Problems:

$$1.4, 1.6, 1.7, 1.10, 1.14, 1.16, 1.18, 1.23, 1.37 \quad (1)$$

When the velocity is small (with respect to c) determine

- $(L - L_o)/L_o$ (i.e. the percent change) to second order in (v/c) from Eq. (3)
- $(f - f_o)/f_o$ (i.e. the percent change) to first order in v/c Eq. (8)

Basic Relativity

1. The speed of light is constant in all reference frames
2. For an observer moving with velocity v relative to a "lab" we use two symbols alot

$$\gamma \equiv \frac{1}{\sqrt{1-(v/c)^2}} \qquad \beta \equiv \frac{v}{c}$$

3. Moving clocks run slow. A time interval $\Delta\tau$ in the rest frame of the clock is measured to be

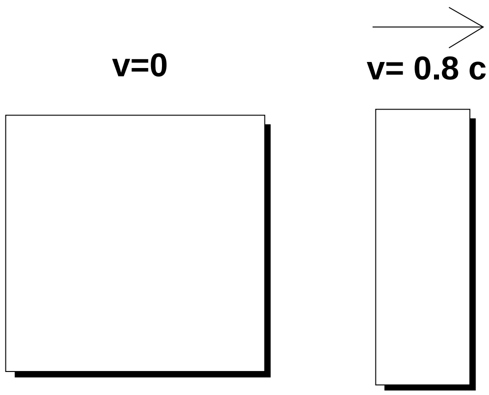
$$\Delta t = \gamma\, \Delta\tau \quad (2)$$

according to an observer moving relative to the clock.

4. An observer moving relative to a ruler stick with rest length L_p will see it length contracted by a factor γ.

$$L = L_p/\gamma \quad (3)$$

The transverse directions are not affected by the motion, i.e. a square becomes a rectangle according to an observer moving quickly with respect to the square.



Lorentz Transformations

1. An event at space time point (t, x, y, z) will appear at a different space time point (t', x', y', z') according to an observer moving with velocity v in the positive x direction.

$$\begin{aligned} ct' &= \gamma\,(ct) - \gamma\beta\, x \\ x' &= -\gamma\beta\,(c\,t) + \gamma\, x \\ y' &= y \\ z' &= z \end{aligned}$$

For an observer moving in the negative x direction the same formula holds with the replacement $\beta \to -\beta$

2. This should be compared to *Classical Galilean* transformations where the space time point (t, x, y, z) appears at a different space point but the same time point (t', x', y', z')

$$\begin{aligned} t' &= t \\ x' &= x - vt \\ y' &= y \\ z' &= z \end{aligned}$$

3. For a given observer in a fixed frame the normal rules of classical physics apply.

4. The Lorentz transformation leaves the following unchanged

$$(\Delta x')^2 - (c\Delta t')^2 = (\Delta x)^2 - (c\Delta t)^2 \tag{4}$$

Other results

1. A space ship moving with velocity $\mathbf{u} = (u_x, u_y, u_z)$ in the "lab" frame will be moving with a different velocity $\mathbf{u}'$ according to an observer moving with speed v in the positive x direction. $\mathbf{u}'$ is related to $\mathbf{u}$

$$u'_x = \frac{u_x - v}{1 - u_x v/c^2} \tag{5}$$

$$u'_y = \frac{u_y}{\gamma(1 - u_x v/c^2)} \tag{6}$$

$$u'_z = \frac{u_z}{\gamma(1 - u_y v/c^2)} \tag{7}$$

2. A source emits light waves with frequency f_o. According to an observer moving directly toward the source with speed v the source has a frequency which is blue shifted

$$f = f_o \sqrt{\frac{1 + v/c}{1 - v/c}} \tag{8}$$

For an observer moving away from the source make the replacement $v \to -v$.

Mathematical Relations

1. The Taylor series for any function around a point x_o is

$$f(x) = f(x_o) + f'(x_o)\,\Delta x + \frac{1}{2!} f''(x_o)\,\Delta x^2 + \ldots \tag{9}$$

with $\Delta x = (x - x_o)$.

2. The following taylor series comes up frequently in relativity

$$(1 + x)^\alpha \approx 1 + \alpha x + \frac{\alpha(\alpha - 1)}{2!} x^2 + \ldots \tag{10}$$

$$\frac{1}{1 - x} = 1 + x + x^2 + x^3 + \ldots \tag{11}$$