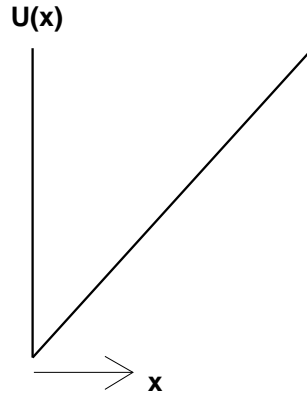


Quantity	Symbol	Value
Coulombs Constant	k_C	$8.98 \times 10^9 \text{ Nm}^2/\text{C}^2$
Electron Mass	m_e	$9.1 \times 10^{-31} \text{ kg}$
Electron Charge	e	$-1.6 \times 10^{-19} \text{ C}$
Electron Volt	eV	$1.6 \times 10^{-19} \text{ J}$
Permittivity	ϵ_o	$8.85 \times 10^{-12} \frac{\text{C}^2}{\text{Nm}^2}$
Magnetic Permeability	μ_o	$4\pi \times 10^{-7} \text{ N} \cdot \text{A}^2$
Speed of Light	c	$3.0 \times 10^8 \text{ m/s}$
Planck's Constant	h	$6.6 \times 10^{-34} \text{ m}^2\text{kg/s}$
Planck's Constant	hc	1240 eV nm
Planck's Constant	$\hbar c$	197 eV nm
Electron Mass	$m_e c^2$	511 keV
Coupling Constant	$\alpha = \frac{k_C e^2}{\hbar c}$	1/137
Bohr Radius	$a_o = \frac{\hbar^2}{m_e k_C e^2}$	$0.5 \text{ \AA} = 0.05 \text{ nm}$
Compton Length	$\frac{\hbar}{m_e c} = a_o \alpha$	0.00036 nm
Compton Length	$\frac{\hbar}{m_e c} = a_o \alpha$	0.0023 nm
1/2 of a Rydberg	$\frac{k_C e^2}{2a_o} = \frac{\hbar^2}{2m_e a_o^2}$	13.6 eV

Integral	Value
$\int_{-\infty}^{\infty} du e^{-\alpha u^2}$	$\sqrt{\frac{\pi}{\alpha}}$
$\int_0^{\infty} du u^n e^{-\alpha u}$	$\frac{n!}{\alpha^{n+1}}$
$\int du \sin^2(\alpha u)$	$\frac{u}{2} - \frac{\sin(2\alpha u)}{4\alpha}$
$\int du \cos^2(\alpha u)$	$\frac{u}{2} + \frac{\sin(2\alpha u)}{4\alpha}$

Consider an electron moving in the electric field of a parallel plate capacitor. The potential energy of the electron is $e\mathcal{E}x$ where x is the distance away from the positive plate, e is the electron charge and \mathcal{E} is the electric field. When the electron reaches the plate the potential energy is very large and may be treated as a wall. A plot of the potential is shown below.



- Estimate the size of the electron wave function in its ground state. (Give your answer in terms of e , \mathcal{E} and the mass of the electron and fundamental constants.)
- Estimate the kinetic energy of the electron in this electric field (Given your answer in term of e , \mathcal{E} , m_e and fundamental constants.)
- Estimate how strong the electric field would need to be before the typical energy of the electron would be comparable to the energy of the ground state of hydrogen. Compare this to the strongest laboratory electric field that is possible $\mathcal{E} \sim 10^7$ V/m .

Consider making a simple model of an atom.

- Estimate the size of an atom
- Suppose we place an electron in a (1 Dimensional) box with which has a Length L exactly equal to the size you estimated in part (a). Determine the energy of the photon emitted when the electron in the box decays from its first excited state down to its ground state. Work symbolically and then substitute numbers.
- Make a sketch of the ground state and first and second excited state wave functions of this electron in the box. Also make a sketch of the associated probability density $P(x)$ for the ground, first, and second excited states. (There is a total of six graphs in this problem).
- Show that the second excited state obeys the Schrödinger equation and determine the corresponding energy. (Work analytically numbers not necessary)
- Consider the electron in its first excited state. Let's agree to place the left hand side of the box at the origin. (We have always done in class and this is assumed on the formula sheet!) Determine the probability that an electron in the first excited state is between $L/4$ and $3L/4$.
- Estimate v/c for the electron in the box. (Work analytically and then substitute numbers)

- Consider the $4d$ state of hydrogen with magnetic quantum number, $m = 2$. Write down the energy, total angular momentum squared, and the z component of angular momentum of this state.
- What is the full wave function of the $1s$ state of hydrogen. Make a qualitative sketch of the radial wave functions for the $1s$ state and the associated radial probability density $P(r)$
- For the $1s$ state of hydrogen determine the average radius of the one orbiting electron.
- Again for the $1s$ state, determine the average radius squared, and the variance Δr in radius of the orbiting electron.
- Make a qualitative sketch of the $2s$ wave function and the associated probability.