

## I. SIMPLE HARMONIC MOTION

- Always use radians for these formulas
- For a mass of mass  $m$  is connected to a spring  $k$  and oscillates back and forth. Its position, velocity, and acceleration are

$$x(t) = A \cos(\omega_o t + \phi) \quad (1)$$

$$v(t) = \frac{dx}{dt} = -A\omega_o \sin(\omega_o t + \phi) \quad (2)$$

$$a(t) = \frac{d^2x}{dt^2} = -A\omega_o^2 \cos(\omega_o t + \phi) \quad (3)$$

- The angular frequency is

$$\omega_o = \sqrt{\frac{k}{m}} \quad (4)$$

This can be related to the period (time per cycle) and the frequency (cycles per second)

$$T = \frac{2\pi}{\omega_o} \quad (5)$$

$$f = 1/T \quad (6)$$

- $A$  is the amplitude and  $\phi$  is the phase and are set by initial conditions. See lectures on this.
- $x(t)$  is the general solution to the differential equation

$$\frac{d^2x}{dt^2} = -\omega_o^2 x(t) \quad (7)$$

- When a spring oscillates back and forth you can use energy conservation. The kinetic and potential energy vary with time

$$PE = \frac{1}{2} kx^2(t) \quad (8)$$

$$KE = \frac{1}{2} mv^2(t) \quad (9)$$

The maximums are

$$PE_{\max} \text{ at max string stretch} = \frac{1}{2} kA^2 \quad (10)$$

$$KE_{\max} \text{ at zero string stretch} = \frac{1}{2} mv_{\max}^2 = \frac{1}{2} kA^2 \quad (11)$$

- These formulas also work for a vertical spring provided you consider the oscillation around the equilibrium position of spring with hanging mass. This equilibrium position is found by balancing the upward force of the spring with the downward gravity force

$$kx_o = mg \rightarrow x_o = \frac{mg}{k} \quad (12)$$

- For a solid body oscillating around a pivot point (for example you are holding a ruler stick at one end and watching it say back and forth) the angular frequency is

$$\omega_o = \sqrt{\frac{mgd}{I}} \quad (13)$$

where  $m$  is the mass,  $d$  is the distance from the pivot to the center of mass, and  $I$  is the moment of inertia around the center of mass. The angle (in radians!) satisfies

$$\theta(t) = A \cos(\omega_o t + \phi) \quad (14)$$

- For the special case of a simple pendulum (like the old clocks) the angular frequency is

$$\omega_o = \sqrt{\frac{g}{l}} \quad (15)$$

## II. UNITS IN THERMO

- When in doubt convert to Kelvin
- To convert from Celsius to Fahrenheit use

$$T_F = \frac{9}{5} T_C + {}^{\circ}32 \quad (16)$$

- To convert from Celsius to Kelvin

$$T_K = T_C + {}^{\circ}273.15 \quad (17)$$

- One mole of a substance is one Avagadro # ( $N_A = 6.022 \times 10^{23}$ ) of molecules.
- One Avagadro's number of protons weighs approximately one gram. More accurately

$$1N_A \times \underbrace{1u}_{\text{Atomic mass unit}} = 1g = 1.66 \times 10^{-27} \text{kg} \quad (18)$$

The mass of a proton is  $1.67 \times 10^{-27} \text{kg}$ .

- Often when converting between moles and number of molecules the relation
- Pressure is force per unit area. We measure pressure in terms of bar and pascals

$$1Pa = \frac{N}{m^2} \quad (19)$$

$$1 \underbrace{\text{atm}}_{\text{atmosphere}} \approx 1\text{bar} = 10^5 Pa \quad (20)$$

- We often measure volume in liters

$$1L = 10^{-3} m^3 \quad (21)$$

- Boltzman constant  $k_B$  is related to the Ideal gas constant

$$N_A k_B = R = 8.314 \frac{J}{mol \text{ } ^\circ K} \quad (22)$$

and therefore

$$N k_B = nR \quad (23)$$

where  $N$  is the total number of molecules in the gas

### III. THERMO

- For an ideal gas

$$PV = nRT \quad (24)$$

where  $P$  is pressure,  $V$  volume,  $n$  the number of moles,  $T$  is the temperature in Kelvin,  $R$  is the ideal gas constant

$$R = 8.314 \frac{J}{mol \text{ } ^\circ K} \quad (25)$$

When using this formula, use standard units but be very careful when converting pressure in bar to  $N/m^2$  and liters to  $m^3$  (see above) .

- For a mono-atomic (marble like) ideal gas the energy in a gas

$$E = \frac{3}{2} nRT \quad (26)$$

- For a di-atomic (two marbles connected by a rod) ideal gas the energy in a gas

$$E = \frac{5}{2} nRT \quad (27)$$

- For a substance not undergoing a phase change adding heat increases the temperature

$$Q = mc\Delta T \quad (28)$$

$Q$  is the heat added to the substance,  $\Delta T$  is the change in temperature,  $m$  is the mass of the gas, and  $c$  is the specific heat. If  $Q$  is negative, it means heat is removed.

- For a substance melting or changing phase, it takes a certain amount of heat either added/taken away to for example melt/freeze a certain mass  $m$  of substance. That amount of heat is

$$Q = \pm mL \quad (29)$$

with  $L$  the latent heat of the substance.

- The work done by an expanding gas is

$$W = \int_{V_i}^{V_f} PdV \quad (30)$$

The work done on the gas is minus this amount.

- For constant pressure (isobaric)

$$W = P\Delta V \quad (31)$$

- For constant volume  $dV = 0$  and

$$W = 0 \quad (32)$$

- For an ideal gas the work done by the gas in an isothermal (constant temperature) expansion

$$W = nRT \ln \left( \frac{V_f}{V_i} \right) \quad (33)$$

where  $V_f$  is the final volume and  $V_i$  is the initial volume.

- For a given step of a process (say an expansion of a gas with input of heat)

$$\Delta E = Q - W \quad (34)$$

Here  $\Delta E$  is the change in the internal energy,  $Q$  is the heat that flows into the gas, and  $W$  is the work done by the gas. The signs are very important. For example, If heat flows out of the gas  $Q$  is negative. If you compress the gas, you do positive work on the gas, and the work that the gas does  $W = -W_{\text{by you}}$  negative.

- For mono-atomic or diatomic ideal gasses the change in energy is zero if the change in temperature is zero.

- In a typical engine there is a cyclic process (this makes the engine go chug-a-chug-a) and there is a certain amount of heat which flows into the gas  $Q_{\text{in}}$  and a certain amount of exhaust heat  $Q_{\text{out}}$ . The efficiency of an engine is the work you got for the amount of heat you put in

$$\epsilon = \frac{W}{Q_{\text{in}}} \quad (35)$$