -a \]

\[ T_3 = T_2 \] Newton's laws
Subtracting Eq (1) from Eq (2)

\[ (-T_1 - m_3 g) + m_5 g = m_3 a + m_5 a \]

\[ T_1 = -m_3 g + m_5 g - m_3 a - m_5 a \]

\[ (-m_3 g + m_5 g - m_3 a - m_5 a) - m_1 g = m_1 a \]

\[ (m_5 g - m_3 g - m_1 g) = (m_1 + m_3 + m_5) a \]

\[ \left( \frac{m_5 - (m_3 + m_1)}{m_1 + m_3 + m_5} \right) g = a \]

Substituting #5:

\[ \frac{5 \text{kg} - (3 \text{kg} + 1 \text{kg})}{(4 \text{kg} + 3 \text{kg} + 5 \text{kg})} \ g = a \]

\[ \frac{1}{q} g = a \]

So the tension is:

\[ T_1 = m_1 g + m_1 a \]

\[ T_1 = (1 \text{kg}) g + (1 \text{kg}) \left( \frac{1}{9} g \right) = \frac{10}{9} \text{kg} \cdot g \]

\[ T_1 = 10.89 \text{ N} \]
1) Imagine moving the 5.4 kg block up an amount \( \Delta y \). Then the "slack" in the rope is taken up by both ropes so its change in position is 
\[
\Delta x = \frac{\Delta y}{2}
\]

The following diagram may help:

**Before:**

**After:**

Clearly, 
\[
2|\Delta x| = |\Delta y|
\]
so if \( \Delta y = 9.8 \text{ cm} \),
\[
\Delta x = 4.9 \text{ cm}
\]

Now since,
\[
V = \frac{\Delta (\text{distance})}{\Delta \text{time}} \quad \text{and} \quad 2|V_x| = |V_y|
\]

Now since
\[
\alpha = \frac{\Delta V}{\Delta t} \quad \text{and} \quad 2|\alpha_x| = |\alpha_y|
\]
Newton's Laws:

\[ \sum F_x = m_2 a_x \]
\[ 2T = m_2 a_x \]

\[ \sum F_y = m_1 a_y \]
\[ T - m_1 g = m_1 a_y \]

So
\[ 2T = m_2 a \quad \Rightarrow \quad T = m_2 a_{x/2} \]

\[ T - m_1 g = +m_1 (-2a) \]

Then
\[ m_2 a - m_1 g = -2m_1 a \]

\[ \frac{m_2 a - m_1 g}{\frac{1}{2}} = m_1 g \quad \Rightarrow \quad a = \frac{m_1 g}{2m_1 + \frac{m_2}{2}} \]

This is clear -- the table supports the weights.
\[ a = \frac{5.1 \text{ kg}}{2(5.4 \text{ kg}) + (20 \text{ kg})} \cdot 9.8 \text{ m/s}^2 \]

\[ a = 2.54 \text{ m/s}^2 \]

\[ T = m_2 a \]

\[ T = \left( \frac{20 \text{ kg}}{2} \right) \left( 2.54 \text{ m/s}^2 \right) \]

\[ T = 25.4 \text{ N} \]

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To find the acceleration break up the forces into \( x \) and \( y \) directions:

\[ \Sigma F_y = m a_y \]

\[ N - mg \cos \theta = m \frac{a_y}{2} \]

\[ -mg \sin \theta + \mu_k N = ma_x \]

\[ -mg \sin \theta + \mu_k N = ma_x \text{ friction} \]
So

\[ N = mg \cos \theta \]

\[-mg \sin \theta + \mu_k mg \cos \theta = xa\]

\[ g (\mu_k \cos \theta - \sin \theta) = a \]

Then to find how far it travels

\[ V_f^2 = V_0^2 + 2a \Delta x \]

Sketch:

[Diagram with vectors and labels]

\[ V_f^0 = V_0^2 + 2a (-L) \]

\[ \frac{V_0^2}{2a} = L \]

we assume \( \mu \cos \theta - \sin \theta > 0 \)
Examination Solutions

1. Increasing

\[ a = \frac{\Delta v}{\Delta t} = \frac{434 \text{ m/s}}{2.63 \text{ s}} = 165.0 \text{ m/s}^2 \approx 16 \text{ g} \]

\[ a = \frac{\Delta v}{\Delta t} = \frac{0 - 434 \text{ m/s}}{3.22 \text{ s}} = -134 \text{ m/s}^2 \approx -13 \text{ g} \]

2. i) Find the velocity at \( t = 5 \text{ s} \):

- At \( t = 3 \text{ s} \), \( v = a t + v_0 \) so \( v = -1 \cdot 3 \text{ s} = -3 \text{ m/s} \)
- At \( t = 5 \text{ s} \)

\[ v = v_0 + a \Delta t \]

So \( v = -7 \text{ m/s} \)

- Find position at \( t = 3 \) \( x(t = 3) = x_0 + v_0 t + \frac{1}{2} a t^2 \)

\[ = \frac{1}{2} (1 \text{ m/s}^2) (3 \text{ s})^2 \]

\[ = -4.5 \text{ m} \]

Then at \( t = 5 \) the

\[ x = x_0 + v_0 \Delta t + \frac{1}{2} a (\Delta t)^2 \]

\[ x = -4.5 \text{ m} + (-3 \text{ m/s}) (2 \text{ s}) + \frac{1}{2} (1 \text{ m/s}^2) (2 \text{ s})^2 \Rightarrow x = -8.5 \text{ m/s} \]